9.1 Waves in Two Dimensions

INVESTIGATION 9.1.1

Transmission, Reflection, and Refraction of Water Waves in a Ripple Tank

How can we view waves to study them? How are waves transmitted, reflected, and refracted in a ripple tank?

It is difficult to study the properties of waves for sound, light, and radio because we cannot view the waves directly. However, if we use a ripple tank, not only can we view the waves directly, but we can create most conditions needed to demonstrate the properties of **transverse waves** in this two-dimension space. Investigation 9.1.1, in the Lab Activities section at the end of this chapter, provides you with an opportunity to study the properties of waves in a ripple tank in order to better understand and predict similar behaviours and relationships for other waves.

Transmission

A wave originating from a point source is circular, whereas a wave originating from a linear source is straight. We confine ourselves for the moment to waves from sources with a constant frequency. As a wave moves away from its constant-frequency source, the spacing between successive crests or successive troughs—the wavelength—remains the same provided the speed of the wave does not change. A continuous crest or trough is referred to as a **wave front**. To show the direction of travel, or transmission, of a wave front, an arrow is drawn at right angles to the wave front (**Figure 1**). This line is called a **wave ray**. Sometimes we refer to wave rays instead of wave fronts when describing the behaviour of a wave.

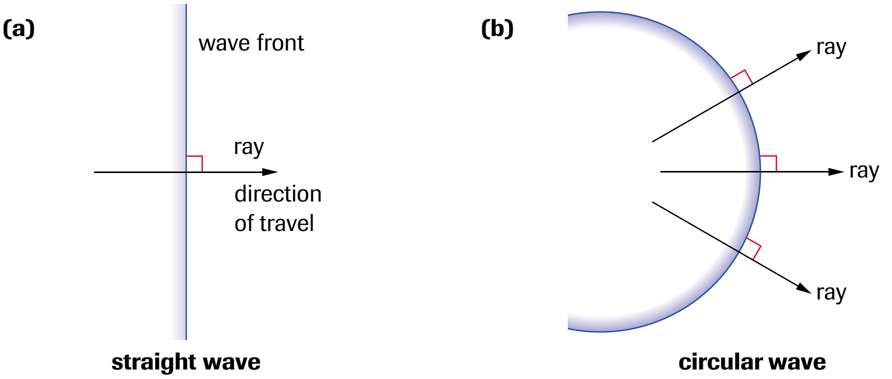

Figure 1
In both cases the wave ray is at 90° to the wave front.

When the speed decreases, as it does in shallow water, the wavelength decreases (**Figure 2**), since wavelength is directly proportional to speed ($\lambda \propto v$). When the frequency of a source is increased, the distance between successive crests becomes smaller, since wavelength is inversely proportional to frequency $\left(\lambda \propto \frac{1}{f}\right)$. Both proportionalities are consequences of the universal wave equation, $v = f\lambda$. This equation holds for all types of waves—one-dimensional, two-dimensional, and three-dimensional.

The wave travelling in deep water has a speed $v_1 = f_1\lambda_1$. Similarly, $v_2 = f_2\lambda_2$ for the wave travelling in shallow water. In a ripple tank, the frequency of a water wave is determined by the wave generator and does not change when the speed changes. Thus $f_1 = f_2$.

If we divide the first equation by the second equation, we get

$$\frac{v_1}{v_2} = \frac{f_1\lambda_1}{f_2\lambda_2}$$

However, $f_1 = f_2$. Therefore,

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

transverse wave periodic disturbance where particles in the medium oscillate at right angles to the direction in which the wave travels

wave front the leading edge of a continuous crest or trough

wave ray a straight line, drawn perpendicular to a wave front, indicating the direction of transmission

