



**Figure 1**

The details of the night sky are enhanced when the surroundings are dark and a telescope is used. The force of gravity has an important influence on all the objects in the universe.

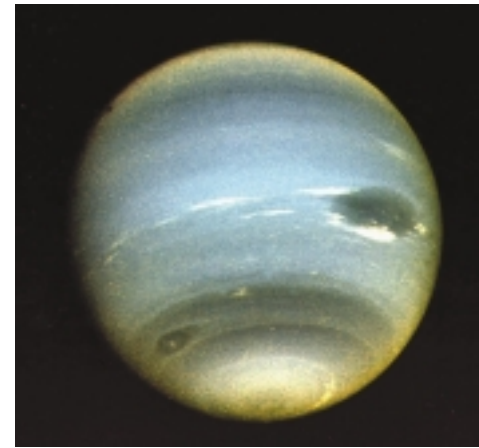
People have always enjoyed viewing stars and planets on clear, dark nights (**Figure 1**). It is not only the beauty and variety of objects in the sky that is so fascinating, but also the search for answers to questions related to the patterns and motions of those objects.

Until the late 1700s, Jupiter and Saturn were the only outer planets identified in our solar system because they were visible to the naked eye. Combined with the inner planets (Mercury, Venus, Earth, and Mars), the solar system was believed to consist of the Sun and six planets, as well as other smaller bodies such as moons. Then in 1781, British astronomer William Herschel (1738–1822), after making careful observations on what other astronomers thought was a star, announced that the “star” appeared to move relative to the background stars over a long period. This wandering star turned out to be the seventh planet, which Herschel named Uranus, after the Greek god of the sky and ruler of the universe. Astronomers studied the motion of Uranus over many years and discovered that its path was not quite as smooth as expected. Some distant hidden object appeared to be “tugging” on Uranus causing a slightly uneven orbit. Using detailed mathematical analysis, they predicted where this hidden object should be, searched for it for many years, and in 1846 discovered Neptune (**Figure 2**). Neptune is so far from the Sun that it takes almost 165 Earth years to complete one orbit; in other words, it will soon complete its first orbit since being discovered.

The force that keeps the planets in their orbits around the Sun and our Moon in its orbit around Earth is the same force tugging on Uranus to perturb its motion—the force of gravity. This force exists everywhere in the universe where matter exists. Sir Isaac Newton first analyzed the effects of gravity throughout the universe. Neptune was discovered by applying Newton’s analysis of gravity.

## Newton’s Law of Universal Gravitation

In his *Principia*, published in 1687, Newton described how he used known data about objects in the solar system, notably the Moon’s orbit around Earth, to discover the factors that affect the force of gravity throughout the universe. The relationships involved are summarized in his *law of universal gravitation*.



**Figure 2**

Neptune, the most distant of the gas giant planets, was named after the Roman god of water. Although earthbound telescopes disclose little detail, photographs taken by the space probe *Voyager 2* in 1989 reveal bright blue and white clouds, and a dark area that may be a large storm.

### LEARNING TIP

#### Perturbations

In physics, a perturbation is a slight alteration in the action of a system that is caused by a secondary influence. Perturbations occur in the orbits of planets, moons, comets, and other heavenly bodies. When astronomers analyze the perturbations in the orbits of heavenly bodies, they search for the secondary influence and sometimes discover another body too small to find by chance.

## DID YOU KNOW?

### Caroline Herschel: An Underrated Astronomer



Uranus was the first planet discovered by telescope. William Herschel, the astronomer who made the discovery, designed and built his own instruments with the assistance of another astronomer, his sister.

Caroline Herschel (1750–1848) spent long hours grinding and polishing the concave mirrors used to make reflecting telescopes. With some of those telescopes, she made many discoveries of her own, including nebulae (clouds of dust or gas in interstellar space) and comets. She also helped William develop a mathematical approach to astronomy, and contributed greatly to permanently valuable catalogues of astronomical data. She was the first woman to be granted a membership in the Royal Astronomical Society in London, UK.

### Newton's Law of Universal Gravitation

The force of gravitational attraction between any two objects is directly proportional to the product of the masses of the objects, and inversely proportional to the square of the distance between their centres.

To express this law in equation form, we use the following symbols for the magnitudes of the variables involved:  $F_G$  is the force of gravitational attraction between any two objects;  $m_1$  is the mass of one object;  $m_2$  is the mass of a second object; and  $r$  is the distance *between the centres* of the two objects, which are assumed to be spherical.

Newton discovered the following proportionalities:

If  $m_2$  and  $r$  are constant,  $F_G \propto m_1$  (direct variation).

If  $m_1$  and  $r$  are constant,  $F_G \propto m_2$  (direct variation).

If  $m_1$  and  $m_2$  are constant,  $F_G \propto \frac{1}{r^2}$  (inverse square variation).

