

The Motion of Charged Particles in Electric Fields

7.6

In the Millikan experiment, when charged oil drops were sprayed between two charged parallel plates, the electric field they experienced caused some of them to move slowly in one direction and others to move more slowly or more quickly in the other direction. By adjusting the magnitude of the electric field between the plates, it was possible to get some drops to remain stationary, balanced by the downward force of gravity and the upward force of the electric field.

The Millikan apparatus does not, however, give a true picture of the motion of charged particles in an electric field because of the presence of air and its resistant effect on the motion of such tiny particles, resulting in a constant terminal velocity. Consider a small positive charge q_1 , with a very small mass m , in a vacuum a distance r from a fixed positive charge q_2 . We will assume that the mass m is so small that gravitational effects are negligible.

In **Figure 1**, the charge q_1 experiences a Coulomb force, to the right in this case, whose magnitude is given by

$$F_E = \frac{kq_1q_2}{r^2}$$

If the charged mass is free to move from its original position, it will accelerate in the direction of this electric force (Newton's second law) with an instantaneous acceleration whose magnitude is given by

$$a = \frac{F_E}{m}$$

Describing the subsequent motion of the charged mass becomes difficult because as it begins to move, r increases, causing F_E to decrease, so that a decreases as well. (You can see that a decrease is inevitable, since the acceleration, like the magnitude of the electrical force, is inversely proportional to the square of the distance from the repelling charge.) Motion with a decreasing acceleration poses a difficult analytical problem if we apply Newton's laws directly.

If we use considerations of energy to analyze its motion, it becomes much simpler. As the separation distance between q_1 and q_2 increases, the electric potential energy decreases, and the charged mass q_1 begins to acquire kinetic energy. The law of conservation of energy requires that the total energy remain constant as q_1 moves away from q_2 . Activity 7.6.1 in the Lab Activities section at the end of this chapter provides the opportunity for you to simulate the motion of charged particles in electric fields.

This aspect of the system is illustrated in **Figure 2**. When q_1 is at r_1 , q_1 is at rest, and so the total energy of the charged mass equals its potential energy:

$$E = E'$$

$$E_E + E_K = E_E' + E_K'$$

$$\frac{kq_1q_2}{r} + \frac{1}{2}mv^2 = \frac{kq_1q_2}{r'} + \frac{1}{2}m(v')^2$$

$$\frac{kq_1q_2}{r} - \frac{kq_1q_2}{r'} = \frac{1}{2}m(v')^2 - \frac{1}{2}mv^2$$

$$-\left(\frac{kq_1q_2}{r'} - \frac{kq_1q_2}{r}\right) = \frac{1}{2}mm(v')^2 - 0$$

$$-\Delta E_K = \Delta E_K$$

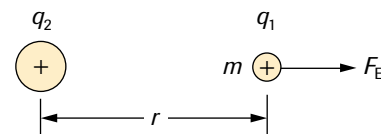


Figure 1

Charge will accelerate in the direction of the electric force according to Newton's second law.

ACTIVITY 7.6.1

The Motion of Charged Particles in Electric Fields (p. 375)

How many ways can you think of to show that the law of conservation of energy governs the motion of charged particles? How do the force, acceleration, and velocity of the charged particles change?

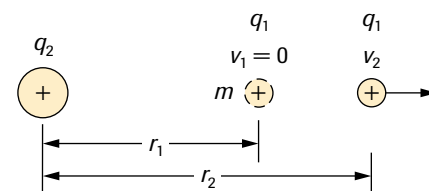


Figure 2

The total energy of the charges remains constant even though the distance between them changes.

The charged particle q_1 thus moves in the electric field of q_2 in such a way that the electric potential energy it loses ($-\Delta E_E$) is equal to the kinetic energy it gains (ΔE_K). A sample problem will illustrate this.

▶ SAMPLE problem 1

Figure 3 shows two small conducting spheres placed on top of insulating pucks. One puck is anchored to the surface, while the other is allowed to move freely on an air table. The mass of the sphere and puck together is 0.15 kg, and the charge on each sphere is $+3.0 \times 10^{-6}$ C and $+5.0 \times 10^{-9}$ C. The two spheres are initially 0.25 m apart. How fast will the sphere be moving when they are 0.65 m apart?

