# Gravitational Potential Energy in General 6.3

To explore such concepts as how much energy a space probe needs to escape from Earth's gravity, we must expand on the topic of gravitational potential energy, which we examined in Section 4.3 for objects at Earth's surface. To calculate the change in gravitational potential energy for a mass that undergoes a vertical displacement near Earth's surface, we developed the following equation:

$$\Delta E_g = mg\Delta y$$

where $\Delta E_g$ is the change in gravitational potential energy, $m$ is the mass, $g$ is the magnitude of the gravitational field constant, and $\Delta y$ is the vertical displacement. This equation is accurate provided that the magnitude of the gravitational field strength $g$ remains reasonably constant during $\Delta y$. This means that we can be fairly accurate for vertical displacements of a few hundred kilometres but inaccurate for vertical displacements beyond that.

The more general problem, however, is to develop an expression for the gravitational potential energy of a system of any two masses a finite distance apart. Recall that the law of universal gravitation is given by

$$F_G = \frac{GMm}{r^2}$$

where $F_G$ is the magnitude of the force of gravitational attraction between any two objects, $M$ is the mass of one object, $m$ is the mass of the second object, and $r$ is the distance between the centres of the two spherical objects (**Figure 1**). To increase the separation of the two masses from $r_1$ to $r_2$ requires work to be done to overcome their force of attraction, just as in stretching a spring. As a result of this work being done, the gravitational potential energy of the system increases. Notice that the work done to change the separation from $r_1$ to $r_2$ is equal to the change in gravitational potential energy from $r_1$ to $r_2$. This applies to an isolated system in which the law of conservation of energy holds.

However, recall that the work done by a varying force is equal to the area under the force-displacement graph for the interval. The force-separation graph, with the shaded area representing the work done to increase the separation from $r_1$ to $r_2$, is shown in **Figure 2**.

You may not recognize this area as a well-known geometric shape, and you have no simple equation to determine its area. The mathematics for an inverse square relationship involves calculus and is beyond the scope of this book. However, instead of using the arithmetic average of $F_1$ and $F_2$, we can use the geometric average $\sqrt{F_1F_2}$, to produce an accurate result. Thus, to determine the area under the force-separation graph from $r_1$ to $r_2$:

$$\text{area} = \sqrt{F_1F_2}\,(r_2 - r_1)$$

$$= \sqrt{\left(\frac{GMm}{r_1^2}\right)\left(\frac{GMm}{r_2^2}\right)}\,(r_2 - r_1)$$

$$= \frac{GMm}{r_1r_2}(r_2 - r_1)$$

$$\text{area} = \frac{GMm}{r_1} - \frac{GMm}{r_2}$$

This area represents the work done in changing the separation of the two masses from $r_1$ to $r_2$ and is an expression for the resulting change in gravitational potential energy.

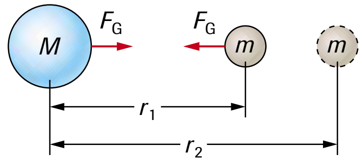


**Figure 1**
The two masses, $M$ and $m$, are moved from a separation $r_1$ to a separation $r_2$ by a force that just overcomes the gravitational attraction between the masses at every point along the path. The masses are at rest at both positions.

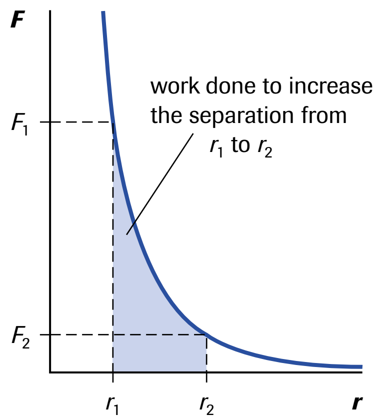


**Figure 2**
In this force-separation graph, the area under the curve for the interval $r_1$ to $r_2$ is equal to the work done in increasing the separation of the two masses.

**DID YOU KNOW?**

**Newton's New Mathematics**
Newton saw the need to accurately calculate areas, such as the area shown in **Figure 2**. To do so, he developed a whole new branch of mathematics called calculus. At approximately the same time, independently of Newton, Gottfried Wilhelm Leibniz (1646–1716), a German natural philosopher, also developed calculus.

**LEARNING TIP**

**Energy of a System**

The equation for the gravitational potential energy between two masses gives the potential energy of the system, such as an Earth-satellite system. Despite this fact, we often say that the potential energy is associated with only the smaller object, in this case the satellite.

