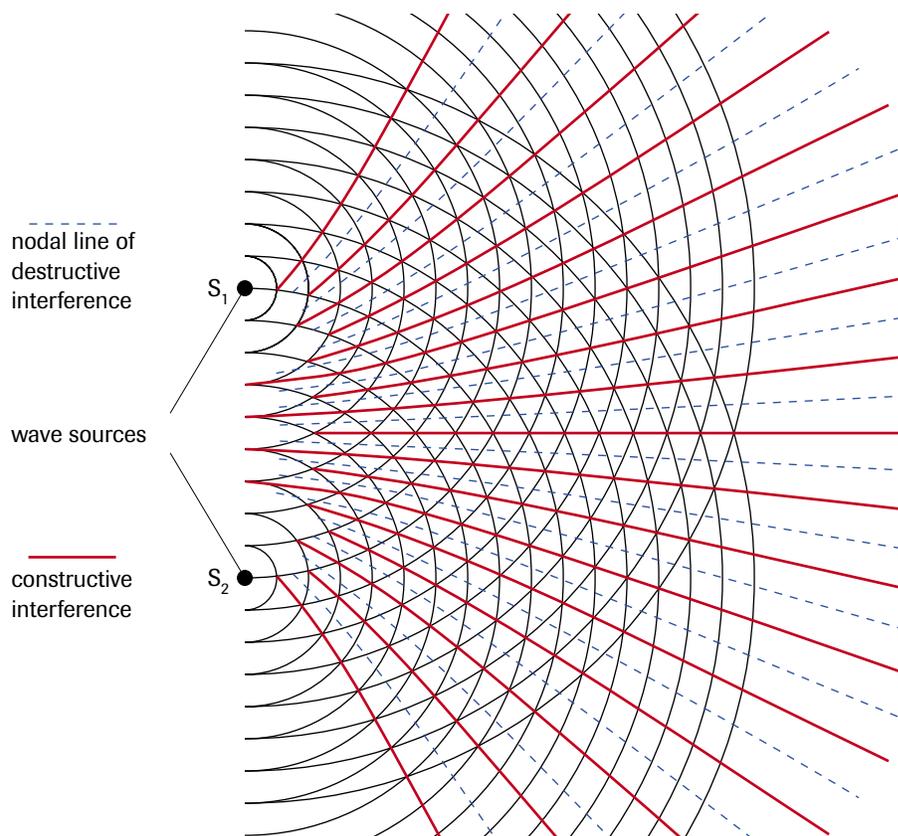


Wave Interference: Young's Double-Slit Experiment

9.5

If light has wave properties, then two light sources oscillating in phase should produce a result similar to the interference pattern in a ripple tank for vibrators operating in phase (**Figure 1**). Light should be brighter in areas of constructive interference, and there should be darkness in areas of destructive interference.



Many investigators in the decades between Newton and Thomas Young attempted to demonstrate interference in light. In most cases, they placed two sources of light side by side. They scrutinized screens near their sources for interference patterns but always in vain. In part, they were defeated by the exceedingly small wavelength of light. In a ripple tank, where the frequency of the sources is relatively small and the wavelengths are large, the distance between adjacent nodal lines is easily observable. In experiments with light, the distance between the nodal lines was so small that no nodal lines were observed.

There is, however, a second, more fundamental, problem in transferring the ripple tank setup into optics. If the relative phase of the wave sources is altered, the interference pattern is shifted. When two incandescent light sources are placed side by side, the atoms in each source emit the light randomly, out of phase. When the light strikes the screen, a constantly varying interference pattern is produced, and no single pattern is observed.

In his experiments over the period 1802–1804, Young used one incandescent body instead of two, directing its light through two pinholes placed very close together. The light was diffracted through each pinhole, so that each acted as a point source of light. Since the sources were close together, the spacing between the nodal lines was large enough to make the pattern of nodal lines visible. Since the light from the two pinholes originated from the same incandescent body (Young chose the Sun), the two interfering

Figure 1

Interference of circular waves produced by two identical point sources in phase

DID YOU KNOW?

