8.3 Magnetic Force on a Conductor

A beam of charged particles moving through a magnetic field in a vacuum experiences a magnetic force. This also occurs if the charged particles are inside a conductor. Under normal circumstances, the charged particles cannot leave the conductor, so the force they experience is transferred to the conductor as the deflected electrons collide with the ions comprising the conducting material.

INVESTIGATION 8.3.1

Force on a Conductor in a Magnetic Field (p. 422)
What factors affect the magnitude of the force on a conductor in a magnetic field? Performing this investigation will allow you to check your answer.

This principle is applied in electric motors and audio speakers. Some countries are even experimenting with *maglev*, or magnetic levitation, trains that use this force to suspend an entire train, moving without wheel-on-rail friction, at speeds over 500 km/h.

The factors that affect the magnitude of the magnetic force on a conductor are similar to those affecting the force on a single charged particle (**Figure 1**). Investigation 8.3.1 in the Lab Activities section at the end of this chapter investigates two of these factors.

Consider a conductor with a current I, placed in a magnetic field of magnitude B. Careful measurements reveal that the force on the conductor is directly proportional to the magnitude of the magnetic field, to the current in the conductor, and to the length of the conductor. In addition, if the angle between the conductor (or current) and the magnetic field lines is θ, then the magnetic force is at a maximum when $\theta = 90°$ and zero when $\theta = 0°$ or $180°$. Hence the magnitude of the magnetic force is directly proportional to $\sin\theta$. Combining these relationships produces an expression for the force acting on the conductor that is very similar to the magnetic force on a single point charge:

$$F \propto IlB\sin\theta \qquad \text{or} \qquad F = kIlB\sin\theta$$

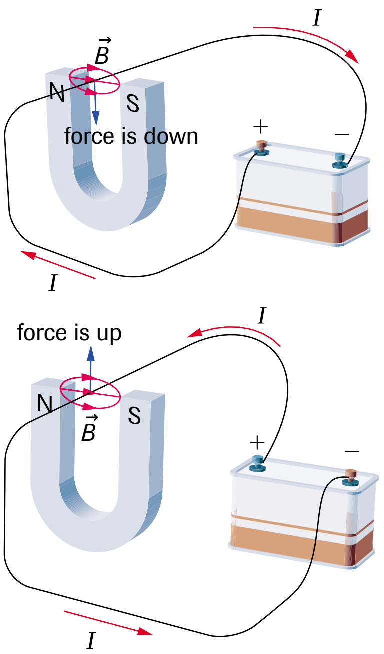


Figure 1
Changing the direction of the current in the conductor changes the direction of the force.

In SI, units have already been chosen for length, current, and force, but if we use this equation as the defining equation for the units of magnetic field strength B, then

$$B = \frac{F}{kIl\sin\theta}$$

The SI unit of magnetic field strength is the tesla (T), defined so that

> 1 T is the magnetic field strength present when a conductor with a current of 1 A and a length of 1 m at an angle of 90° to the magnetic field experiences a force of 1 N; 1 T = 1 N/A·m.

Then in the defining equation for B we have

$$B = \frac{1\text{ N}}{k(1\text{ A})(1\text{ m})(1)}$$

$$1\text{ T} = \frac{1\text{ N/(A·m)}}{k}$$

As a result, the value of k will always be 1 (when appropriate units are used for B, I, F, and l), so the expression for the magnitude of the force on a conductor with a current in it in a magnetic field becomes

$$F = IlB\sin\theta$$

where F is the force on the conductor, in newtons; B is the magnitude of the magnetic field strength, in teslas; I is the current in the conductor, in amperes; l is the length of the conductor in the magnetic field, in metres; and θ is the angle between I and B.

There are other important observations to be made from the use of a conductor in a magnetic field:

- The force on the conductor $\vec{F}$ is in a direction perpendicular to both the magnetic field $\vec{B}$ and the direction of the current I.
- Reversing either the current direction or the magnetic field reverses the direction of the force.

Another simple right-hand rule, equivalent to the one for charges moving in a magnetic field, can be used to determine the relative directions of $\vec{F}$, I, and $\vec{B}$ (**Figure 2**):

Right-Hand Rule for the Motor Principle

If the right thumb points in the direction of the current (flow of positive charge), and the extended fingers point in the direction of the magnetic field, the force is in the direction in which the right palm pushes.

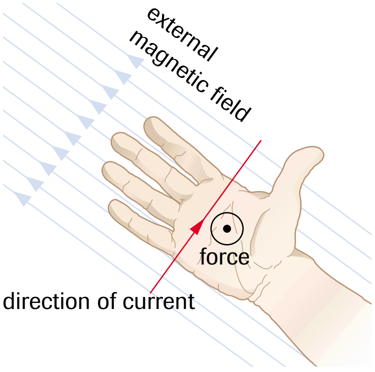


Figure 2
The right-hand rule for determining the direction of the magnetic force

SAMPLE problem

A straight conductor 10.0 cm long with a current of 15 A moves through a uniform 0.60-T magnetic field. Calculate the magnitude of the force on the conductor when the angle between the current and the magnetic field is (a) 90°, (b) 45°, and (c) 0°.