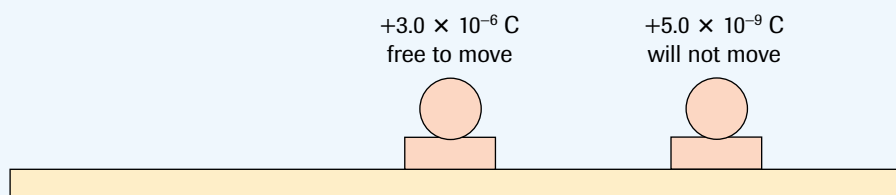


Figure 3

Solution

$$\begin{aligned}
 m &= 0.15 \text{ kg} & r &= 0.25 \text{ m} \\
 q_1 &= +3.0 \times 10^{-6} \text{ C} & r' &= 0.65 \text{ m} \\
 q_2 &= +5.0 \times 10^{-9} \text{ C} & v' &= ?
 \end{aligned}$$

To find the speed, the kinetic energy at $r' = 0.65$ m is required. Since the initial kinetic energy is 0, the final kinetic energy is equal to the change in kinetic energy. To determine the change in kinetic energy, we first find the change in electric potential energy:

$$\begin{aligned}
 E_K' &= \Delta E_K \\
 &= -\Delta E_E \\
 &= -\left(\frac{kq_1q_2}{r'} - \frac{kq_1q_2}{r} \right) \\
 &= -\frac{(9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})(5.0 \times 10^{-9} \text{ C})}{0.65 \text{ m}} + \frac{(9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})(5.0 \times 10^{-9} \text{ C})}{0.25 \text{ m}}
 \end{aligned}$$

$$E_K' = 3.3 \times 10^{-4} \text{ J}$$

We can now find the speed:

$$\begin{aligned}
 v' &= \sqrt{\frac{2E_K'}{m}} \\
 &= \sqrt{\frac{2(0.33 \text{ J})}{0.15 \text{ kg}}} \\
 v' &= 0.07 \text{ m/s}
 \end{aligned}$$

The sphere will be moving with a speed of 0.07 m/s.

When the electric field in which the charged particle is moving is uniform, its motion is much simpler. In a uniform electric field

$$\vec{F}_E = q\vec{E} = \text{constant}$$

Therefore,

$$\vec{a} = \frac{\vec{F}_E}{m} = \text{constant}$$

Thus, the charged particle moves with uniform acceleration. This will be the case for small charged particles (such as ions, electrons, and protons) where gravitational effects are negligible and they are moving between two parallel plates in a vacuum.

The work done by a constant force in the same direction as the displacement is the scalar product of the force and the displacement. In a parallel-plate apparatus with plate separation r , the work done by the electric force in moving a charge q from one plate to the other is

$$\begin{aligned} W &= \vec{F}_E \cdot \vec{r} \\ &= \varepsilon q r \quad (\text{since } \vec{E} \text{ and } \vec{r} \text{ are in the same direction}) \\ &= \frac{\Delta V}{d} q r \\ W &= \Delta V q \end{aligned}$$

This amount of work is equal in magnitude to the change in electric potential energy and the change in kinetic energy of the particle as it moves from one plate to the other. A sample problem will illustrate many of these relationships.

▶ SAMPLE problem 2

The cathode in a typical cathode-ray tube (**Figure 4**), found in a computer terminal or an oscilloscope, is heated, which makes electrons leave the cathode. They are then attracted toward the positively charged anode. The first anode has only a small potential rise while the second is at a large potential with respect to the cathode. If the potential difference between the cathode and the second anode is $2.0 \times 10^4 \text{ V}$, find the final speed of the electron.

Solution

The mass and charge of an electron can be found in Appendix C.

$$\begin{aligned} \Delta V &= 2.0 \times 10^4 \text{ V} & q &= 1.6 \times 10^{-19} \text{ C} \\ m &= 9.1 \times 10^{-31} \text{ kg} & v &= ? \end{aligned}$$

For the free electron,

$$\begin{aligned} -\Delta E_E &= \Delta E_K \\ q\Delta V &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2q\Delta V}{m}} \\ &= \sqrt{\frac{2(1.6 \times 10^{-19} \text{ C})(2.0 \times 10^4 \text{ V})}{9.1 \times 10^{-31} \text{ kg}}} \\ v &= 8.4 \times 10^7 \text{ m/s} \end{aligned}$$

The final speed of the electron is $8.4 \times 10^7 \text{ m/s}$.