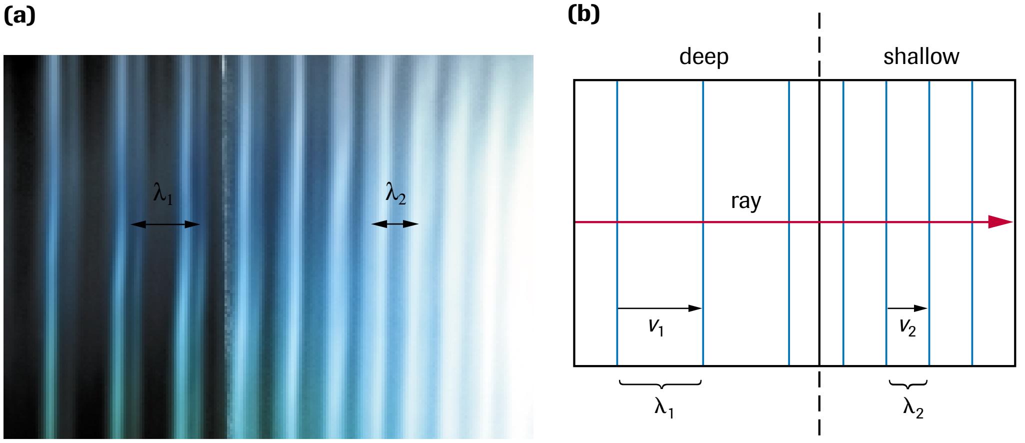

Figure 2
Periodic straight waves travelling from deep water to shallow water (left to right)

SAMPLE problem 1

A water wave has a wavelength of 2.0 cm in the deep section of a tank and 1.5 cm in the shallow section. If the speed of the wave in the shallow water is 12 cm/s, what is its speed in the deep water?

Solution

$\lambda_1 = 2.0$ cm

$\lambda_2 = 1.5$ cm

$v_2 = 12$ cm/s

$v_1 = ?$

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$v_1 = \left(\frac{\lambda_1}{\lambda_2}\right)v_2$$

$$= \left(\frac{2.0\text{ cm}}{1.5\text{ cm}}\right)12\text{ cm/s}$$

$$v_1 = 16\text{ cm/s}$$

The speed of the wave in deep water is 16 cm/s.

Practice

Understanding Concepts

1. The speed and the wavelength of a water wave in deep water are 18.0 cm/s and 2.0 cm, respectively. The speed in shallow water is 10.0 cm/s. Find the corresponding wavelength.
2. A wave travels 0.75 times as fast in shallow water as it does in deep water. Find the wavelength of the wave in deep water if its wavelength is 2.7 cm in shallow water.
3. In question 1, what are the respective frequencies in deep and shallow water?

Answers

1. 1.1 cm
2. 3.6 cm
3. 9.0 Hz; 9.0 Hz

Reflection from a Straight Barrier

A straight wave front travels in the "wave ray" direction perpendicular to the wave front, but how will it behave when encountering obstacles? When a straight wave front runs into a straight reflective barrier, head on, it is reflected back along its original path (**Figure 3**). If a wave encounters a straight barrier obliquely (i.e., at an angle other than 90°), the wave front is likewise reflected obliquely. The angle formed by the incident wave front and the normal is equal to the angle formed by the reflected wave front and the normal. These angles are called the **angle of incidence** (θ_i) and the **angle of reflection** (θ_r), respectively (**Figure 4**). Reflection leaves wavelength, speed, and frequency unchanged.

angle of incidence (θ_i) the angle between the incident wave front and the barrier, or the angle between the incident ray and the normal

angle of reflection (θ_r) the angle between the reflected wave front and the barrier, or the angle between the reflected ray and the normal

refraction the bending effect on a wave's direction that occurs when the wave enters a different medium at an angle

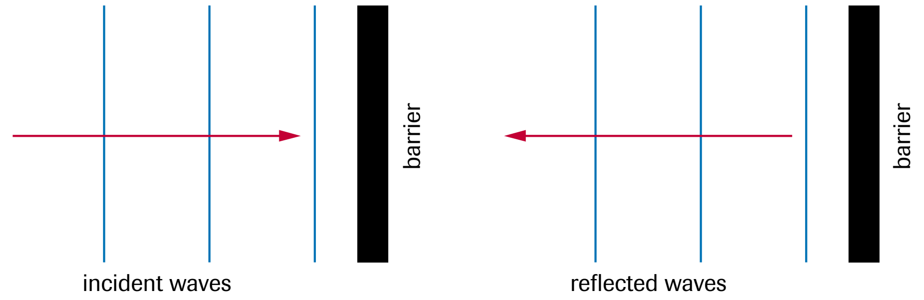

Figure 3
A straight wave front meeting a straight barrier head on is reflected back along its original path.