Solution

$I = 15$ A $B = 0.60$ T

$l = 10.0$ cm $F = ?$

In the general case, the magnitude of the force is given by

$$F = IlB \sin\theta$$
$$= (15 \text{ A})(0.60 \text{ T})(0.10 \text{ m}) \sin\theta$$
$$F = (0.90 \text{ N}) \sin\theta$$

(a) when $\theta = 90°$, $\sin\theta = 1$ and $F = 0.90$ N

(b) when $\theta = 45°$, $\sin\theta = 0.707$ and $F = (0.90 \text{ N})(0.707) = 0.64$ N

(c) when $\theta = 0°$, $\sin\theta = 0$ and $F = 0$ N

The magnitude of the force is 0.90 N at $\theta = 90°$, 0.64 N at $\theta = 45°$, and 0 N at $\theta = 0°$. In each case, the direction of the force is given by the right-hand rule for the motor principle.

Practice

Understanding Concepts

1. A wire in the armature of an electric motor is 25 cm long and remains in, and perpendicular to, a uniform magnetic field of 0.20 T. Calculate the force exerted on the wire when it has a current of 15 A.
2. What length of conductor, running at right angles to a 0.033-T magnetic field and with a current of 20.0 A, experiences a force of 0.10 N?
3. A straight 1.0-m wire connects the terminals of a motorcycle battery to a taillight. The motorcycle is parked so that the wire is perpendicular to Earth's magnetic field. The wire experiences a force of magnitude 6.0×10^{-5} N when there is a current of 1.5 A. Calculate the magnitude of Earth's magnetic field at that location.
4. Two electrical line poles are situated 50.0 m apart, one directly north of the other. A horizontal wire running between them carries a DC current of 2.0×10^2 A. If Earth's magnetic field in the vicinity has a magnitude of 5.0×10^{-5} T and the magnetic inclination is 45°, calculate the magnitude of the magnetic force on the wire.

Answers

1. 0.75 N
2. 0.15 m
3. 4.0×10^{-5} T
4. 0.35 N

Deriving the Equation for the Magnetic Force

The connection between the two equations for the magnetic force on a moving point charge and a conductor with a current can be made algebraically as follows. The force on a single point charge is

$$F_M = qvB \sin \theta$$

If there are n such charges in a conductor with length l in the magnetic field and current I, then the net magnetic force on the conductor due to all the charges is

$$F_M = n(qvB \sin \theta) \qquad \text{Equation (1)}$$

We could use this equation to specify the magnetic force on a conductor with a current, but it would not be very useful, since some of the quantities in the equation are difficult to measure directly. However, we can measure the current easily. If n charged particles pass a point in a conductor in time Δt, then the electric current is given by

$$I = \frac{nq}{\Delta t}$$

$$q = \frac{I\Delta t}{n} \qquad \text{Equation (2)}$$

The speed of the charges in the conductor can be found by dividing the distance they travel through the magnetic field, or l, the length of the conductor, by the time Δt:

$$v = \frac{l}{\Delta t} \qquad \text{Equation (3)}$$

Substituting Equations (2) and (3) into Equation (1), and simplifying:

$$F_M = n(qvB \sin \theta)$$

$$= n\left(\frac{I\Delta t}{n}\right)\left(\frac{l}{\Delta t}\right)B \sin \theta$$

$$F_M = IlB \sin \theta$$

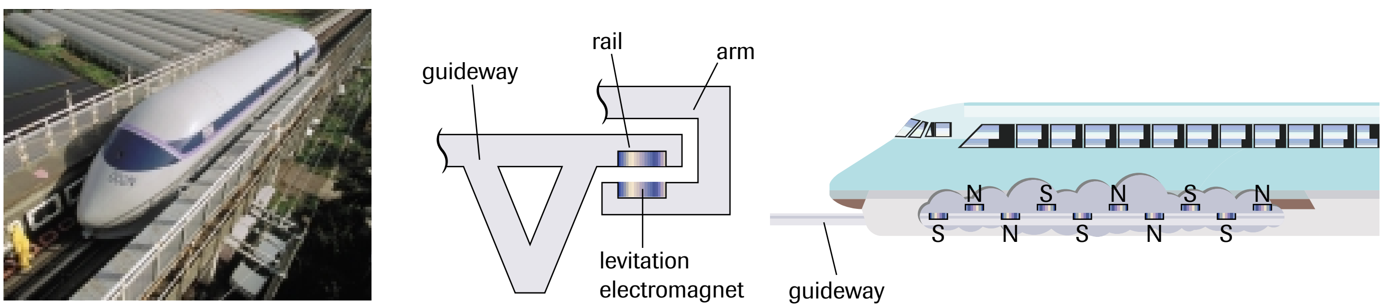
This yields the same equation as that determined experimentally.