Combining these statements, we obtain a joint variation:

$$F_G \propto \frac{m_1 m_2}{r^2}$$

Finally, we can write the equation for the law of universal gravitation:

$$F_G = \frac{Gm_1 m_2}{r^2}$$

where  $G$  is the universal gravitation constant.

In applying the law of universal gravitation, it is important to consider the following observations:

- There are two equal, but opposite forces present. For example, Earth pulls on you and you pull on Earth with a force of equal magnitude.
- For the force of attraction to be noticeable, at least one of the objects must be very large.
- The inverse square relationship between  $F_G$  and  $r$  means that the force of attraction diminishes rapidly as the two objects move apart. On the other hand, there is no value of  $r$ , no matter how large, that would reduce the force of attraction to zero. Every object in the universe exerts a force of attraction on every other object.
- The equation for the law of universal gravitation applies only to two spherical objects (such as Earth and the Sun), to two objects whose sizes are much smaller than their separation distance (for example, you and a friend separated by 1.0 km), or to a small object and a very large sphere (such as you and Earth).

### ▶ SAMPLE problem 1

Earth's gravitational pull on a spacecraft some distance away is  $1.2 \times 10^2$  N in magnitude. What will the magnitude of the force of gravity be on a second spacecraft with 1.5 times the mass of the first spacecraft, at a distance from Earth's centre that is 0.45 times as great?

#### **Solution**

Let  $m_E$  represent the mass of Earth, and the subscripts 1 and 2 represent the first and second spacecraft, respectively.

$$F_1 = 1.2 \times 10^2 \text{ N}$$

$$m_2 = 1.5m_1$$

$$r_2 = 0.45r_1$$

$$F_2 = ?$$

By ratio and proportion:

$$\frac{F_2}{F_1} = \frac{\left(\frac{Gm_2m_2}{r_2^2}\right)}{\left(\frac{Gm_1m_1}{r_1^2}\right)}$$

$$F_2 = F_1 \left(\frac{m_2}{r_2^2}\right) \left(\frac{r_1^2}{m_1}\right)$$

$$= F_1 \left(\frac{1.5m_1}{(0.45r_1)^2}\right) \left(\frac{r_1^2}{m_1}\right)$$

$$= 1.2 \times 10^2 \text{ N} \left(\frac{1.5}{(0.45)^2}\right)$$

$$F_2 = 8.9 \times 10^2 \text{ N}$$

The force of gravity on the spacecraft is  $8.9 \times 10^2 \text{ N}$  in magnitude.

### Practice

#### Understanding Concepts

1. Relate Newton's third law of motion to his law of universal gravitation.
2. What is the direction of the gravitational force of attraction of object A on object B?
3. The magnitude of the force of gravitational attraction between two uniform spherical masses is 36 N. What would the magnitude of the force be if one mass were doubled, and the distance between the objects tripled?
4. Mars has a radius and mass 0.54 and 0.11 times the radius and mass of Earth. If the force of gravity on your body is  $6.0 \times 10^2 \text{ N}$  in magnitude on Earth, what would it be on Mars?
5. The magnitude of the force of gravity between two uniform spherical masses is 14 N when their centres are 8.5 m apart. When the distance between the masses is changed, the force becomes 58 N. How far apart are the centres of the masses?

#### Applying Inquiry Skills

6. Sketch a graph showing the relationship between the magnitude of the gravitational force and the distance separating the centres of two uniform spherical objects.

#### Making Connections

7. In the past, Pluto has been known as the ninth planet in the solar system. Recently, however, it has been suggested that Pluto should be classified as a body other than a planet. Research and write a brief report on Pluto's discovery, and also the reasons for the recent controversy over Pluto's planetary status.