Thus, $\Delta E_g = E_2 - E_1$

$$\Delta E_g = \frac{GMm}{r_1} - \frac{GMm}{r_2}$$

$$\Delta E_g = \left(-\frac{GMm}{r_2}\right) - \left(-\frac{GMm}{r_1}\right)$$

where $\Delta E_g$ is the change in gravitational potential energy in joules. The term involving $r_1$ is changed to negative, which places the term involving $r_2$ first. Thus, the first term in the expression depends only on $r_2$ and the second term only on $r_1$. As $r_2 \rightarrow \infty$, $E_{g2} \rightarrow 0$. Since $m$ is now outside the gravitational field of $M$, the expression simplifies to

$$\Delta E_g = 0 - E_{g1}$$

$$= -\left(-\frac{GMm}{r_1}\right)$$

Thus, $\Delta E_g = \frac{GMm}{r_1}$ or $E_g = -\frac{GMm}{r}$

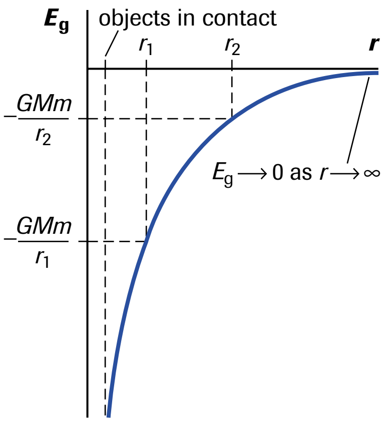


**Figure 3**
A graph of gravitational potential energy $E_g$ as a function of $r$ for two masses $M$ and $m$

Note that $r$ is the distance between the centres of two objects and that the expression is not valid at points inside either object. As with the law of universal gravitation, objects must be spherical or far enough apart that they can be considered as small particles.

The equation for $E_g$ always produces a negative value. As $r$ increases—that is, as the masses get farther apart—$E_g$ increases by becoming less negative. Also, as $r \rightarrow \infty$, $E_g \rightarrow 0$. The zero value of gravitational potential energy between two masses occurs when they are infinitely far apart; this is a reasonable assumption since the point at which $r = \infty$ is the only point when the masses will have no gravitational attraction force between them. A graph of $E_g$ as a function of $r$ for two masses is shown in **Figure 3**.

We can show that the equation for the change in gravitational potential energy at Earth's surface is just a special case of the general situation. Near Earth's surface

$r_1 = r_E$ and $r_2 = r_E + \Delta y$

so that $r_1 r_2 \approx r_E^2$ (because $\Delta y << r_E$ close to the surface of Earth)

and $\Delta y = r_2 - r_1$

$$\text{Thus, } \Delta E_g = \left(-\frac{GMm}{r_2}\right) - \left(-\frac{GMm}{r_1}\right)$$

$$= \frac{GMm}{r_1 r_2}(r_2 - r_1)$$

$$\Delta E_g \approx \frac{GMm\Delta y}{r_E^2}$$

However, from the law of universal gravitation,

$$F_G = \frac{GMm}{r_E^2} = mg$$

Therefore, $\Delta E_g \approx mg\Delta y$ for a mass near the surface of Earth.

## SAMPLE problem 1

What is the change in gravitational potential energy of a 64.5-kg astronaut, lifted from Earth's surface into a circular orbit of altitude $4.40 \times 10^2$ km?

### Solution

$G = 6.67 \times 10^{-11}$ N·m$^2$/kg$^2$ $\quad m = 64.5$ kg

$M_E = 5.98 \times 10^{24}$ kg $\quad r_E = 6.38 \times 10^6$ m

$$r_2 = r_E + 4.40 \times 10^2 \text{ km}$$
$$= 6.38 \times 10^6 \text{ m} + 4.40 \times 10^5 \text{ m}$$
$$r_2 = 6.82 \times 10^6 \text{ m}$$

On Earth's surface,

$$E_{g1} = -\frac{GM_E m}{r_E}$$
$$= -\frac{(6.67 \times 10^{-11} \text{ N·m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(64.5 \text{ kg})}{6.38 \times 10^6 \text{ m}}$$
$$E_{g1} = -4.03 \times 10^9 \text{ J}$$

In orbit,

$$E_{g2} = -\frac{GM_E m}{r_2}$$
$$= -\frac{(6.67 \times 10^{-11} \text{ N·m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(64.5 \text{ kg})}{6.82 \times 10^6 \text{ m}}$$
$$E_{g2} = -3.77 \times 10^9 \text{ J}$$

$$\Delta E_g = E_{g2} - E_{g1}$$
$$= (-3.77 \times 10^9 \text{ J}) - (-4.03 \times 10^9 \text{ J})$$
$$\Delta E_g = 2.6 \times 10^8 \text{ J}$$

The change in gravitational potential energy is $2.6 \times 10^8$ J.