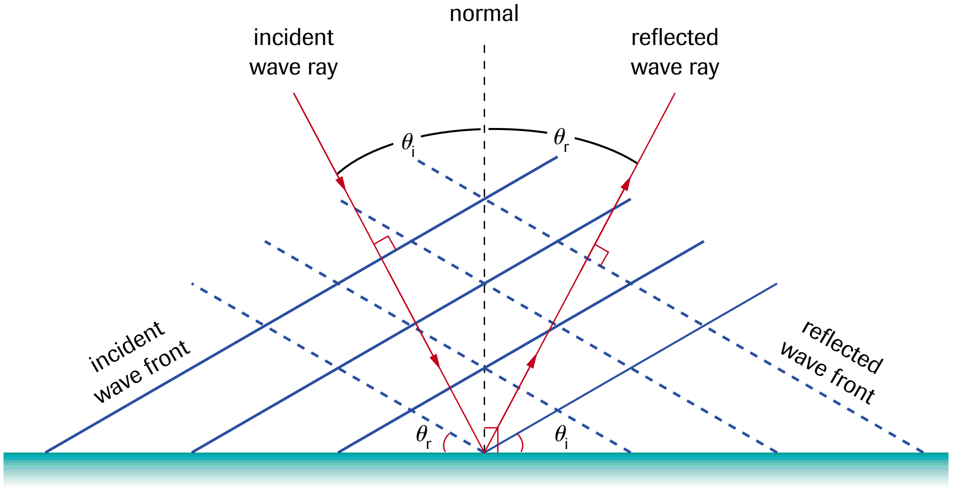

Figure 4
When a wave encounters a straight barrier obliquely, rather than head on, the angle of incidence equals the angle of reflection.

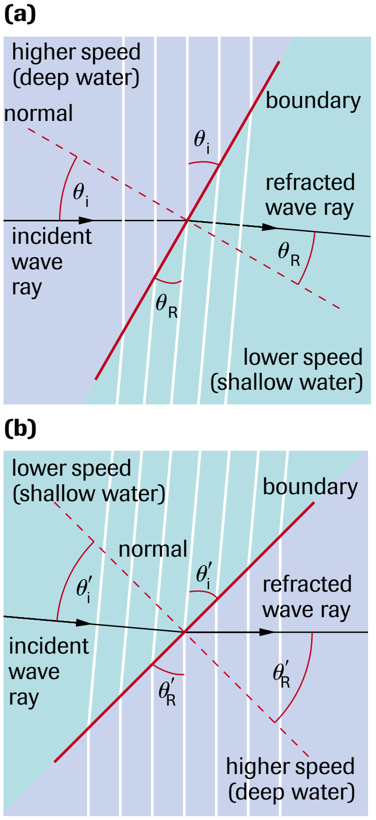

Figure 5
(a) When water waves travel obliquely into a slower medium, the wave ray bends toward the normal.
(b) If the new medium is a faster one, the wave ray bends away from the normal.

Refraction

When a wave travels from deep water to shallow water in such a way that it meets the boundary between the two depths straight on, no change in direction occurs. On the other hand, if a wave meets the boundary at an angle, the direction of travel does change. This phenomenon is called **refraction** (**Figure 5**).

We usually use wave rays to describe refraction. The **normal** is a line drawn at right angles to a boundary at the point where an incident wave ray strikes the boundary. The angle formed by an incident wave ray and the normal is called the angle of incidence, θ_i. The angle formed by the normal and the refracted wave ray is called the **angle of refraction**, θ_R.

When a wave travels at an angle into a medium in which its speed decreases, the refracted wave ray is bent (refracted) toward the normal, as in **Figure 5(a)**. If the wave travels at an angle into a medium in which its speed increases, the refracted wave ray is bent away from the normal, as in **Figure 5(b)**.

Figure 6 shows geometrically that θ_i is equal to the angle between the incident wave front and the normal and that θ_R is equal to the angle between the refracted wave front and the normal. In the ripple tank, it is easier to measure the angles between the wave rays and the boundary, that is, θ_i' and θ_R'.