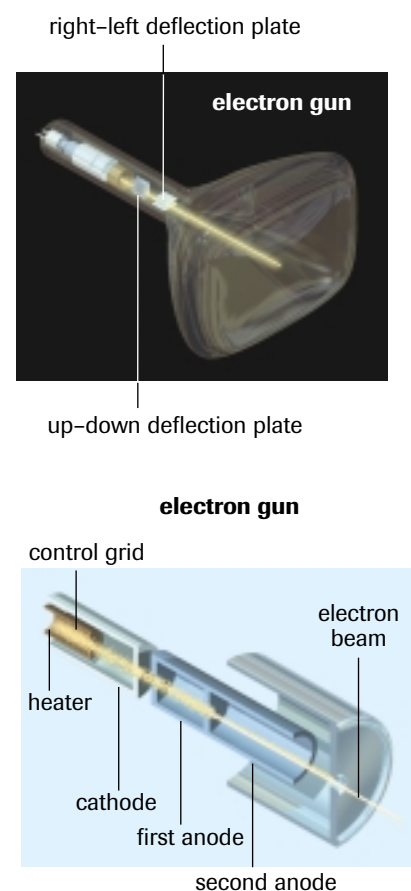


Figure 4
A typical cathode ray tube

Answers

- $1.6 \times 10^{-3} \text{ C}$
- (a) $4.4 \times 10^5 \text{ m/s}$
(b) $3.1 \times 10^5 \text{ m/s}$
- (a) $3.2 \times 10^{-4} \text{ J}$
(b) $6.4 \times 10^{-4} \text{ N}$
(c) $3.2 \times 10^{-4} \text{ J}$
(d) 8.0 m/s
- $2.3 \times 10^{-16} \text{ J}; 1.6 \times 10^7 \text{ m/s}$
- $2.6 \times 10^5 \text{ V}$

Note that this speed is not negligible compared to the speed of light (about 28%). The high velocity is due to the high electric potential difference and very low mass of the electron. This analysis becomes unsatisfactory for electrons accelerated by potentials of more than a few thousand volts because they begin to show relativistic effects (see Chapter 11).