Notice that even though the values for the astronaut's gravitational potential energy are negative at both positions, the change in $E_g$, when the astronaut's distance from Earth increases, is positive, indicating an increase in gravitational potential energy. Note also that, even for an altitude of $4.40 \times 10^2$ km, the approximation assuming a constant value of $g$ is quite good.

$$\Delta E_g \approx mg\Delta y$$
$$= (64.5 \text{ kg})(9.80 \text{ N/kg})(4.40 \times 10^5 \text{ m})$$
$$\Delta E_g \approx 2.8 \times 10^8 \text{ J}$$

## Practice

### Understanding Concepts

**1.** Determine the gravitational potential energy of the Earth–Moon system, given that the average distance between their centres is $3.84 \times 10^5$ km, and the mass of the Moon is 0.0123 times the mass of Earth.

**2.** (a) Calculate the change in gravitational potential energy for a 1.0-kg mass lifted $1.0 \times 10^2$ km above the surface of Earth.
(b) What percentage error would have been made in using the equation $\Delta E_g = mg\Delta y$ and taking the value of $g$ at Earth's surface?
(c) What does this tell you about the need for the more exact treatment in most normal Earth-bound problems?

**3.** With what initial speed must an object be projected vertically upward from the surface of Earth to rise to a maximum height equal to Earth's radius? (Neglect air resistance.) Apply energy conservation.

**Answers**

1. $-7.64 \times 10^{28}$ J
2. (a) $1.0 \times 10^6$ J
   (b) 2%
3. $7.91 \times 10^3$ m/s

**Answers**

4. (a) $1.8 \times 10^{32}$ J
   (b) perihelion; $1.8 \times 10^{32}$ J
5. (a) $-1.56 \times 10^{10}$ J; $-1.04 \times 10^{10}$ J
   (b) $5.2 \times 10^{9}$ J
   (c) $5.2 \times 10^{9}$ J

**LEARNING TIP**

**"Apo" and "Peri"**
The prefix "apo" means away from and "geo" represents Earth, so apogee refers to the point in a satellite's elliptical orbit farthest from Earth. Furthermore, since "helios" represents the Sun, aphelion refers to the point in a planet's elliptical orbit farthest from the Sun. The prefix "peri" means around, so perihelion refers to the point in a planet's orbit closest to the Sun. What does perigee mean?

**4.** The distance from the Sun to Earth varies from $1.47 \times 10^{11}$ m at perihelion (closest approach) to $1.52 \times 10^{11}$ m at aphelion (farthest distance away).
   (a) What is the maximum change in the gravitational potential energy of Earth during one orbit of the Sun?
   (b) At what point in its orbit is Earth moving the fastest? What is its maximum change in kinetic energy during one orbit? (Think about energy conservation.)

**Making Connections**

**5.** A satellite of mass $5.00 \times 10^{2}$ kg is in a circular orbit of radius $2r_E$ around Earth. Then it is moved to a circular orbit of radius $3r_E$.
   (a) Determine the satellite's gravitational potential energy in each orbit.
   (b) Determine the change in gravitational potential energy from the first orbit to the second orbit.
   (c) Determine the work done in moving the satellite from the first orbit to the second orbit. Apply energy conservation.

## Escape from a Gravitational Field

We have seen that any two masses have a gravitational potential energy of $E_g = -\frac{GMm}{r}$ at a separation distance $r$. The negative value of this potential energy is characteristic of a *potential well*, a name derived from the shape of the graph of the gravitational potential energy as a function of separation distance (**Figure 4**).

For example, a rocket at rest on Earth's surface has the value of $E_g$, given by point A on the graph in **Figure 4**. Since the kinetic energy $E_K$ of the rocket is zero, its total energy $E_T$ would also be represented by point A, and the rocket would not leave the ground. However, suppose the rocket is launched at a speed such that its kinetic energy is represented by the distance AB on the graph. Now its total energy $E_T = E_g + E_K$ is represented by point B, and the rocket begins to rise. As its altitude increases, $E_g$ increases

