Sec. 7.6 - Motion of Charged Particles

Learning Goal: By the end of today, I will be able to apply kinematics, force, and energy principles to solve for the motion of charged particles.

Case 1 - Motion with Particles

In **Figure 1**, the charge q_1 experiences a Coulomb force, to the right in this case, whose magnitude is given by

$$F_E = \frac{kq_1q_2}{r^2}$$

If the charged mass is free to move from its original position, it will accelerate in the direction of this electric force (Newton's second law) with an instantaneous acceleration whose magnitude is given by

$$a = \frac{F_E}{m}$$

F=ma

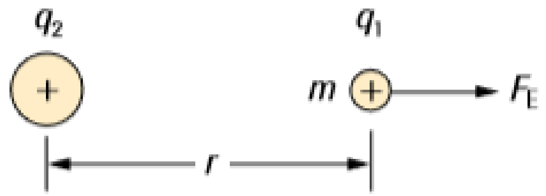


Figure 1
Charge will accelerate in the direction of the electric force according to Newton's second law.

As q_1 moves away, "r" changes as does "a", this makes it difficult to describe using our kinematics equations.

An energy analysis is much easier to use in this case.

$$E = E' \quad \text{(total energy is constant)}$$

$$E_E + E_K = E_E' + E_K'$$

$$\frac{kq_1q_2}{r} + \frac{1}{2}mv^2 = \frac{kq_1q_2}{r'} + \frac{1}{2}m(v')^2$$

$$\frac{kq_1q_2}{r} - \frac{kq_1q_2}{r'} = \frac{1}{2}m(v')^2 - \frac{1}{2}mv^2$$

$$-\left(\frac{kq_1q_2}{r'} - \frac{kq_1q_2}{r}\right) = \frac{1}{2}mm(v')^2 - 0$$

$$-\Delta E_E = \Delta E_K$$

typo in text

change in potential energy is equal to the change in kinetic energy

Figure 3 shows two small conducting spheres placed on top of insulating pucks. One puck is anchored to the surface, while the other is allowed to move freely on an air table. The mass of the sphere and puck together is 0.15 kg, and the charge on each sphere is $+3.0 \times 10^{-6}$ C and $+5.0 \times 10^{-9}$ C. The two spheres are initially 0.25 m apart. How fast will the sphere be moving when they are 0.65 m apart?

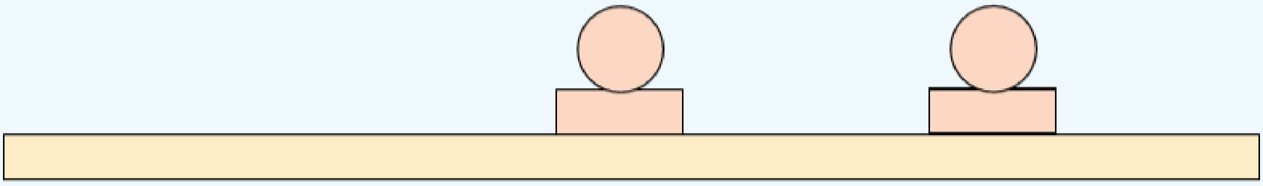


Figure 3

$$E_E + E_K = E_E' + E_K'$$

Solution

$m = 0.15$ kg $r = 0.25$ m

$q_1 = +3.0 \times 10^{-6}$ C $r' = 0.65$ m

$q_2 = +5.0 \times 10^{-9}$ C $v' = ?$

k=9.0x10^9

$$E_E - E_E' = E_K' - E_K$$

$$\frac{kq_1q_2}{r} - \frac{kq_1q_2}{r'} = \frac{1}{2}m(v')^2 - \frac{1}{2}mv^2$$

Apr 29-1:23 PM

Case 2 - Particles in Uniform Electric Fields

When the electric field in which the charged particle is moving is uniform, its motion is much simpler. In a uniform electric field

