6.1 Gravitational Fields

Humans have a natural curiosity about how Earth and the solar system formed billions of years ago, and whether or not we are alone in the universe. The search for answers to these questions begins with an understanding of *gravity*, the force of attraction between all objects with mass in the universe.

Concepts and equations related to gravity have been introduced earlier in the text. In this section, we will build on the concepts most directly related to planetary and satellite motion.

A *force field* exists in the space surrounding an object in which a force is exerted on objects. Thus, a **gravitational field** exists in the space surrounding an object in which the force of gravity is exerted on objects. The strength of the gravitational field is directly proportional to the mass of the central body and inversely proportional to the square of the distance from the centre of that body. To understand this relationship, we combine the law of universal gravitation and Newton's second law of motion, as illustrated in **Figure 1**.

From the law of universal gravitation,

$$F_G = \frac{GMm}{r^2}$$

where F_G is the magnitude of the force of gravity, G is the universal gravitation constant, m is the mass of a body influenced by the gravitational field of the central body of mass M, and r is the distance between the centres of the two bodies. (Remember that this law applies to spherical bodies, which is basically what we are considering in this topic of celestial mechanics.)

LEARNING TIP

Review of Important Concepts

To help you remember and understand the ideas presented in this chapter, you may want to briefly review some concepts from earlier sections (as indicated in parentheses):

- force, force of gravity (2.1)
- weight, force field, gravitational field strength (2.2)
- uniform circular motion, centripetal acceleration (3.1)
- centripetal force, rotating frame of reference (3.2)
- law of universal gravitation, universal gravitation constant (3.3)
- satellite, space station, apparent weight, artificial gravity, speed of a satellite in circular motion, black hole (3.4)
- kinetic energy (4.2)
- gravitational potential energy at Earth's surface (4.3)
- law of conservation of energy (4.4)

gravitational field exists in the space surrounding an object in which the force of gravity exists

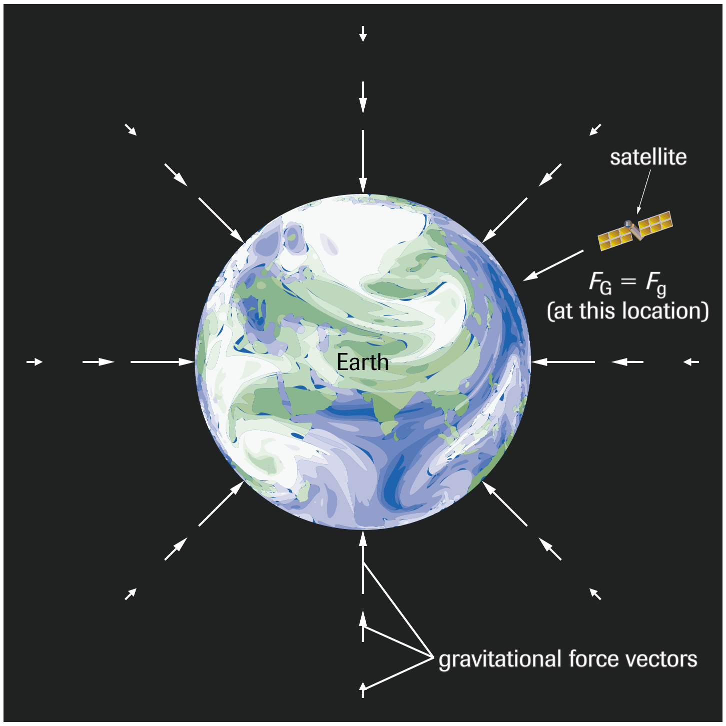

Figure 1
The magnitudes of the force vectors surrounding Earth show how the strength of the gravitational field diminishes inversely as the square of the distance to Earth's centre. To derive the equation for the gravitational field strength in terms of the mass of the central body (Earth in this case), we equate the magnitudes of the forces F_G and F_g.