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#### Answers

3. 8.0 N
4.  $2.3 \times 10^2 \text{ N}$
5. 4.2 m

## DID YOU KNOW?

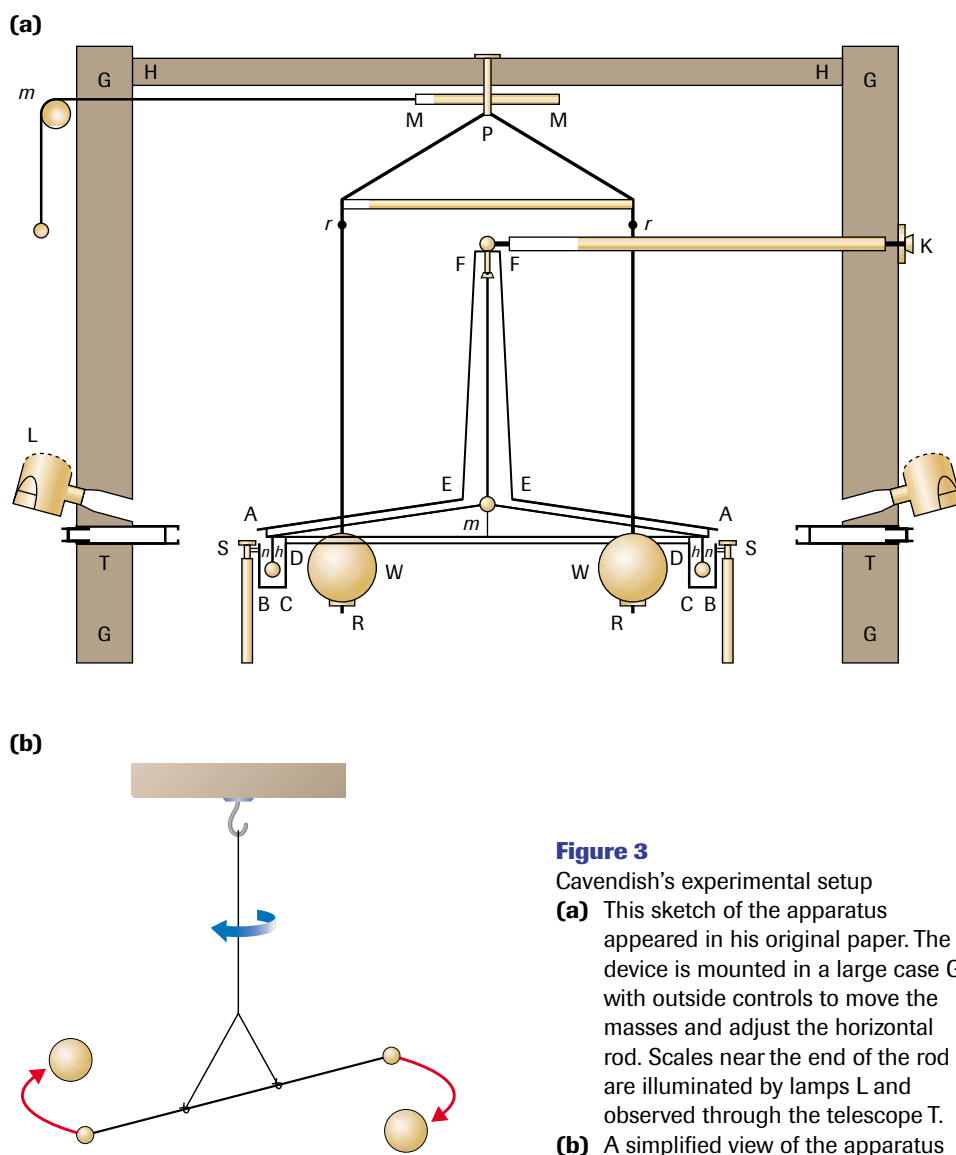
### Universal Laws

Newton's law of universal gravitation was among the first of the "universal truths," or laws of nature that could be applied everywhere.

Scientists in the 18th and 19th centuries introduced an analytical and scientific approach to searching for answers to questions in other fields. By the turn of the 20th century, however, scientific investigation showed that nature was not as exact and predictable as everyone had believed. For example, as you will see in Unit 5, the tiny physical world of the atom does not obey strict, predictive laws.

## Determining the Universal Gravitation Constant

The numerical value of the universal gravitation constant  $G$  is extremely small; experimental determination of the value did not occur until more than a century after Newton formulated his law of universal gravitation. In 1798, British scientist Henry Cavendish (1731–1810), using the apparatus illustrated in **Figure 3**, succeeded in measuring the gravitational attraction between two small spheres that hung on a rod approximately 2 m long and two larger spheres mounted independently. Using this equipment, he derived a value of  $G$  that is fairly close to today's accepted value of  $6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ . His experiment showed that gravitational force exists even for relatively small objects and, by establishing the value of the constant of proportionality  $G$ , he made it possible to use the law of universal gravitation in calculations. Cavendish's experimental determination of  $G$  was a great scientific triumph. Astronomers believe that its magnitude may influence the rate at which the universe is expanding.



**Figure 3**

Cavendish's experimental setup

- (a)** This sketch of the apparatus appeared in his original paper. The device is mounted in a large case G, with outside controls to move the masses and adjust the horizontal rod. Scales near the end of the rod are illuminated by lamps L and observed through the telescope T.
- (b)** A simplified view of the apparatus

### ▶ SAMPLE problem 2

Determine the magnitude of the force of attraction between two uniform metal balls, of mass 4.00 kg, used in women's shot-putting, when the centres are separated by 45.0 cm.

#### Solution

$$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 \quad r = 0.450 \text{ m}$$

$$m_1 = m_2 = 4.00 \text{ kg} \quad F_G = ?$$

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(4.00 \text{ kg})(4.00 \text{ kg})}{(0.450 \text{ m})^2}$$

$$F_G = 5.27 \times 10^{-9} \text{ N}$$

The magnitude of the force of attraction is  $5.27 \times 10^{-9} \text{ N}$ , an extremely small value.