$\vec{F}_E = q\vec{\varepsilon} =$ constant

Therefore,

$\vec{a} = \frac{\vec{F}_E}{m} =$ constant

uniform acceleration - kinematics work again

The work done by a constant force in the same direction as the displacement is the scalar product of the force and the displacement. In a parallel-plate apparatus with plate separation r, the work done by the electric force in moving a charge q from one plate to the other is

$$W = \vec{F}_E \cdot \vec{r}$$

$$= \varepsilon q r$$

(since $\vec{\varepsilon}$ and $\vec{r}$ are in the same direction)

$$\varepsilon = \frac{\Delta V}{d}$$

$$= \frac{\Delta V}{d} q r$$

$$W = \Delta V q$$

r = d for a parallel-plate apparatus (cancels)

or

$$\Delta E = \Delta V \cdot q$$

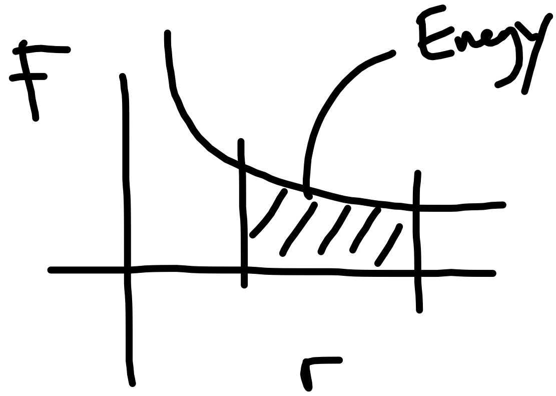
Apr 29-1:26 PM

Spheer

$\Delta V = V_B - V_A$

$= \frac{kq}{r_B} - \frac{kq}{r_A}$

$= kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$

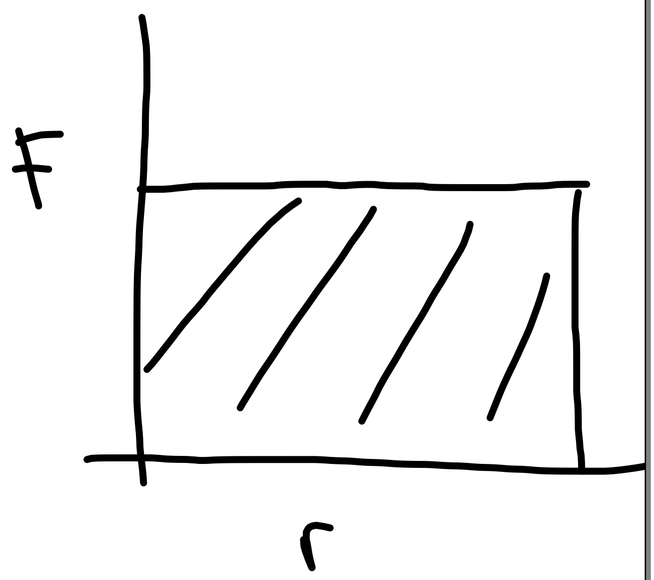


Uniform

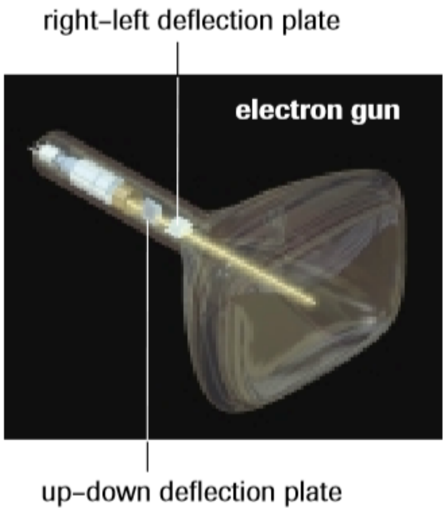
$\Delta V = \mathcal{E} r$

$\Delta V = \frac{F}{q} \cdot r$

$q\Delta V = F \cdot r$



Example



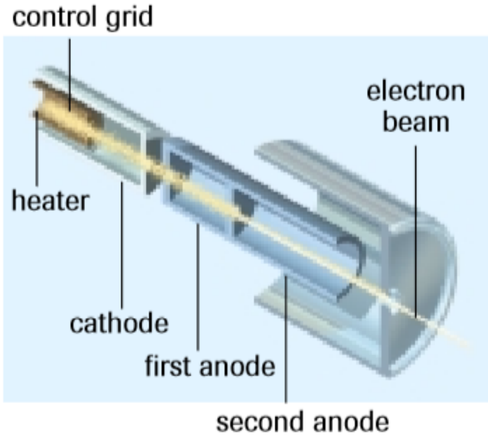


Figure 4
A typical cathode ray tube

The cathode in a typical cathode-ray tube (**Figure 4**), found in a computer terminal or an oscilloscope, is heated, which makes electrons leave the cathode. They are then attracted toward the positively charged anode. The first anode has only a small potential rise while the second is at a large potential with respect to the cathode. If the potential difference between the cathode and the second anode is 2.0×10^4 V, find the final speed of the electron.

Solution

The mass and charge of an electron can be found in Appendix C.

$\Delta V = 2.0 \times 10^4 \text{ V}$ $\quad q = 1.6 \times 10^{-19} \text{ C}$

$m = 9.1 \times 10^{-31} \text{ kg}$ $\quad v = ?$

For the free electron,

$$-\Delta E_E = \Delta E_K$$

$$q\Delta V = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

$$= \sqrt{\frac{2(1.6 \times 10^{-19} \text{ C})(2.0 \times 10^4 \text{ V})}{9.1 \times 10^{-31} \text{ kg}}}$$

$$v = 8.4 \times 10^7 \text{ m/s}$$

The final speed of the electron is 8.4×10^7 m/s.