Thomas Young



Thomas Young (1773–1829), an English physicist and physician, was a child prodigy who could read at the age of two. While studying the human voice, he became interested in the physics of waves and was able to demonstrate that the wave theory explained the behaviour of light. His work met with initial hostility in England because the particle theory was considered to be “English” and the wave theory “European.” Young’s interest in light led him to propose that any colour in the spectrum could be produced by using the primary colours, red, green, and blue. In 1810, he first used the term energy in the sense that we use it today, that is, the property of a system that gives it the ability to do work. His work on the properties of elasticity was honoured by naming the constant used in these equations “Young’s modulus.”

beams of light were always in phase, and a single, fixed interference pattern could be created on a screen.

Young's experiment resolved the two major problems in observing the interference of light: the two sources were in phase, and the distance between sources was small enough that a series of light and dark bands was created on a screen placed in the path of the light. These bands of bright and dark are called **interference fringes** or **maxima** and **minima**. This experiment, now commonly called Young's experiment, provided very strong evidence for the wave theory of light.

The sense in which Young's experiment confirms the wave nature of light becomes evident in **Figure 2**, where water waves falling upon a barrier with two slits are diffracted and produce an interference pattern similar to that produced by two point sources vibrating in phase in a ripple tank.

interference fringes the light (**maxima**) and dark (**minima**) bands produced by the interference of light

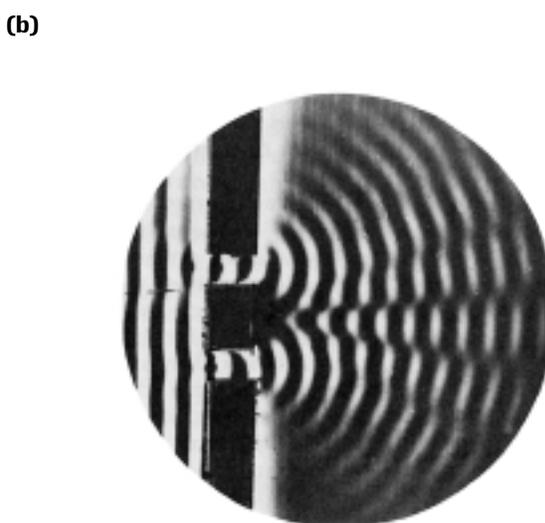
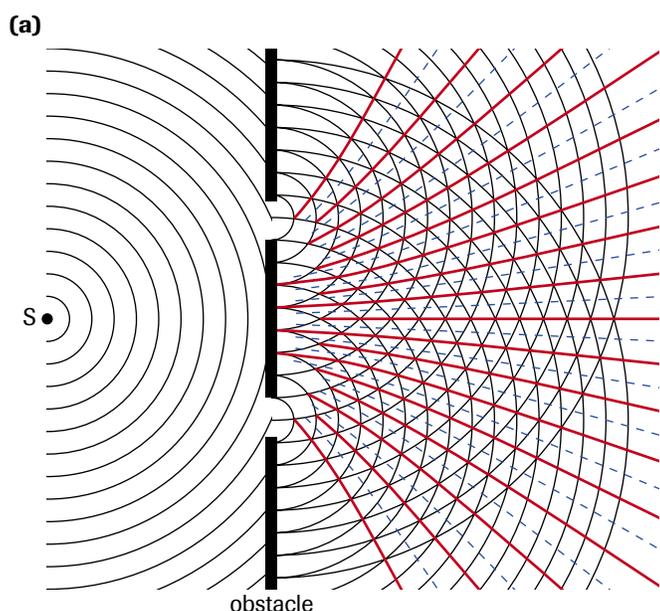