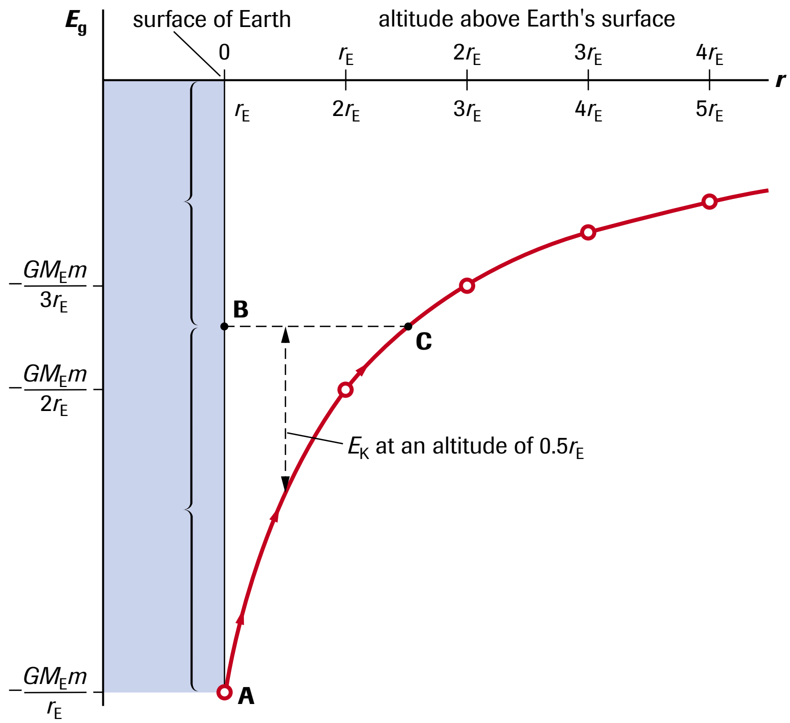


**Figure 4**
This graph of the gravitational potential energy as a function of the altitude above Earth's surface illustrates Earth's potential well.

along the curve AC and $E_T$ remains constant along the line BC. The kinetic energy decreases and, at any point, is given by the length of the vertical line from the curve to the horizontal line BC. When the rocket reaches an altitude corresponding to point C, $E_K$ has decreased to zero, and the rocket can go no higher. Instead, it falls back down, with $E_K$ and $E_g$ governed by the same constraints as on the upward trip.

It is an interesting exercise to determine what minimum speed this rocket would have to be given at Earth's surface to "escape" the potential well of Earth. To "escape," the rocket's initial kinetic energy must just equal the depth of the potential well at Earth's surface, thereby making its total energy zero. This also means the rocket must reach an infinite distance, where $E_g = 0$, before coming to rest. At this infinite distance, the gravitational force is zero and hence the rocket remains at rest there.

$$E_T = E_K + E_g = 0$$
$$E_K = -E_g$$
$$\frac{1}{2}mv^2 = -\left(-\frac{GM_Em}{r_E}\right)$$
$$v = \sqrt{\frac{2GM_E}{r_E}}$$
$$= \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{6.38 \times 10^6\ \text{m}}}$$
$$v = 1.12 \times 10^4\ \text{m/s, or } 11.2\ \text{km/s}$$

This speed is called the **escape speed**, which is the minimum speed needed to project a mass $m$ from the surface of mass $M$ to just escape the gravitational force of $M$ (with a final speed of zero). The **escape energy** is the kinetic energy needed to give an object its escape speed. A rocket launched from Earth with a speed greater than the escape speed moves away from Earth, losing $E_K$ and gaining $E_g$ as it does so. Since its $E_K$ is greater than the depth of its gravitational potential well at any point, its total energy will always be positive. This rocket will reach an infinite separation distance from Earth with some $E_K$ left. For a launch speed less than the escape speed, the rocket will come to rest at some finite distance and then fall back to Earth.

**escape speed** the minimum speed needed to project a mass $m$ from the surface of mass $M$ to just escape the gravitational force of $M$

**escape energy** the minimum kinetic energy needed to project a mass $m$ from the surface of mass $M$ to just escape the gravitational force of $M$

In practice, a space vehicle does not achieve its highest speed upon launch. Its speed increases after launch as its rocket engines continue to be fired. If a satellite is launched from the cargo hold of an orbiting space shuttle, it is already travelling at the speed of the shuttle (about $8 \times 10^3$ m/s), so the small rocket engines on the satellite need to supply a relatively small amount of energy to propel the satellite into its higher orbit.

A rocket whose total energy is negative will not be able to escape from Earth's potential well and is "bound" to Earth. The **binding energy** of any mass is the amount of additional kinetic energy it needs to just escape (with a final speed of zero) to an infinite distance away. For a rocket of mass $m$ at rest on Earth's surface (of mass $M_E$), the total energy is equal to the gravitational potential energy:

**binding energy** the amount of additional kinetic energy needed by a mass $m$ to just escape from a mass $M$

$$E_T = E_K + E_g$$
$$= 0 + \left(-\frac{GM_Em}{r_E}\right)$$
$$E_T = -\frac{GM_Em}{r_E}$$

Thus, the binding energy must be $\frac{GM_Em}{r_E}$ to give the rocket enough energy to escape.