From Newton's second law,

$$F_g = mg$$

where F_g is the magnitude of the force of gravity acting on a body of mass m and g is the magnitude of the gravitational field strength. At any specific location, F_g and F_G are equal, so we have:

$$F_g = F_G$$
$$mg = \frac{GMm}{r^2}$$
$$g = \frac{GM}{r^2}$$

SAMPLE problem 1

Determine the mass of Earth using the magnitude of the gravitational field strength at the surface of the Earth, the distance r between Earth's surface and its centre (6.38×10^6 m), and the universal gravitation constant.

Solution

$g = 9.80$ N/kg $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$
$r = 6.38 \times 10^6$ m $M = ?$

$$g = \frac{GM}{r^2}$$
$$M = \frac{gr^2}{G}$$
$$= \frac{(9.80\ \text{N/kg})(6.38 \times 10^6\ \text{m})^2}{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}$$
$$M = 5.98 \times 10^{24}\ \text{kg}$$

The mass of Earth is 5.98×10^{24} kg.

Notice that the relation $g = \frac{GM}{r^2}$ is valid not only for objects on Earth's surface, but also for objects above Earth's surface. For objects above Earth's surface, r represents the distance from an object to Earth's centre and, except for objects close to Earth's surface, g is not 9.80 N/kg. The magnitude of the gravitational field strength g decreases with increasing distance r from Earth's centre according to $g = \frac{GM}{r^2}$. We can also use the same equation for other planets and stars by substituting the appropriate mass M.

SAMPLE problem 2

(a) Calculate the magnitude of the gravitational field strength on the surface of Mars.

(b) What is the ratio of the magnitude of the gravitational field strength on the surface of Mars to that on the surface of Earth?

Solution

Appendix C contains the required data.

(a) $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ $r = 3.40 \times 10^6$ m
$M = 6.37 \times 10^{23}$ kg $g = ?$

LEARNING TIP

Units of *g*

Recall that the magnitude of g at Earth's surface is 9.8 N/kg, which is equivalent to the magnitude of the acceleration due to gravity (9.8 m/s^2). In all calculations involving forces and energies in this topic, it is important to remember to use the SI base units of metres (m), kilograms (kg), and seconds (s).

$$g = \frac{GM}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2)(6.37 \times 10^{23}\ \text{kg})}{(3.40 \times 10^6\ \text{m})^2}$$

$$g = 3.68\ \text{N/kg}$$

The magnitude of the gravitational field strength on the surface of Mars is 3.68 N/kg.

(b) The required ratio is:

$$\frac{g_{\text{Mars}}}{g_{\text{Earth}}} = \frac{3.68\ \text{N/kg}}{9.80\ \text{N/kg}} = 0.375:100$$

The ratio of the magnitudes of the gravitational field strengths is 0.375:100. This means that the gravitational field strength on the surface of Mars is 37.5% of the gravitational field strength on the surface of Earth.

Table 1 Magnitude of the Gravitational Field Strength of Planets Relative to Earth's Value (g = 9.80 N/kg)

Planet	Surface Gravity (Earth = 1.00)
Mercury	0.375
Venus	0.898
Earth	1.00
Mars	0.375
Jupiter	2.53
Saturn	1.06
Uranus	0.914
Neptune	1.14
Pluto	0.067

The magnitude of the gravitational field strength on the surface of Mars is only 0.375 of that on the surface of Earth. The corresponding values for all the planets in the solar system are listed in **Table 1**.

Practice

Understanding Concepts

Refer to Appendix C for required data.

1. What keeps the International Space Station and other satellites in their orbits around Earth?
2. Determine the magnitude and direction of the gravitational force exerted (a) on the Moon by Earth and (b) on Earth by the Moon.
3. If we represent the magnitude of Earth's surface gravitational field strength as 1g, what are the magnitudes of the gravitational field strengths (in terms of g) at the following distances above Earth's surface: (a) 1.0 Earth radii, (b) 3.0 Earth radii, and (c) 4.2 Earth radii?
4. If a planet has the same mass as Earth, but a radius only 0.50 times the radius of Earth, what is the magnitude of the planet's surface gravitational field strength as a multiple of Earth's surface g?
5. The Moon has a surface gravitational field strength of magnitude 1.6 N/kg.
   (a) What is the mass of the Moon? (*Hint:* The Moon's radius is in the appendix.)
   (b) What would be the magnitude of your weight if you were on the Moon?
6. The magnitude of the total gravitational field strength at a point in interstellar space is 5.42×10^{-9} N/kg. What is the magnitude of the gravitational force at this point on an object (a) of mass 1.00 kg and (b) of mass 8.91×10^5 kg?