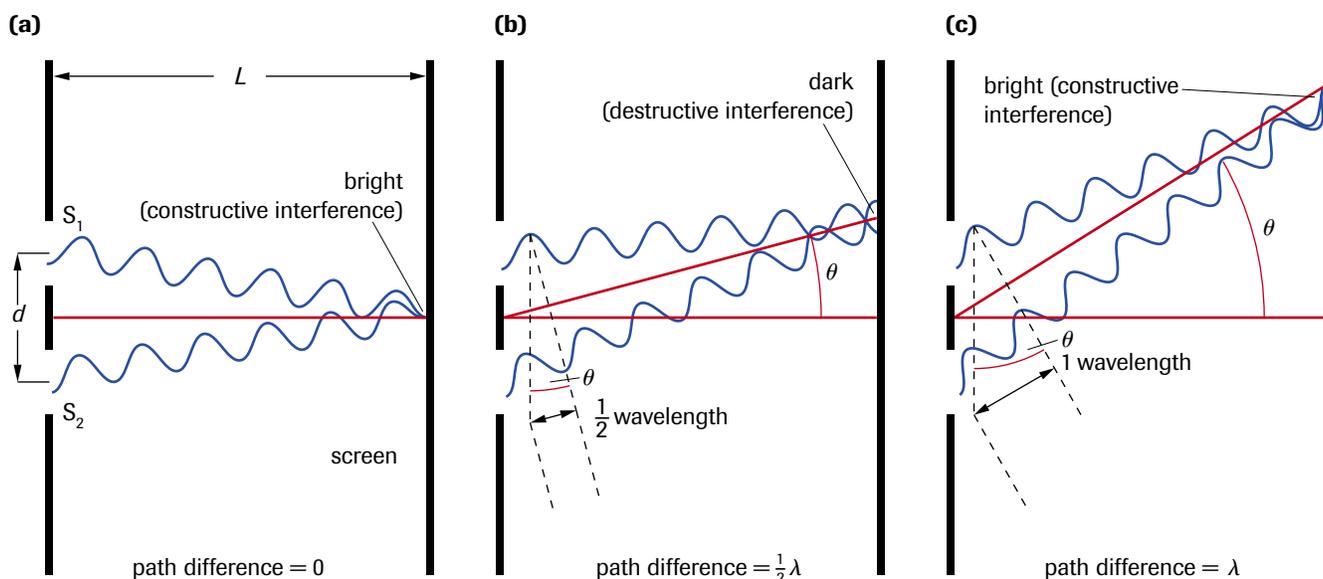
Figure 2

- (a) Interference in a ripple tank produced by waves from a single source passing through adjacent openings
- (b) Photograph of interference in a ripple tank

Figure 3 shows light waves, in phase, emerging from slits S_1 and S_2 , a distance d apart. Although the waves spread out in all directions after emerging from the slits, we will analyze them for only three different angles, θ . In **Figure 3(a)**, where $\theta = 0$, both waves reach the centre of the screen in phase, since they travel the same distance. Constructive interference therefore occurs, producing a bright spot at the centre of the screen. When the waves from slit S_2 travel an extra distance of $\frac{\lambda}{2}$ to reach the screen in **(b)**, the waves from the two sources arrive at the screen 180° out of phase. Destructive interference occurs, and the screen is dark in this region (corresponding to nodal line $n = 1$). As we move still farther from the centre of the screen, we reach a point at which the path difference is λ , as in **(c)**. Since the two waves are back in phase, with the waves from S_2 one whole wavelength behind those from S_1 , constructive interference occurs (causing this region, like the centre of the screen, to be bright).