An example of a bound object is a satellite moving in a circular orbit of radius $r$ in the potential well of Earth. The net force (of magnitude $\Sigma F$) necessary to sustain the circular orbit is provided by the force of gravitational attraction between the satellite and Earth. Using the magnitudes of the forces, for a satellite of mass $m$ and orbital speed $v$:

$$\Sigma F = F_G$$
$$\frac{mv^2}{r} = \frac{GM_Em}{r^2}$$
$$mv^2 = \frac{GM_Em}{r}$$

The total energy of the satellite is constant and is given by:

$$E_T = E_K + E_g$$
$$E_T = \frac{1}{2}mv^2 - \frac{GM_Em}{r}$$

Substituting $mv^2 = \dfrac{GM_Em}{r}$ into the equation:

$$E_T = \frac{1}{2}\frac{GM_Em}{r} - \frac{GM_Em}{r}$$
$$= -\frac{1}{2}\frac{GM_Em}{r}$$
$$E_T = \frac{1}{2}E_g$$

This is a very significant result. The total energy of a satellite in circular orbit is negative and is equal to one-half the value of the gravitational potential energy at the separation corresponding to the radius of its orbit. **Figure 5** shows the potential well for Earth and the position of this orbiting satellite in the well. This satellite is bound to Earth and its binding energy is $\frac{1}{2}\frac{GM_Em}{r}$.

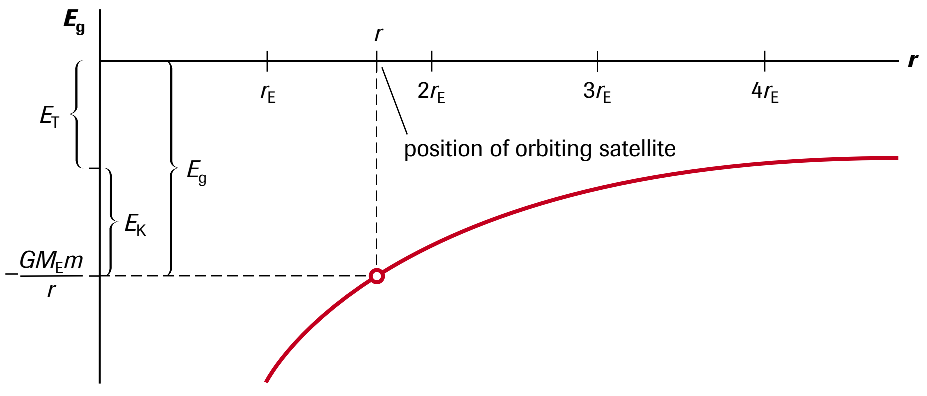


**Figure 5**
The gravitational potential energy of a satellite in Earth's potential well

In summary, the total energy of any object in Earth's gravitational field is composed of kinetic energy and gravitational potential energy. The graphs shown in **Figure 6** illustrate the three general cases possible for such an object.