To analyze wave fronts refracted at a boundary, the angles of incidence and refraction can be determined using the equations $\sin\theta_i = \frac{\lambda_1}{xy}$ and $\sin\theta_R = \frac{\lambda_2}{xy}$, respectively (**Figure 7**).

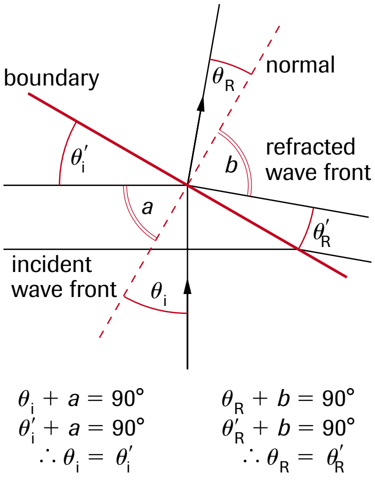

Figure 6

normal a straight line drawn perpendicular to a barrier struck by a wave

angle of refraction (θ_R) the angle between the normal and the refracted ray, or between the refracted wave front and the boundary

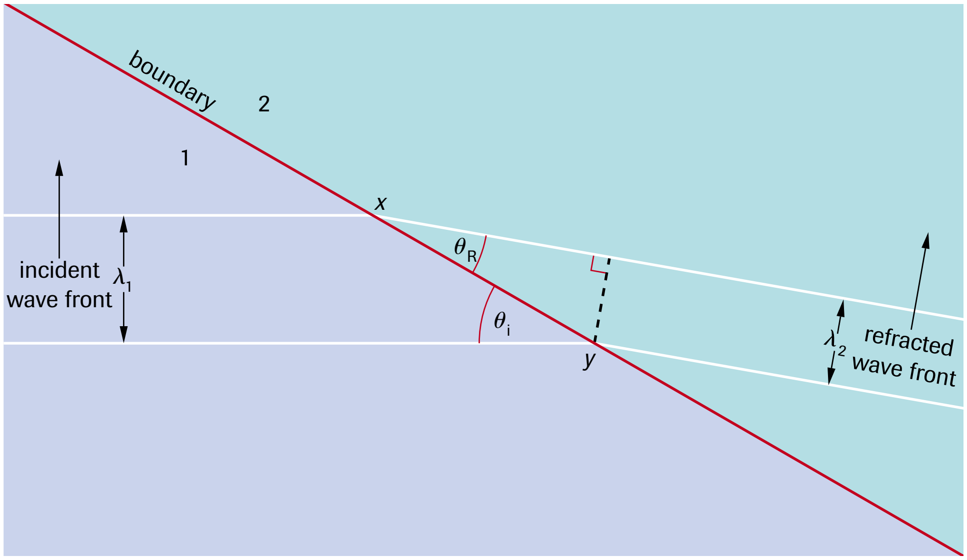

Figure 7

The ratio of the sines gives

$$\frac{\sin\theta_i}{\sin\theta_R} = \frac{\left(\frac{\lambda_1}{xy}\right)}{\left(\frac{\lambda_2}{xy}\right)}$$

which reduces to

$$\frac{\sin\theta_i}{\sin\theta_R} = \frac{\lambda_1}{\lambda_2}$$

For a specific change in medium, the ratio $\frac{\lambda_1}{\lambda_2}$ has a constant value. Recall Snell's law from optics, $\sin\theta_i \propto \sin\theta_R$. This equation can be converted to $\sin\theta_i = n\sin\theta_R$. The constant of proportionality (n) and the index of refraction (n) are one and the same thing.

Consequently, we can write

$$\frac{\sin \theta_i}{\sin \theta_R} = n$$

absolute index of refraction the index of refraction for light passing from air or a vacuum into a substance

This relationship holds for waves of all types, including light, which we will see shortly. When light passes from a vacuum into a substance, n is called the **absolute index of refraction**. (See **Table 1** for a list of absolute indexes of refraction.) The value for the absolute index of refraction is so close to the value from air to a substance that we rarely distinguish between them. In this text, when we refer to the index of refraction, we will be referring to the absolute index of refraction.