As in the ripple tank, we find destructive interference for appropriate values of the path difference, $d \sin \theta_n$:

$$\sin \theta_n = \left(n - \frac{1}{2} \right) \frac{\lambda}{d} \quad \text{where } n = 1, 2, 3, \dots$$



Another set of values of $d \sin \theta$ yields constructive interference:

$$\sin \theta_m = \frac{m\lambda}{d} \quad \text{where } m = 0, 1, 2, 3, \dots$$

Since the dark fringes created on the screen by destructive interference are very narrow in comparison with the bright areas, measurements are made with those nodal lines. Also, $\sin \theta_n$ is best determined not by trying to measure θ_n directly but by using the ratio $\frac{x}{L}$, where x is the distance of the nodal line from the centre line on the screen, and L is the distance to the screen from the midpoint between the slits.

In general, the equations used for the two-point interference pattern in the ripple tank can be used for light, that is,

$$\sin \theta_n = \frac{x_n}{L} = \left(n - \frac{1}{2}\right) \frac{\lambda}{d}$$

where x_n is the distance to the n th nodal line, measured from the right bisector, as in Figure 4.

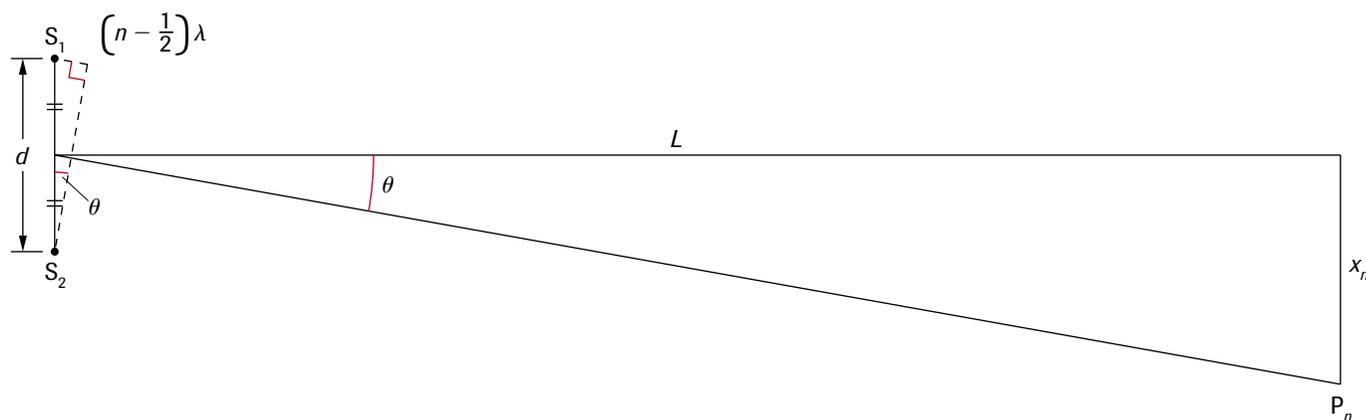


Figure 4

Figure 3

- (a) Path difference = 0
- (b) Path difference = $\frac{1}{2}\lambda$
- (c) Path difference = λ

LEARNING TIP

A Good Approximation

Actually, $\frac{x_n}{L} = \tan \theta_n$. However, for $L \gg x$, $\tan \theta_n$ is very nearly equal to $\sin \theta_n$. Thus $\frac{x_n}{L}$ serves as a good approximation to $\sin \theta_n$ in this instance.

LEARNING TIP

The “Order” of Minima and Maxima

For a two-source, in-phase interference pattern, the nodal line numbers, or minima, can be called first-order minimum, second-order minimum, etc., for $n = 1, 2, \dots$. The maximum numbers can be called zero-order maximum, first-order maximum, second-order maximum, etc., for $m = 0, 1, 2, \dots$. The zero-order maximum is called the central maximum.