Applying Inquiry Skills

7. A space probe orbits Jupiter, gathering data and sending the data to Earth by electromagnetic waves. The probe then travels away from Jupiter toward Saturn.
   (a) As the probe gets farther away from Jupiter (in an assumed straight-line motion), sketch the shape of the graph of the magnitude of the force of Jupiter on the probe as a function of the distance between the centres of the two bodies.
   (b) Repeat (a) for the magnitude of the force of the probe on Jupiter.

Answers

2. (a) 1.99×10^{20} N [toward Earth's centre]
   (b) 1.99×10^{20} N [toward the Moon's centre]
3. (a) $\frac{g}{4}$
   (b) $\frac{g}{16}$
   (c) $\frac{g}{27}$
4. 4.0g
5. (a) 7.3×10^{22} kg
6. (a) 5.42×10^{-9} N
   (b) 4.83×10^{-3} N

Making Connections

8. If the density of Earth were much greater than its actual value, but its radius was the same, what would be the effect on
   (a) Earth's surface gravitational field strength?
   (b) the evolution of human bone structure?
   (c) some other aspects of nature or human activity? (Use your imagination.)

SUMMARY Gravitational Fields

- A gravitational field exists in the space surrounding an object in which the force of gravity is exerted on objects.
- The magnitude of the gravitational field strength surrounding a planet or other body (assumed to be spherical) is directly proportional to the mass of the central body, and inversely proportional to the square of the distance to the centre of the body.
- The law of universal gravitation applies to all bodies in the solar system, from the Sun to planets, moons, and artificial satellites.

Section 6.1 Questions

Understanding Concepts

Refer to Appendix C for required data.

1. How does the weight of a space probe change as it travels from Earth to the Moon? Is there any location at which the weight is zero? Does its mass change? Explain.
2. A satellite of mass 225 kg is located 8.62×10^6 m above Earth's surface.
   (a) Determine the magnitude and direction of the gravitational force on the satellite.
   (b) Determine the magnitude and direction of the resulting acceleration of the satellite.
3. Determine the magnitude and direction of the gravitational field strength at a point in space 7.4×10^7 m from the centre of Earth.
4. A 6.2×10^2-kg satellite above Earth's surface experiences a gravitational field strength of magnitude 4.5 N/kg.
   (a) Knowing the gravitational field strength at Earth's surface and Earth's radius, how far above Earth's surface is the satellite? (Use ratio and proportion.)
   (b) Determine the magnitude of the gravitational force on the satellite.
5. Calculate the magnitude of Neptune's surface gravitational field strength, and compare your answer to the value in **Table 1**.
6. A 456-kg satellite in circular orbit around Earth has a speed of 3.9 km/s and is 2.5×10^7 m from Earth's centre.
   (a) Determine the magnitude and direction of the acceleration of the satellite.
   (b) Determine the magnitude and direction of the gravitational force on the satellite.
7. (a) On the surface of Titan, a moon of Saturn, the gravitational field strength has a magnitude of 1.3 N/kg. Titan's mass is 1.3×10^{23} kg. What is the radius of Titan in kilometres?
   (b) What is the magnitude of the force of gravity on a 0.181-kg rock on Titan?
8. Given that Earth's surface gravitational field strength has a magnitude of 9.80 N/kg, determine the distance (as a multiple of Earth's radius r_E) above Earth's surface at which the magnitude of the field strength is 3.20 N/kg.

Applying Inquiry Skills

9. Use free-body diagrams of a 1.0-kg mass at increasingly large distances from Earth to illustrate that the strength of the gravitational field is inversely proportional to the square of the distance from Earth's centre.

Making Connections

10. Based solely on the data in **Table 1**, speculate on at least one reason why some astronomers argue that Pluto should not be classified as a planet.