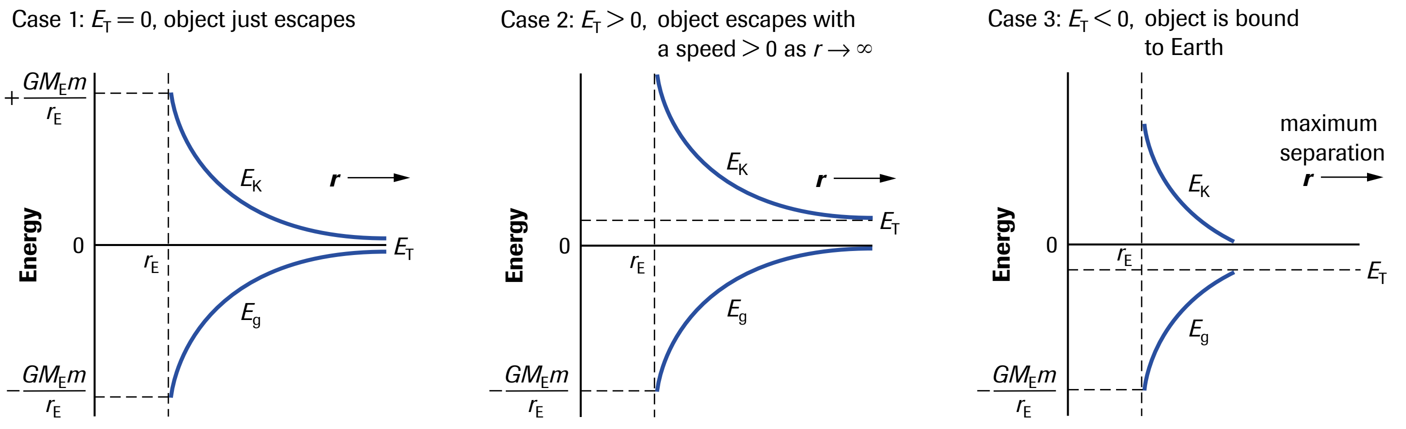


**Figure 6**
Comparing the energies of the same object given different amounts of kinetic energy at Earth's surface

## SAMPLE problem 2

A $5.00 \times 10^2$-kg communications satellite is to be placed into a circular geosynchronous orbit around Earth. (A geosynchronous satellite remains in the same relative position above Earth because it has a period of 24.0 h, the same as that of Earth's rotation on its axis.)

(a) What is the radius of the satellite's orbit?

(b) What is the gravitational potential energy of the satellite when it is attached to its launch rocket, at rest on Earth's surface?

(c) What is the total energy of the satellite when it is in geosynchronous orbit?

(d) How much work must the launch rocket do on the satellite to place it into orbit?

(e) Once in orbit, how much additional energy would the satellite require to escape from Earth's potential well?

### Solution

(a) $G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$

$T = 24.0\ \text{h} = 8.64 \times 10^4\ \text{s}$

$M_E = 5.98 \times 10^{24}\ \text{kg}$

As for any satellite:

$$\sum F = F_G$$

$$\frac{4\pi^2 mr}{T^2} = \frac{GM_Em}{r^2}$$

$$r = \sqrt[3]{\frac{GM_ET^2}{4\pi^2}}$$

$$= \sqrt[3]{\frac{(6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(8.64 \times 10^4\ \text{s})^2}{4\pi^2}}$$

$$r = 4.22 \times 10^7\ \text{m}$$

The radius of the satellite's orbit is $4.22 \times 10^7$ m. This radius represents an altitude of $3.58 \times 10^4$ km above Earth's surface.

(b) $r_E = 6.38 \times 10^6\ \text{m}$

$m = 5.00 \times 10^2\ \text{kg}$

At the surface of Earth,

$$E_g = -\frac{GM_Em}{r_E}$$

$$= -\frac{(6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(5.00 \times 10^2\ \text{kg})}{6.38 \times 10^6\ \text{m}}$$

$$E_g = -3.13 \times 10^{10}\ \text{J}$$

The gravitational potential energy of the satellite when it is attached to its launch rocket at rest on Earth's surface is $-3.13 \times 10^{10}$ J.

(c) $r = 4.22 \times 10^7$ m

The total energy of a satellite in circular orbit, bound to Earth, is given by:

$$E_T = E_K + E_g$$

$$= \frac{1}{2}mv^2 - \frac{GM_Em}{r}$$

$$= -\frac{1}{2}\frac{GM_Em}{r} \quad \text{(based on the theory related to } \textbf{Figure 5}\text{)}$$

$$= -\frac{1}{2}\frac{(6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(5.00 \times 10^2\ \text{kg})}{4.22 \times 10^7\ \text{m}}$$

$$E_T = -2.36 \times 10^9\ \text{J}$$

The total energy of the satellite when in geosynchronous orbit is $-2.36 \times 10^9$ J.

(d)
$$W = \Delta E = E_T\ \text{(in orbit)} - E_T\ \text{(on Earth)}$$

$$= -2.36 \times 10^9\ \text{J} - (-3.13 \times 10^{10}\ \text{J})$$

$$W = 2.89 \times 10^{10}\ \text{J}$$

The launch rocket must do $2.89 \times 10^{10}$ J of work on the satellite to place it into orbit.

(e) To escape Earth's potential well, the total energy of the satellite must be zero or greater. In orbit, $E_T = -2.36 \times 10^9$ J. Therefore, to escape Earth's potential well, the satellite must acquire at least $2.36 \times 10^9$ J of additional energy.

**black hole** a very dense body in space with a gravitational field so strong that nothing can escape from it

**event horizon** the surface of a black hole

**singularity** the dense centre of a black hole

**Schwartzschild radius** the distance from the centre of the singularity to the event horizon

**LAB EXERCISE 6.3.1**

**Graphical Analysis of Energies** **(p. 295)**

A detailed analysis of the energies involved in launching a space vehicle and its payload from another body must be carried out before the mission is undertaken. How can graphing be used to analyze the energy data related to a spacecraft launch?