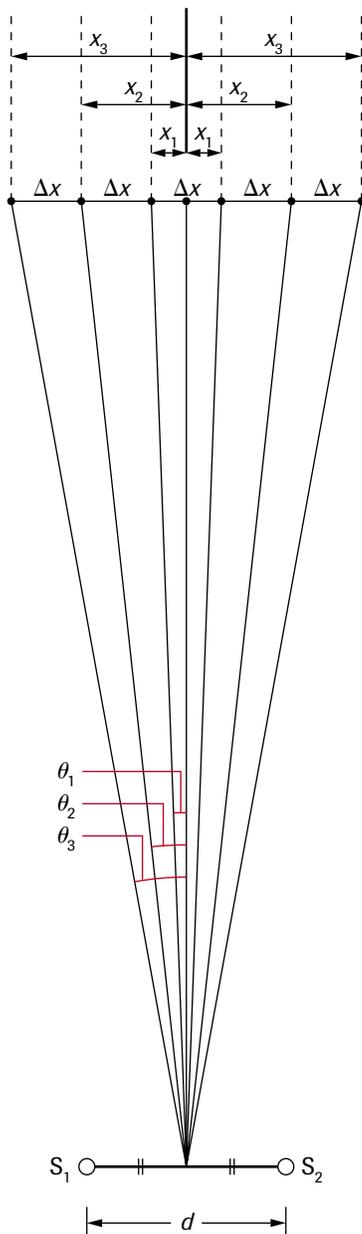


Figure 5

For each nodal line, we can derive a separate value for x (Figure 4) from this equation, as follows:

$$x_n = \left(n - \frac{1}{2}\right) \frac{L\lambda}{d}$$

$$x_1 = \left(1 - \frac{1}{2}\right) \frac{L\lambda}{d} = \frac{L\lambda}{2d}$$

$$x_2 = \left(2 - \frac{1}{2}\right) \frac{L\lambda}{d} = \frac{3L\lambda}{2d}$$

$$x_3 = \left(3 - \frac{1}{2}\right) \frac{L\lambda}{d} = \frac{5L\lambda}{2d}$$

etc.

Although L was different in the earlier equation, in this case L is so large compared to d and the values of L for the various nodal lines are so similar, that we can treat L as a constant, being essentially equal to the perpendicular distance from the slits to the screen.

Figure 5 shows that the displacement between adjacent nodal lines (Δx) is given by $(x_2 - x_1)$, $(x_3 - x_2)$, and $(x_3 + x_1)$. In each case the value is

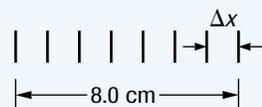
$$\frac{\Delta x}{L} = \frac{\lambda}{d}$$

where Δx is the distance between adjacent nodal lines on the screen, d is the separation of the slits, and L is the perpendicular distance from the slits to the screen.

SAMPLE problem 1

You are measuring the wavelength of light from a certain single-colour source. You direct the light through two slits with a separation of 0.15 mm, and an interference pattern is created on a screen 3.0 m away. You find the distance between the first and the eighth consecutive dark lines to be 8.0 cm. At what wavelength is your source radiating?

Solution



$$L = 3.0 \text{ m}$$

$$\lambda = ?$$

$$8 \text{ nodal lines} = 7\Delta x$$

$$\Delta x = \frac{8.0 \text{ cm}}{7} = 1.14 \text{ cm} = 1.14 \times 10^{-2} \text{ m}$$

$$d = 0.15 \text{ mm} = 1.5 \times 10^{-4} \text{ m}$$

$$\frac{\Delta x}{L} = \frac{\lambda}{d}$$

$$\lambda = \frac{d\Delta x}{L}$$

$$= \frac{(1.14 \times 10^{-2} \text{ m})(1.5 \times 10^{-4} \text{ m})}{3.0 \text{ m}}$$

$$\lambda = 5.7 \times 10^{-7} \text{ m}$$

The wavelength of the source is $5.7 \times 10^{-7} \text{ m}$, or $5.7 \times 10^2 \text{ nm}$.

▶ SAMPLE problem 2

The third-order dark fringe of 652-nm light is observed at an angle of 15.0° when the light falls on two narrow slits. How far apart are the slits?