page 368 - read inkjet printing

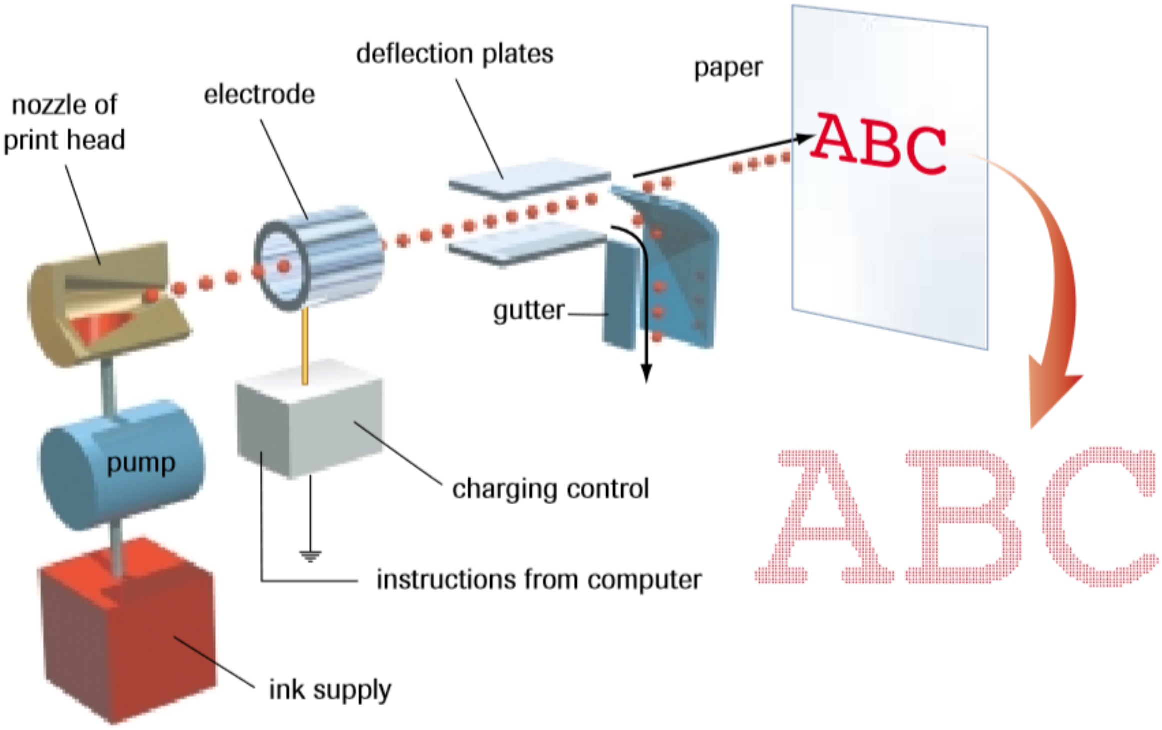


Figure 5
The print head emits a steady flow of ink droplets. Uncharged ink droplets pass straight through the deflection plates to form letters. Charged droplets are deflected into the gutter when the paper is to be blank. Notice that the evidence of the ink drops can be seen when the letters are enlarged.

An electron is fired horizontally at 2.5×10^6 m/s between two horizontal parallel plates 7.5 cm long, as shown in **Figure 7**. The magnitude of the electric field is 130 N/C. The plate separation is great enough to allow the electron to escape. Edge effects and gravitation are negligible. Find the velocity of the electron as it escapes from between the plates.

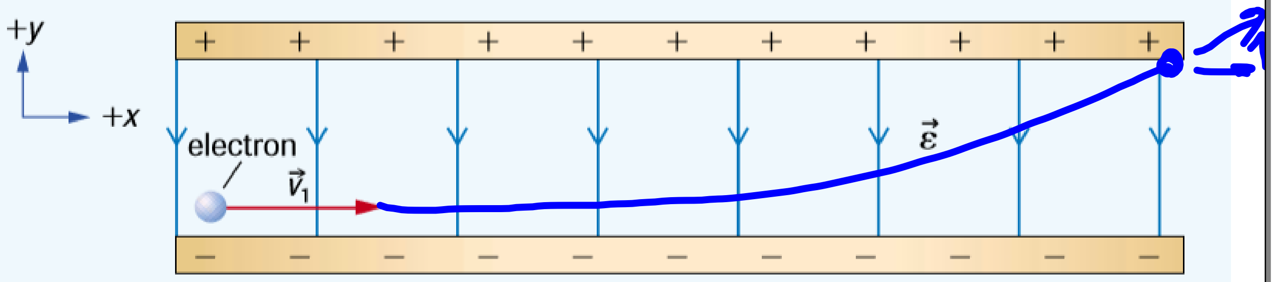


Figure 7

Solution

$q = 1.6 \times 10^{-19}$ C

$\vec{v}_1 = 2.5 \times 10^6$ m/s [horizontally]

$l = 7.5$ cm

$\varepsilon = 130$ N/C

$\vec{v}_2 = ?$

$m_e = 9.1 \times 10^{-31}$ kg

Note that $\vec{v}_2$ has two components, v_{2x} and v_{2y}.

- horizontal velocity will remain the same (think projectile motion)
- vertical velocity is subject to acceleration from an unbalanced force

Vertical Acceleration

$F_{net} = F_E$

$F_{net} = q\varepsilon$

$F_{net} = m a_y$

$$a_y = \frac{q\varepsilon}{m}$$

Time in between plates $v = d/t$

Vertical Velocity

Resultant and Angle

SUMMARY ***The Motion of Charged Particles in Electric Fields***

- A charged particle in a uniform electric field moves with uniform acceleration.
- From conservation principles, any changes to a particle's kinetic energy result from corresponding changes to its electric potential energy (when moving in any electric field and ignoring any gravitational effects).

Homework

Read 365 - 371

page 368 #1, 2, 3, 4, 5

page 371 #1, 2, 5

Apr 29-1:46 PM