Practice

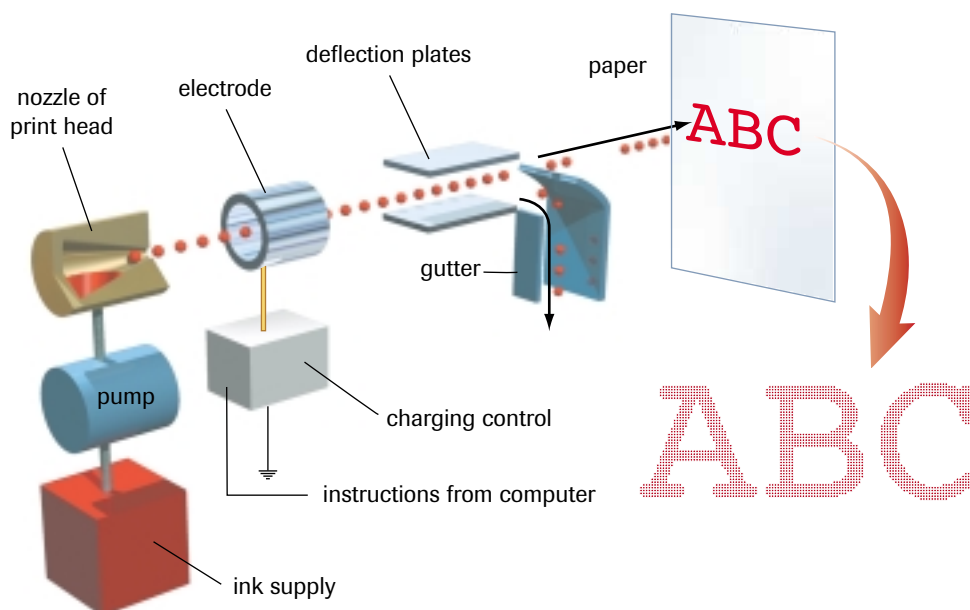
Understanding Concepts

- The potential difference between two parallel plates is $1.5 \times 10^2 \text{ V}$. You would have to do 0.24 J of work if you were to move a small charge, in opposition to the electric force, from one plate to the other. Calculate the magnitude of the charge.
- An α particle has a positive charge of $2e$ and a mass of $6.6 \times 10^{-27} \text{ kg}$. With what velocity would an α particle reach the negative plate of a parallel-plate apparatus with a potential difference of $2.0 \times 10^3 \text{ V}$
 - if it started from rest at the positive plate
 - if it started from rest at a point halfway between the plates
- A pith ball of mass $1.0 \times 10^{-5} \text{ kg}$ with a positive charge of $4.0 \times 10^{-7} \text{ C}$ is slowly pulled at a constant speed by a string a distance of 50.0 cm through a potential difference of $8.0 \times 10^2 \text{ V}$. It is then released from rest and “falls” back to its original position.
 - Calculate the work done by the string in moving the pith ball.
 - Calculate the magnitude of the average force required to do this work.
 - Calculate the kinetic energy of the pith ball when it arrives back at its original position.
 - Calculate the speed of the pith ball just as it reaches its final position.
- Two electrons are held, at rest, $1.0 \times 10^{-12} \text{ m}$ apart, then released. With what kinetic energy and speed is each moving when they are a “large” distance apart?
- What is the potential difference required to accelerate a deuteron of mass $3.3 \times 10^{-27} \text{ kg}$ and charge $1.6 \times 10^{-19} \text{ C}$ from rest to a speed of $5.0 \times 10^6 \text{ m/s}$?

Inkjet Printers

An inkjet printer uses charged parallel plates to deflect droplets of ink headed toward the paper. One type of inkjet print head ejects a thin stream of small ink droplets while it is moving back and forth across the paper (**Figure 5**). Typically, a small nozzle breaks up the stream of ink into droplets $1 \times 10^{-4} \text{ m}$ in diameter at a rate of 150 000 droplets per second and moving at 18 m/s . When the print head passes over an area of the paper that should have no ink on it, the charging electrode is turned on, creating an electric field between the print head and the electrode. The ink droplets acquire an electric charge by induction (the print head itself is grounded), and the deflection plates prevent the charged droplets from reaching the paper by diverting them into a gutter. When the print head passes over an area where ink is to be placed, the electrode is turned off and the uncharged ink droplets pass through the deflection plates in a straight line, landing on the paper. The movement of the print head across the paper determines the form of what appears on the paper.

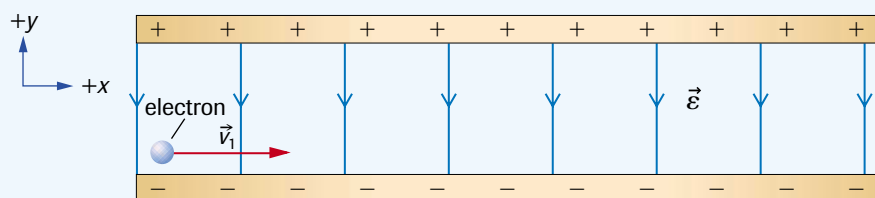
Another type of inkjet printer places a charge on all the ink droplets. Two parallel plates are charged proportionately to a control signal from a computer, steering the droplets vertically as the paper moves horizontally (**Figure 6**). In the areas where no ink is to be placed, the droplets are deflected into a gutter as in the previous design.

**Figure 5**

The print head emits a steady flow of ink droplets. Uncharged ink droplets pass straight through the deflection plates to form letters. Charged droplets are deflected into the gutter when the paper is to be blank. Notice that the evidence of the ink drops can be seen when the letters are enlarged.

▶ SAMPLE problem 3

An electron is fired horizontally at 2.5×10^6 m/s between two horizontal parallel plates 7.5 cm long, as shown in **Figure 7**. The magnitude of the electric field is 130 N/C. The plate separation is great enough to allow the electron to escape. Edge effects and gravitation are negligible. Find the velocity of the electron as it escapes from between the plates.