Solution

$$n = 3 \qquad \theta_3 = 15.0^\circ$$

$$\lambda = 652 \text{ nm} = 6.52 \times 10^{-7} \text{ m} \qquad d = ?$$

$$\sin \theta_n = \left(n - \frac{1}{2}\right) \frac{\lambda}{d}$$

$$d = \frac{\left(n - \frac{1}{2}\right) \lambda}{\sin \theta_n}$$

$$= \frac{\left(3 - \frac{1}{2}\right)(6.52 \times 10^{-7} \text{ m})}{\sin 15.0^\circ}$$

$$d = 6.30 \times 10^{-6} \text{ m}$$

The slit separation is $6.30 \times 10^{-6} \text{ m}$.

▶ Practice

Understanding Concepts

1. A student performing Young's experiment with a single-colour source finds the distance between the first and the seventh nodal lines to be 6.0 cm. The screen is located 3.0 m from the two slits. The slit separation is $2.2 \times 10^2 \mu\text{m}$. Calculate the wavelength of the light.
2. Single-colour light falling on two slits 0.042 mm apart produces the fifth-order fringe at a 3.8° angle. Calculate the wavelength of the light.
3. An interference pattern is formed on a screen when helium-neon laser light ($\lambda = 6.3 \times 10^{-7} \text{ m}$) is directed toward it through two slits. The slits are $43 \mu\text{m}$ apart. The screen is 2.5 m away. Calculate the separation of adjacent nodal lines.
4. In an interference experiment, reddish light of wavelength $6.0 \times 10^2 \text{ nm}$ passes through a double slit. The distance between the first and eleventh dark bands, on a screen 1.5 m away, is 13.2 cm.
 - (a) Calculate the separation of the slits.
 - (b) Calculate the spacing between adjacent nodal lines using blue light of wavelength $4.5 \times 10^2 \text{ nm}$.
5. A parallel beam of light from a laser, with a wavelength 656 nm, falls on two very narrow slits, 0.050 mm apart. How far apart are the fringes in the centre of the pattern thrown upon a screen 2.6 m away?
6. Light of wavelength $6.8 \times 10^2 \text{ nm}$ falls on two slits, producing an interference pattern where the fourth-order dark fringe is 48 mm from the centre of the interference pattern on a screen 1.5 m away. Calculate the separation of the two slits.
7. Reddish light of wavelength $6.0 \times 10^{-7} \text{ m}$ passes through two parallel slits. Nodal lines are produced on a screen 3.0 m away. The distance between the first and the tenth nodal lines is 5.0 cm. Calculate the separation of the two slits.
8. In an interference experiment, reddish light of wavelength $6.0 \times 10^{-7} \text{ m}$ passes through a double slit, hitting a screen 1.5 m away. The distance between the first and eleventh dark bands is 2.0 cm.
 - (a) Calculate the separation of the slits.
 - (b) Calculate the spacing between adjacent nodal lines using blue light ($\lambda_{\text{blue}} = 4.5 \times 10^{-7} \text{ m}$).

Answers

1. $7.3 \times 10^{-7} \text{ m}$
2. $6.2 \times 10^2 \text{ nm}$
3. 3.7 cm
4. (a) $68 \mu\text{m}$
(b) 1.0 cm
5. 3.4 cm
6. $7.4 \times 10^{-2} \text{ mm}$
7. $3.2 \times 10^{-4} \text{ m}$
8. (a) $1.5 \times 10^{-3} \text{ m}$
(b) $4.5 \times 10^{-4} \text{ m}$

monochromatic composed of only one colour; possessing only one wavelength

INVESTIGATION 9.5.1

Young's Double-Slit Experiment (p. 484)

Can you predict interference patterns using Young's method?

DID YOU KNOW?

Augustin Fresnel



Augustin Fresnel (1788–1827) spent most of his life working for the French government as a civil engineer. His mathematical analysis provided the theoretical basis for the transverse wave model of light. Fresnel applied his analysis to design a lens of nearly uniform thickness for use in lighthouses as a replacement for the less efficient mirror systems of his day. Today, the Fresnel lens is found in a wide range of devices, including overhead projectors, beacon lights, and solar collectors.