Table 1 Approximate Absolute Indexes of Refraction for Various Substances*

Substance	Absolute Refractive Index
vacuum	1.000 000
air	1.000 29
ice	1.31
water	1.333
ethyl alcohol	1.36
turpentine	1.472
glass	1.50
Plexiglas	1.51
crown glass	1.52
polystyrene	1.59
carbon disulphide	1.628
flint glass	1.66
zircon	1.923
diamond	2.417
gallium phosphide	3.50

*Measured with a wavelength of 589 nm. Values may vary with physical conditions.

You will also recall that we derived a general equation for Snell's law that applies to any two substances:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where n_1 is the index of refraction in the first medium, n_2 is the index of refraction in the second medium, and θ_1 and θ_2 are angles in each respective medium.

For waves we found that $\frac{\sin \theta_i}{\sin \theta_R} = \frac{\lambda_1}{\lambda_2}$, which we can generalize to $\frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2}$. But from the universal wave equation, $v = f\lambda$, we can show that $\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$ since f is constant. Therefore, we can write

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

The following sample problems will illustrate the application of these relationships in both in the ripple tank and for light.

SAMPLE problem 2

A 5.0 Hz water wave, travelling at 31 cm/s in deep water, enters shallow water. The angle between the incident wave front in the deep water and the boundary between the deep and shallow regions is 50°. The speed of the wave in the shallow water is 27 cm/s. Find

(a) the angle of refraction in the shallow water

(b) the wavelength in shallow water

Solution

(a) $f = 5.0\text{ Hz}$ $\qquad \theta_1 = 50.0°$

$v_1 = 31\text{ cm/s}$ $\qquad \theta_2 = ?$

$v_2 = 27\text{ cm/s}$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

$$\sin\theta_2 = \left(\frac{v_2}{v_1}\right)\sin\theta_1$$

$$\sin\theta_2 = \left(\frac{27\text{ cm/s}}{31\text{ cm/s}}\right)\sin 50.0°$$

$$\theta_2 = 41.9,\text{ or } 42°$$

The angle of refraction is 42°.

(b) $$\lambda_2 = \frac{v_2}{f_2} \quad \text{but } f_2 = f_1 = 5.0\text{ Hz}$$

$$= \frac{27\text{ cm/s}}{5.0\text{ Hz}}$$

$$\lambda_2 = 5.4\text{ cm}$$

The wavelength in shallow water is 5.4 cm.

SAMPLE problem 3

For a light ray travelling from glass into water, find

(a) the angle of refraction in water, if the angle of incidence in glass is 30.0°

(b) the speed of light in water

Solution

From **Table 1**,

$n_g = n_1 = 1.50$ $\qquad \theta_g = \theta_1 = 30.0°$

$n_w = n_2 = 1.333$ $\qquad \theta_w = \theta_2 = ?$

(a) $$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$\frac{\sin\theta_g}{\sin\theta_w} = \frac{n_w}{n_g}$$

$$\frac{\sin 30.0°}{\sin\theta_w} = \frac{1.333}{1.50}$$

$$\sin\theta_w = \frac{1.50\sin 30.0°}{1.333}$$

$$\theta_w = 34.3°$$

The angle of refraction in water is 34.3°.

(b) $n_a = n_1 = 1.00$

$n_w = n_2 = 1.333$

$v_1 = c = 3.00 \times 10^8$ m/s

$v_2 = ?$

$$\frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$v_2 = \frac{n_1 v_1}{n_2}$$

$$= \frac{(1.00)(3.00 \times 10^8 \text{ m/s})}{1.333}$$

$$v_2 = 2.26 \times 10^8 \text{ m/s}$$

The speed of light in water is 2.26×10^8 m/s.

Practice

Understanding Concepts

4. A wave in a ripple tank passes from a deep to a shallow region with $\theta_1 = 60°$ and $\theta_2 = 45°$. Calculate the ratios in the two media of (a) the wavelengths, (b) the speeds, and (c) the frequencies.

5. Water waves travelling at a speed of 28 cm/s enter deeper water at $\theta_1 = 40°$. Determine the speed in the deeper water if $\theta_2 = 46°$.