An important goal of future space missions will be to mine minerals on distant bodies, such as moons and asteroids, in the solar system. Once the minerals are mined, some will be used for manufacturing on the moon or asteroid, while others will be brought back to Earth or to the International Space Station for research and manufacturing. You can learn about the energies associated with this application by performing Lab Exercise 6.3.1 in the Lab Activities section at the end of this chapter.

Among the most interesting objects in the universe are extremely dense bodies that form at the end of a massive star's life. A **black hole** is a small, very dense body with a gravitational field so strong that nothing can escape from it. Even light cannot be radiated away from its surface, which explains the object's name.

The surface of a black hole is called its **event horizon** because no "event" can be observed from outside this surface. Inside the event horizon, at the very core of the black hole, is an unbelievably dense centre called a **singularity**. The distance from the centre of the singularity to the event horizon is the **Schwartzschild radius**, named after German astronomer Karl Schwartzschild (1873–1916), who was the first person to solve Einstein's equations of general relativity.

Since the speed of light $c$ is $3.00 \times 10^8$ m/s, we can use that value in the equation for escape speed to determine the Schwartzschild radius of a black hole of known mass.

As an example, assume that a certain black hole results from the collapse of a star that has a mass 28 times the Sun's mass. Since the minimum escape speed is $v_e = c$, we have

$$\frac{mv_e^2}{2} = \frac{GMm}{r}$$

$$v_e^2 = \frac{2GM}{r}$$

$$r = \frac{2GM}{v_e^2}$$

$$= \frac{2GM}{c^2}$$

$$= \frac{2(6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2)(28 \times 1.99 \times 10^{30}\ \text{kg})}{(3.00 \times 10^8\ \text{m/s})^2}$$

$$= 8.26 \times 10^4\ \text{m}$$

$$r = 82.6\ \text{km}$$

Since light cannot escape from a black hole, the only way a black hole can be detected is indirectly. Material that is close enough to the black hole gets sucked in, and as it does so, the material emits X rays that can be detected and analyzed.

The celestial mechanics analyzed in this chapter is not a complete picture. You will learn more about high-speed and high-energy particles when you study Einstein's special theory of relativity in Chapter 11.

**DID YOU KNOW ?**

**First Black Hole Discovery**

In 1972, Professor Tom Bolton, while working at the University of Toronto's David Dunlap Observatory in Richmond Hill, Ontario, was investigating a point in space, Cygnus X-1, because it was a source of X rays. It turned out to be one of the most significant discoveries in astronomy: a black hole. This was the first evidence to support the existence of black holes, which were previously hypothetical objecs.

## Practice

### Understanding Concepts

**6.** Does the escape speed of a space probe depend on its mass? Why or why not?

**7.** Jupiter's mass is 318 times that of Earth, and its radius is 10.9 times that of Earth. Determine the ratio of the escape speed from Jupiter to the escape speed from Earth.

**8.** The Moon is a satellite of mass $7.35 \times 10^{22}$ kg, with an average distance of $3.84 \times 10^8$ m from the centre of Earth.
- (a) What is the gravitational potential energy of the Moon–Earth system?
- (b) What is the Moon's kinetic energy and speed in circular orbit?
- (c) What is the Moon's binding energy to Earth?

**9.** What is the total energy needed to place a $2.0 \times 10^3$-kg satellite into circular Earth orbit at an altitude of $5.0 \times 10^2$ km?

**10.** How much additional energy would have to be supplied to the satellite in question 9 once it was in orbit, to allow it to escape from Earth's gravitational field?

**11.** Consider a geosynchronous satellite with an orbital period of 24 h.
- (a) What is the satellite's speed in orbit?
- (b) What speed must the satellite reach during launch to attain the geosynchronous orbit? (Assume all fuel is burned in a short period. Neglect air resistance.)

**12.** Determine the Schwarztschild radius, in kilometres, of a black hole of mass 4.00 times the Sun's mass.

### Applying Inquiry Skills

**13.** Sketch the general shape of the potential wells of both Earth and the Moon on a single graph. Label the axes and use colour coding to distinguish the line for Earth from the line for the Moon.

### Making Connections

**14.** (a) Calculate the binding energy of a 65.0-kg person on Earth's surface.
- (b) How much kinetic energy would this person require to just escape from the gravitational field of Earth?
- (c) How much work is required to raise this person by 1.00 m at Earth's surface?
- (d) Explain why one of NASA's objectives in designing launches into space is to minimize the mass of the payload (including the astronauts).