Further Developments in the Wave Theory of Light

In Section 9.3 we analyzed the interference pattern between two-point sources in a ripple tank and derived equations that permitted the calculation of the wavelength of the source. But with light, all we see are the results of light interference on a screen; we cannot see what is happening when the light passes through two slits. If Young's hypothesis is correct, we should be able to use the same equations to measure the wavelength of light. But we must have a source of light that has a fixed wavelength. There are a limited number of **monochromatic** (composed of only one colour and having one wavelength) sources of light. Traditionally the sodium vapour lamp was used for this purpose, but today we have lasers that emit a bright single wavelength light, usually in the red part of the spectrum. Investigation 9.5.1, in the Lab Activities section at the end of this chapter, gives you the opportunity to use a helium–neon laser or a light-emitting diode (LED) as your source and, based on the above analysis, predict the interference pattern using a procedure similar to that used by Young.

When Young announced his results in 1807, he reminded his audience that Newton had made several statements favouring a theory with some wave aspects. Nevertheless, Newton's influence was still dominant, and the scientific establishment did not take Young and his work seriously. It was not until 1818, when the French physicist Augustin Fresnel proposed his own mathematical wave theory, that Young's research was accepted. Fresnel's work was presented to a group of physicists and mathematicians, most of them strong supporters of the particle theory. One mathematician, Simon Poisson, showed that Fresnel's wave equations predict a unique diffraction pattern when light is directed past a small solid disk. If light really did behave like a wave, argued Poisson, the light diffracting around the edges of the disk should interfere constructively, producing a small bright spot at the exact centre of the diffraction pattern (**Figure 6**). (According to the particle theory, constructive interference at this position was impossible.) Poisson did not observe a bright spot and felt he had refuted the wave theory.

However, in 1818, Dominique Arago tested Poisson's prediction experimentally, and the bright spot was seen at the centre of the shadow (**Figure 7**). Even though Poisson refuted the wave theory, his prediction that there would be a bright spot at the centre of the shadow if the wave theory proved valid, led to this phenomenon being known as "Poisson's Bright Spot." (Note that there is also interference near the edge of the disk.)

By 1850, the validity of the wave theory of light had been generally accepted. For some time afterward, the mathematical consequences of the wave theory were applied to numerous aspects of the properties of light, including dispersion, polarization, single-slit diffraction, and the development of the electromagnetic spectrum, which we will discuss in the next section and in Chapter 10.

But the wave theory was not adequate to explain the movement of light through the vacuum of space, since waves required a material medium for their transmission. The power of the wave theory was now so great, however, that scientists theorized a "fluid" filling all space, from the space between atoms to the space between planets. They called it "ether." Many experiments were attempted to detect this ether, but none were successful (see Section 11.1).

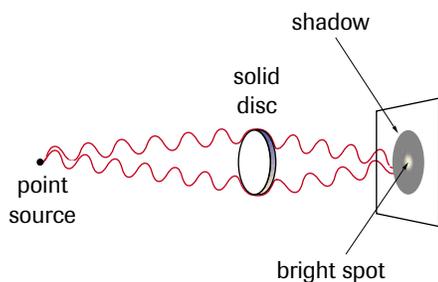


Figure 6

If light is a wave, a bright spot should appear at the centre of the shadow of a solid disk illuminated by a point source of monochromatic light.

Although in our exploration of the wave theory to this point, we have treated light as a transverse wave, in fact no evidence has been provided that it is a transverse, not a longitudinal, wave. One property of light not discussed previously is polarization. When we understand polarization and apply the wave theory in the next chapter we will have convincing evidence that validates the wave theory of light, leading to the study of electromagnetic waves.