6. A 10.0-Hz water wave travels from deep water, where its speed is 38.0 cm/s, to shallow water, where its speed is 28.0 cm/s and $\theta_1 = 30°$. Find (a) the index of refraction, (b) the wavelengths in the two media, and (c) the angle of refraction in the shallow water.

7. A plane wave generator with a frequency of 6.0 Hz creates a water wave of wavelength 2.0 cm in region A of a ripple tank (**Figure 8**). The angle between the wave crests and the straight boundary between regions A and B is 30°. In region B the angle is 20°.
(a) Use Snell's law to determine the refractive index of the two regions.
(b) Find the speed in each region.

8. Light travels from crown glass into air. (Refer to **Table 1** for the indexes of refraction.) The angle of refraction in air is 60.0°. Calculate the angle of incidence in the crown glass.

9. If the index of refraction for diamond is 2.42, what will the angle of refraction be in diamond for an angle of incidence of 60.0° in water?

Answers

4. (a) 1.2
   (b) 1.2
   (c) 1.0
5. 31 cm/s
6. (a) 1.36
   (b) 3.8 cm, 2.8 cm
   (c) 21.6°
7. (a) 1.46
   (b) 12 cm/s; 8.2 cm/s
8. 34.7°
9. 28.0°

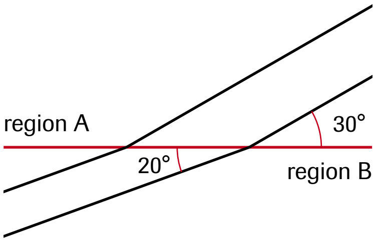

Figure 8
For question 7

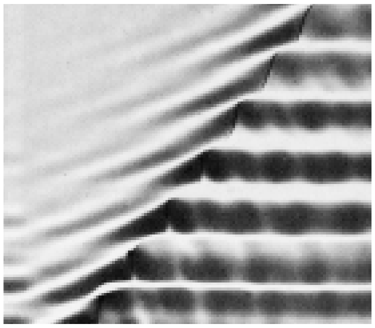

Figure 9
At higher angles of incidence, there is reflection as well as refraction. You can see such partial reflection–partial refraction on the right.

Partial Reflection–Partial Refraction

When refraction occurs, some of the energy usually reflects as well as refracts. This phenomenon was referred to in optics as partial reflection–partial refraction, a description that can also be used when referring to this behaviour in waves. We can demonstrate the same behaviour in a ripple tank, with waves travelling from deep to shallow water, provided we make the angle of incidence large, as in **Figure 9**.

The amount of reflection is more noticeable when a wave travels from shallow to deep water, where the speed increases and again becomes more pronounced as the angle of incidence increases. **Figure 10** shows that an incident angle is reached where the wave is refracted at an angle approaching 90°. For still larger incident angles there is no refraction at all, with all the wave energy being reflected; this behaviour of light is referred to as **total internal reflection**. This phenomenon is analogous to the total internal reflection of light.

total internal reflection the reflection of light in an optically denser medium; it occurs when the angle of incidence in the denser medium is greater than a certain critical angle