**Figure 7**

Solution

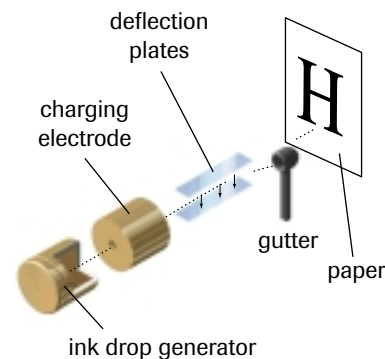
$$q = 1.6 \times 10^{-19} \text{ C} \quad \epsilon = 130 \text{ N/C}$$

$$\vec{v}_1 = 2.5 \times 10^6 \text{ m/s [horizontally]} \quad \vec{v}_2 = ?$$

$$l = 7.5 \text{ cm}$$

Note that \vec{v}_2 has two components, v_{2x} and v_{2y} .

The magnitude of the electric field is constant, and the field is always straight down; therefore, the electric force on the electron is constant, meaning its acceleration is constant and vertical. We can break the problem up into a horizontal part, which involves just uniform motion, and a vertical part, which involves constant acceleration. There is no need to use energy here (although you could). We will use forces and kinematics instead.

**Figure 6**

Charged droplets are deflected vertically to form the letters.

The magnitude of the net force on the electron is

$$\begin{aligned} F_{\text{net}} &= F_E \\ &= q\mathcal{E} \\ F_{\text{net}} &= e\mathcal{E} \end{aligned}$$

Therefore, the magnitude of the acceleration of the electron is given by

$$\begin{aligned} a_y &= \frac{e\mathcal{E}}{m_e} \\ &= \frac{(-1.6 \times 10^{-19} \text{ C})(-130 \text{ N/C})}{9.1 \times 10^{-31} \text{ kg}} \\ a_y &= 2.3 \times 10^{13} \text{ m/s}^2 \\ \vec{a} &= 2.3 \times 10^{13} \text{ m/s}^2 \text{ [up]} \end{aligned}$$

This acceleration is upward because the electron is repelled by the lower plate and attracted to the upper plate, as indicated by the direction of the electric field.

The initial velocity in the vertical direction is zero. To find the final vertical velocity, it suffices to find the time spent on the vertical movement, which equals the time spent passing through the plates. That time, in turn, is given to us by the horizontal velocity and the width of the plates. (Remember that the electron moves with uniform motion in the horizontal direction.)

$$\begin{aligned} \Delta t &= \frac{\Delta l}{v_x} \\ &= \frac{7.5 \times 10^{-2} \text{ m}}{2.5 \times 10^6 \text{ m/s}} \\ \Delta t &= 3.0 \times 10^{-8} \text{ s} \end{aligned}$$

The final vertical component of the velocity is

$$\begin{aligned} v_{2y} &= v_{1y} + a_y \Delta t \\ &= 0 + (2.3 \times 10^{13} \text{ m/s}^2)(3.0 \times 10^{-8} \text{ s}) \\ v_{2y} &= 6.9 \times 10^5 \text{ m/s [up]} \end{aligned}$$

Adding these two components head-to-tail and using the Pythagorean theorem, as in **Figure 8**, gives

$$\begin{aligned} v_2 &= \sqrt{(6.9 \times 10^5 \text{ m/s})^2 + (2.5 \times 10^6 \text{ m/s})^2} \\ v_2 &= 2.6 \times 10^6 \text{ m/s} \end{aligned}$$

We determine the angle upward from the horizontal:

$$\begin{aligned} \theta &= \tan^{-1} \left(\frac{6.9 \times 10^5 \text{ m/s}}{2.5 \times 10^6 \text{ m/s}} \right) \\ \theta &= 15^\circ \end{aligned}$$

The final velocity is $2.6 \times 10^6 \text{ m/s}$ [right 15° up from the horizontal].

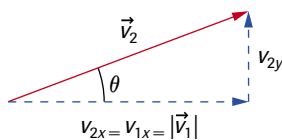


Figure 8

SUMMARY

The Motion of Charged Particles in Electric Fields

- A charged particle in a uniform electric field moves with uniform acceleration.
- From conservation principles, any changes to a particle's kinetic energy result from corresponding changes to its electric potential energy (when moving in any electric field and ignoring any gravitational effects).