SUMMARY

Wave Interference: Young's Double-Slit Experiment

- Early attempts to demonstrate the interference of light were unsuccessful because the two sources were too far apart and out of phase, and the wavelength of light is very small.
- Thomas Young's crucial contribution consisted of using one source illuminating two closely spaced openings in an opaque screen, thus using diffraction to create two sources of light close together and in phase.
- In Young's experiment a series of light and dark bands, called interference fringes, was created on a screen, placed in the path of light, in much the same way as those created in the ripple tank.
- The relationships $\sin \theta_n = \frac{x_n}{L} = \left(n - \frac{1}{2}\right) \frac{\lambda}{d}$ and $\frac{\Delta x}{L} = \frac{\lambda}{d}$ permit unknowns to be calculated, given any three of λ , Δx , L , θ , d , and n .
- Young's experiment supported the wave theory of light, explaining all the properties of light except transmission through a vacuum.

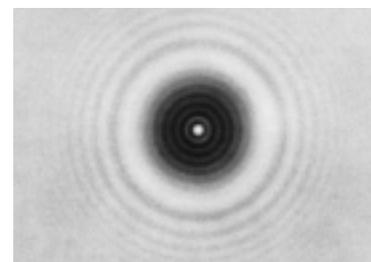


Figure 7
Poisson's Bright Spot

DID YOU KNOW?

Dominique Arago

Dominique Arago (1786–1853) made contributions in many fields of science. He supported the particle theory at first, but later converted to the wave theory. He introduced Fresnel to the work of Young. Arago did some pioneer work in electromagnetism based on the discoveries of Oersted. He was also a fiery political figure, involved in the French revolutions of 1830 and 1852.

Section 9.5 Questions

Understanding Concepts

1. Explain why the discoveries of Grimaldi were so important to Young's work.
2. Explain why the observation of the double-slit interference pattern was more convincing evidence for the wave theory of light than the observation of diffraction.
3. Monochromatic red light is incident on a double slit and produces an interference pattern on a screen some distance away. Explain how the fringe pattern would change if the red light source is replaced with a blue light source.
4. If Young's experiment were done completely under water, explain how the interference pattern would change from that observed in air, using the same equipment and experimental setup.
5. In a Young's double-slit experiment, the angle that locates the second dark fringe on either side of the central bright fringe is 5.4° . Calculate the ratio of the slit separation d to the wavelength λ of the light.
6. In measuring the wavelength of a narrow, monochromatic source of light, you use a double slit with a separation of 0.15 mm. Your friend places markers on a screen 2.0 m in front of the slits at the positions of successive dark bands in the pattern. Your friend finds the dark bands to be 0.56 cm apart.
 - (a) Calculate the wavelength of the source in nanometres.
 - (b) Calculate what the spacing of the dark bands would be if you used a source of wavelength 6.0×10^2 nm.
7. Monochromatic light from a point source illuminates two parallel, narrow slits. The centres of the slit openings are 0.80 mm apart. An interference pattern forms on a screen placed parallel to the plane of the slits and 49 cm away. The distance between two adjacent dark interference fringes is 0.30 mm.
 - (a) Calculate the wavelength of the light.
 - (b) What would the separation of the nodal lines be if the slit centre were narrowed to 0.60 mm?
8. Monochromatic light falls on two very narrow slits 0.040 mm apart. Successive nodal points on a screen 5.00 m away are 5.5 cm apart near the centre of the pattern. Calculate the wavelength of the light.
9. A Young's double-slit experiment is performed using light that has a wavelength of 6.3×10^2 nm. The separation between the slits is 3.3×10^{-5} m. Find the angles, with respect to the slits, that locate the first-, second-, and third-order bright (not dark) fringes on the screen.

Applying Inquiry Skills

10. A thin piece of glass is placed in front of one of the two slits in a Young's apparatus so that the waves exit that slit 180° out of phase with respect to the other slit. Describe, using diagrams, the interference pattern on the screen.