Maglev Trains

Maglev trains eliminate wheels-on-rails friction by using electromagnetic force to levitate the cars. Other electromagnets, mounted on the train itself and on a track guideway (**Figure 3**), then move the train forward through attraction between dissimilar poles and repulsion between similar poles. In reality, the train is flying, encountering only fluid friction from the air. Consequently, these trains can reach top speeds that are much higher than is possible with conventional rail. When the time comes to slow the train down, the currents are reversed, so that the magnetic attraction and repulsion are opposite to the direction of motion.

Figure 3
(a) A maglev train
(b) Electromagnets support the maglev train.
(c) Another set of magnets is used to drive the train forward and to slow it down.

(a) **(b)** **(c)**



SUMMARY Magnetic Force on a Conductor

- The magnitude of the force on the conductor F is in a direction perpendicular to both the magnitude of the magnetic field B and the direction of the current I: in SI units, $F = IlB \sin \theta$.
- Reversing either the current direction or the magnetic field reverses the direction of the force.

Section 8.3 Questions

Understanding Concepts

1. A 1.8-m long, straight wire experiences a maximum force of magnitude 1.8 N as it rotates in a uniform magnetic field of magnitude 1.5 T.
   (a) Calculate the angle between the magnetic field and the current in the wire when the force is a maximum.
   (b) Calculate the current in the wire.
   (c) What is the magnitude of the minimum force on the conductor in this magnetic field with the current found in (b)? Explain your answer.
2. A straight horizontal wire 2.0 m long has a current of 2.5 A toward the east. The local magnetic field of Earth is 5.0×10^{-5} T [N, horizontal]. Calculate the magnitude and direction of the force on the conductor.
3. What is the magnitude of the force on a straight 1.2-m wire with a current of 3.0 A and inclined at an angle of 45° to a 0.40-T uniform magnetic field?
4. Examine the experimental setup shown in **Figure 4**.
   (a) Describe what will happen when the circuit is complete.
   (b) How will the following changes, considered one at a time, affect what is observed? (i) The current is increased. (ii) The magnet is inverted. (iii) A stronger magnet is used.
   (c) A student claims that the force on the conductor will decrease if the bar is not horizontal. Discuss the validity of this statement.

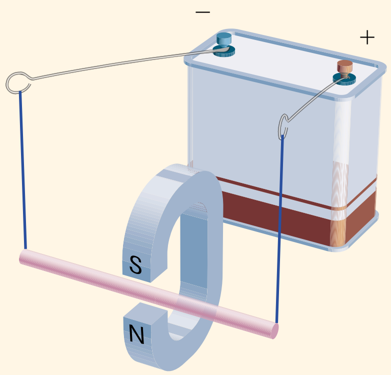


Figure 4

5. Explain how a maglev train uses electromagnets (a) to move forward and (b) to slow down.

Applying Inquiry Skills

6. Some students design an experiment to investigate the properties of Earth's magnetic field in their area using a long, straight wire suspended from string. The teacher tells the class that the accepted value of the magnetic field of Earth is 5.0×10^{-5} T [N, horizontal] in their area. Using a string, the students hang a wire in a north–south direction, connect the wire to a power supply, and begin investigating by varying the amount of current. They intend to measure the angle between the string and the horizontal to determine the force.
   (a) What is wrong with the design of this experiment?
   (b) Can the experimental method be adjusted so that it will work? Explain your answer.
   (c) Try to think of a better design using similar principles.

Making Connections

7. Faraday devised a primitive motor (a "rotator") by immersing wires and bar magnets in mercury (**Figure 5**). There were two versions of this motor, one in which the wire was fixed and a bar magnet rotated around it (left) and another where the magnet was fixed and the wire rotated around it (right). Explain how this motor works.
   (a) Trace the path of the current.
   (b) Explain how this motor works.

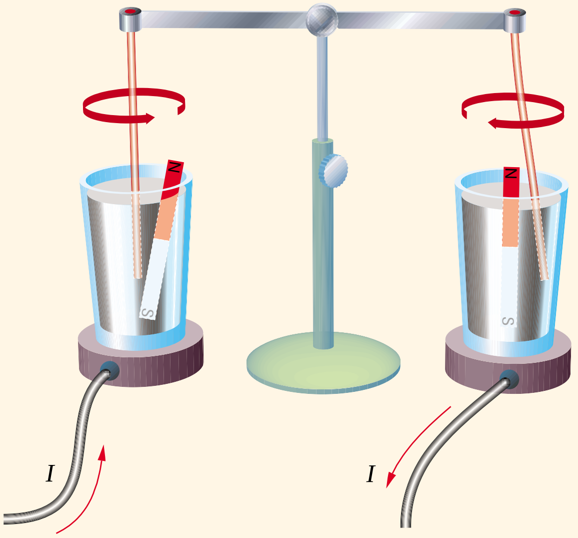


Figure 5