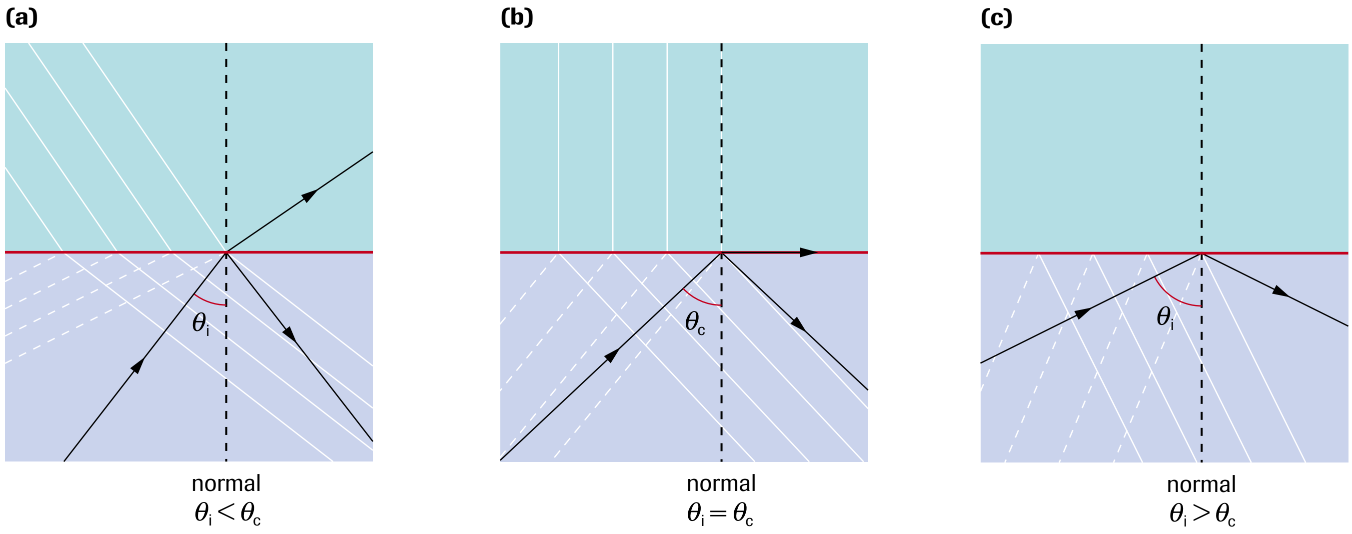

Figure 10
(a) Partial refraction–partial reflection
(b) At the critical angle
(c) Total internal reflection

We have remarked that the frequency of a wave does not in general change when its speed changes. Since $\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$, you might expect that the index of refraction and the amount of bending would not change for waves of different frequencies, provided the medium remains the same (e.g., water of the same depth in both cases).

Figure 11, however, shows that indexes of refraction do, in general, depend on wavelength. In **Figure 11(a)**, the low-frequency (long-wavelength) waves are refracted, as indicated by a rod placed on the screen below the transparent ripple tank. The rod is exactly parallel to the refracted wave fronts. In **Figure 11(b)**, the frequency has been increased (the wavelength decreased), with the rod left in the same position. The rod is no longer parallel to the refracted wave fronts. It appears that the amount of bending, and hence the index of refraction, is affected slightly by the frequency of a wave. We can conclude that, since the index of refraction represents a ratio of speeds in two media, the speed of the waves in at least one of those media must depend on their frequency. Such a medium, in which the speed of the waves depends on the frequency, is called a *dispersive medium*.