**Answers**

7. 5.40 : 1
8. (a) $-7.63 \times 10^{28}$ J
   (b) $3.82 \times 10^{28}$ J; $1.02 \times 10^3$ m/s
   (c) $3.82 \times 10^{28}$ J
9. $6.7 \times 10^{10}$ J
10. $5.80 \times 10^{10}$ J
11. (a) $3.1 \times 10^3$ m/s
    (b) $1.1 \times 10^4$ m/s
12. 11.8 km
14. (a) $4.06 \times 10^9$ J
    (b) $4.06 \times 10^9$ J
    (c) $6.37 \times 10^2$ J

## SUMMARY Gravitational Potential Energy in General

- The gravitational potential energy of a system of two (spherical) masses is directly proportional to the product of their masses, and inversely proportional to the distance between their centres.
- A gravitational potential energy of zero is assigned to an isolated system of two masses that are so far apart (i.e., their separation is approaching infinity) that the force of gravity between them has dropped to zero.
- The change in gravitational potential energy very close to Earth's surface is a special case of gravitational potential energy in general.
- Escape speed is the minimum speed needed to project a mass $m$ from the surface of mass $M$ to just escape the gravitational force of $M$.
- Escape energy is the minimum kinetic energy needed to project a mass $m$ from the surface of mass $M$ to just escape the gravitational force of $M$.
- Binding energy is the amount of additional kinetic energy needed by a mass $m$ to just escape from a mass $M$.

## Section 6.3 Questions

### Understanding Concepts

1. How does the escape energy of a 1500-kg rocket compare to that of a 500-kg rocket, both initially at rest on Earth?
2. Do you agree or disagree with the statement, "No satellite can orbit Earth in less than about 80 min"? Give reasons. (*Hint:* The greater the altitude of an Earth satellite, the longer it takes to complete one orbit.)
3. A space shuttle ejects a $1.2 \times 10^3$-kg booster tank so that the tank is momentarily at rest, relative to Earth, at an altitude of $2.0 \times 10^3$ km. Neglect atmospheric effects.
   (a) How much work is done on the booster tank by the force of gravity in returning it to Earth's surface?
   (b) Determine the impact speed of the booster tank.
4. A space vehicle, launched as a lunar probe, arrives above most of Earth's atmosphere. At this point, its kinetic energy is $5.0 \times 10^9$ J and its gravitational potential energy is $-6.4 \times 10^9$ J. What is its binding energy?
5. An artificial Earth satellite, of mass $2.00 \times 10^3$ kg, has an elliptical orbit with an average altitude of $4.00 \times 10^2$ km.
   (a) What is its average gravitational potential energy while in orbit?
   (b) What is its average kinetic energy while in orbit?
   (c) What is its total energy while in orbit?
   (d) If its perigee (closest position) is $2.80 \times 10^2$ km, what is its speed at perigee?
6. A $5.00 \times 10^2$-kg satellite is in circular orbit $2.00 \times 10^2$ km above Earth's surface. Calculate
   (a) the gravitational potential energy of the satellite
   (b) the kinetic energy of the satellite
   (c) the binding energy of the satellite
   (d) the percentage increase in launching energy required for the satellite to escape from Earth
7. (a) Calculate the escape speed from the surface of the Sun: mass = $1.99 \times 10^{30}$ kg, radius = $6.96 \times 10^8$ m.
   (b) What speed would an object leaving Earth need to escape from our solar system?
8. The mass of the Moon is $7.35 \times 10^{22}$ kg, and its radius is $1.74 \times 10^6$ m. With what speed must an object be projected from the its surface to reach an altitude equal to its radius?
9. A black hole has a Schwartzschild radius of 15.4 km. What is the mass of the black hole in terms of the Sun's mass?

### Applying Inquiry Skills

10. Mars is a planet that could be visited by humans in the future.
    (a) Generate a graph of Mars' potential well (using data from Appendix C) for a spacecraft of mass $2.0 \times 10^3$ kg that is launched from Mars. Draw the graph up to $5r_M$.
    (b) On your graph, draw
       (i) the line for the kinetic energy needed for the craft to just escape from Mars
       (ii) the line of the total energy from Mars' surface to $5r_M$
11. (a) What is the theoretical Schwartzschild radius of a black hole whose mass is equal to the mass of Earth. Express your answer in millimetres.
    (b) What does your answer imply about the density of a black hole?

### Making Connections

12. How would the amount of fuel required to send a spacecraft from Earth to the Moon compare with the amount needed to send the same spacecraft from the Moon back to Earth? Explain. (Numerical values are not required.)