Section 7.6 Questions

Understanding Concepts

- Figure 9** shows a common technique used to accelerate electrons, usually released from a hot filament from rest. The small hole in the positive plate allows some electrons to escape providing a source of fast-moving electrons for experimentation. The magnitude of the potential difference between the two plates is 1.2×10^3 V. The distance between the plates is 0.12 m.
 - At what speed will an electron pass through the hole in the positive plate?
 - Is the electron pulled back to the positive plate once it passes through the hole? Explain your answer.
 - How could the apparatus be modified to accelerate protons?
 - Find the speed of the emerging protons in an appropriate apparatus.

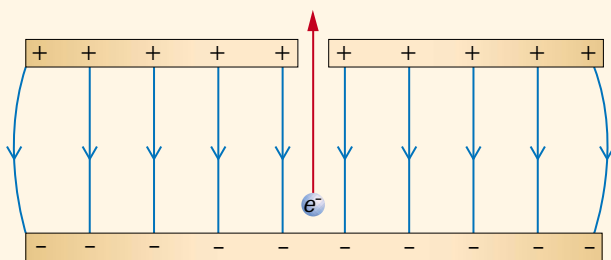


Figure 9

- An electron is accelerated through a uniform electric field of magnitude 2.5×10^2 N/C with an initial speed of 1.2×10^6 m/s parallel to the electric field, as shown in **Figure 10**.
 - Calculate the work done on the electron by the field when the electron has travelled 2.5 cm in the field.
 - Calculate the speed of the electron after it has travelled 2.5 cm in the field.
 - If the direction of the electric field is reversed, how far will the electron move into the field before coming to rest?

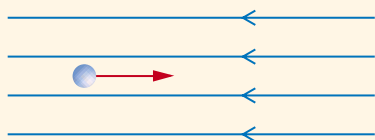


Figure 10

- Two electrons are fired at 3.5×10^6 m/s directly at each other.
 - Calculate the smallest possible distance between the two electrons.
 - Is it likely that two electrons in this situation will actually get this close to each other if the experiment is performed? Explain your answer.

- Ernest Rutherford in his lab at McGill University, Montreal, fired α particles of mass 6.64×10^{-27} kg at gold foil to investigate the nature of the atom. What initial energy must an α particle (charge $+2e$) have to come within 4.7×10^{-15} m of a gold nucleus (charge $+79e$) before coming to rest? This distance is approximately the radius of the gold nucleus.
- An electron with a velocity of 3.00×10^6 m/s [horizontally] passes through two horizontal parallel plates, as in **Figure 11**. The magnitude of the electric field between the plates is 120 N/C. The plates are 4.0 cm across. Edge effects in the field are negligible.
 - Calculate the vertical deflection of the electron.
 - Calculate the vertical component of the final velocity.
 - Calculate the angle at which the electron emerges.

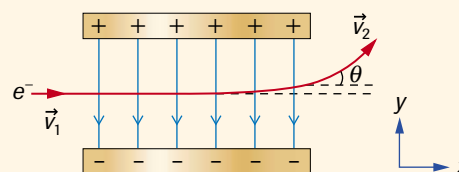


Figure 11

Making Connections

- An oscilloscope is a device that deflects a beam of electrons vertically and horizontally across a screen. Its many applications (e.g., electrocardiography) rely on its high sensitivity to electric potential differences.
 - How can we deflect the beam of electrons in the oscilloscope vertically, whether upward or downward, by various amounts?
 - How can we deflect the beam horizontally from left to right, and then horizontally again, more quickly, from right to left?
 - If an oscilloscope is to be used to monitor the heartbeat of a patient, what will determine the amount of vertical and horizontal deflection of the electrons?
 - Design an oscilloscope that can deflect the electrons 5° vertically up and down and 10° horizontally left and right. Discuss the changes in potential that will be required in its operation if heartbeats are to be measured.
- Two oppositely charged objects, of some nonnegligible mass m_1 , are placed in deep interstellar space, in a region essentially free of gravitational forces from other objects.
 - Discuss the resulting motion of the two objects and the energy transformations.
 - Compare the motion of these two charged objects with the motion of two neutral objects, of some nonnegligible mass m_2 , under similar circumstances.