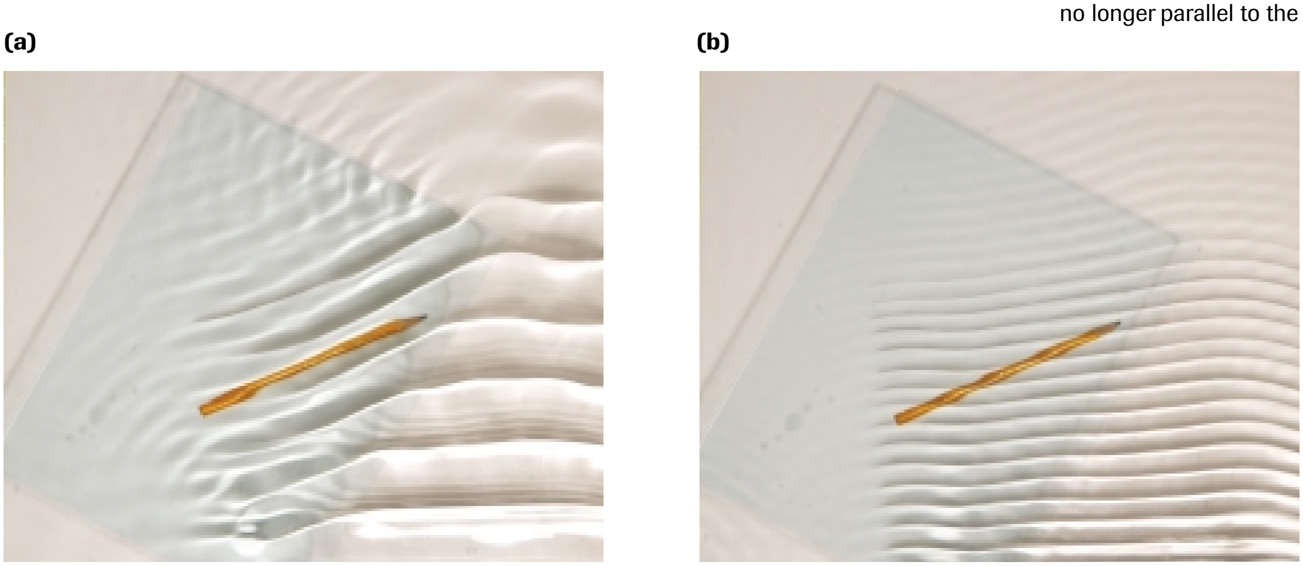

Figure 11
(a) The refraction of straight waves, with a rod marker placed parallel to the refracted wave fronts.
(b) The refracted wave fronts of the higher frequency waves are no longer parallel to the marker.

We stated previously that the speed of waves depends only on the medium. This statement now proves to be an idealization. Nevertheless, the idealization is a good approximation of the actual behaviour of waves, since the dispersion of a wave is the result of minute changes in its speed. For many applications, it is acceptable to make the assumption that frequency does not affect the speed of waves.

SUMMARY Waves in Two Dimensions

- The wavelength of a periodic wave is directly proportional to its speed.
- The frequency of a periodic wave is determined by the source and does not change as the wave moves through different media or encounters reflective barriers.
- All periodic waves obey the universal wave equation, $v = f\lambda$.
- The index of refraction for a pair of media is the ratio of the speeds or the ratio of the wavelengths in the two media $\left(\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}\right)$.
- Snell's law $\left(n = \frac{\sin \theta_i}{\sin \theta_R}\right)$ holds for waves and for light.
- When a wave passes from one medium to another, the wavelength changes and partial reflection–partial refraction can occur.

Section 9.1 Questions

Understanding Concepts

1. Straight wave fronts in the deep region of a ripple tank have a speed of 24 cm/s and a frequency of 4.0 Hz. The angle between the wave fronts and the straight boundary of the deep region is 40°. The wave speed in the shallow region beyond the boundary is 15 cm/s. Calculate
   (a) the angle the refracted wave front makes with the boundary
   (b) the wavelength in the shallow water
2. The following observations are made when a straight periodic wave crosses a boundary between deep and shallow water: 10 wave fronts cross the boundary every 5.0 s, and the distance across 3 wave fronts is 24.0 cm in deep water and 18.0 cm in shallow water.
   (a) Calculate the speed of the wave in deep water and in shallow water.
   (b) Calculate the refractive index.
3. Straight wave fronts with a frequency of 5.0 Hz, travelling at 30 cm/s in deep water, move into shallow water. The angle between the incident wave front in the deep water and the straight boundary between deep and shallow water is 50°. The speed of the wave in the shallow water is 27 cm/s.
   (a) Calculate the angle of refraction in the shallow water.
   (b) Calculate the index of refraction.
   (c) Calculate the wavelength in the shallow water.
4. Straight wave fronts in the deep end of a ripple tank have a wavelength of 2.0 cm and a frequency of 11 Hz. The wave fronts strike the boundary of the shallow section of the tank at an angle of 60° and are refracted at an angle of 30° to the boundary. Calculate the speed of the wave in the deep water and in the shallow water.
5. The speed of a sound wave in cold air (−20°C) is 320 m/s; in warm air (37°C), the speed is 354 m/s. If the wave front in cold air is nearly linear, find θ_R in the warm air if θ_i is 30°.
6. A straight boundary separates two bodies of rock. Longitudinal earthquake waves, travelling through the first body at 7.75 km/s, meet the boundary at an angle of incidence of 20.0°. The wave speed in the second body is 7.72 km/s. Calculate the angle of refraction.
7. Under what conditions do wave rays in water and light rays exhibit total internal reflection?
8. Light travels from air into a certain transparent material of refractive index 1.30. The angle of refraction is 45°. What is the angle of incidence?
9. A ray of light passes from water, with index of refraction 1.33, into carbon disulphide, with index of refraction 1.63. The angle of incidence is 30.0°. Calculate the angle of refraction.