### ▶ Practice

#### Understanding Concepts

- What is the magnitude of the force of gravitational attraction between two  $1.8 \times 10^8$ -kg spherical oil tanks with their centres 94 m apart?
- A 50.0-kg student stands  $6.38 \times 10^6$  m from Earth's centre. The mass of Earth is  $5.98 \times 10^{24}$  kg. What is the magnitude of the force of gravity on the student?
- Jupiter has a mass of  $1.90 \times 10^{27}$  kg and a radius of  $7.15 \times 10^7$  m. Calculate the magnitude of the acceleration due to gravity on Jupiter.
- A space vehicle, of mass 555 kg, experiences a gravitational pull from Earth of 255 N. The mass of Earth is  $5.98 \times 10^{24}$  kg. How far is the vehicle (a) from the centre of Earth and (b) above the surface of Earth?
- Four masses are located on a plane, as in **Figure 4**. What is the magnitude of the net gravitational force on  $m_1$  due to the other three masses?

#### Making Connections

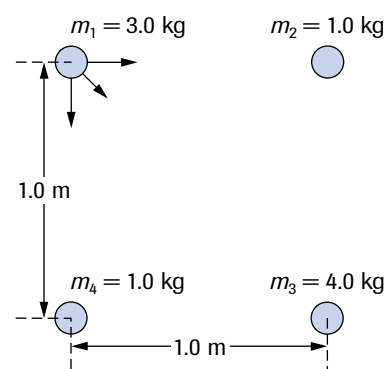
- The mass of Earth can be calculated by applying the fact that an object's weight is equal to the force of gravity between Earth and the object. The radius of Earth is  $6.38 \times 10^6$  m.
  - Determine the mass of Earth.
  - At what stage in the historical development of science would physicists first have been able to calculate Earth's mass accurately? Explain your answer.
  - What effect on society is evident now that we accurately know Earth's mass?

## SUMMARY Universal Gravitation

- Newton's law of universal gravitation states that the force of gravitational attraction between any two objects is directly proportional to the product of the masses of the objects and inversely proportional to the square of the distance between their centres.
- The universal gravitation constant,  $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ , was first determined experimentally by Henry Cavendish in 1798.
- The law of universal gravitation is applied in analyzing the motions of bodies in the universe, such as planets in the solar system. (This analysis can lead to the discovery of other celestial bodies.)

#### Answers

- $2.4 \times 10^2 \text{ N}$
- $4.90 \times 10^2 \text{ N}$
- $24.8 \text{ m/s}^2$
- (a)  $2.95 \times 10^7 \text{ m}$   
(b)  $2.31 \times 10^7 \text{ m}$
- 6.8 N
- (a)  $5.98 \times 10^{24} \text{ kg}$



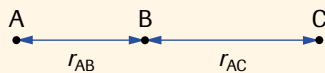
**Figure 4**  
For question 12

## Section 3.3 Questions

### Understanding Concepts

- Do you agree with the statement, "There is no location anywhere in the universe where a body can exist with no force acting on it"? Explain.
- The force of attraction between masses  $m_1$  and  $m_2$  is 26 N in magnitude. What will the magnitude of the force become if  $m_2$  is tripled, and the distance between  $m_2$  and  $m_1$  is halved?
- You are an astronaut. At what altitude above the surface of Earth is your weight one-half your weight on the surface? Express your answer as a multiple of Earth's radius  $r_E$ .
- Calculate the magnitude of the gravitational attraction between a proton of mass  $1.67 \times 10^{-27}$  kg and an electron of mass  $9.11 \times 10^{-31}$  kg if they are  $5.0 \times 10^{-11}$  m apart (as they are in a hydrogen atom).
- Uniform spheres A, B, and C have the following masses and centre-to-centre separations:  $m_A = 55$  kg,  $m_B = 75$  kg,  $m_C = 95$  kg;  $r_{AB} = 0.68$  m,  $r_{BC} = 0.95$  m. If the only forces acting on B are the forces of gravity due to A and C, determine the net force acting on B with the spheres arranged as in **Figures 5(a)** and **(b)**.

(a)



(b)

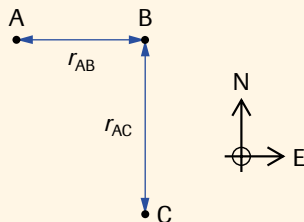


Figure 5

- At a certain point between Earth and the Moon, the net gravitational force exerted on an object by Earth and the Moon is zero. The Earth-Moon centre-to-centre separation is  $3.84 \times 10^5$  km. The mass of the Moon is 1.2% the mass of Earth.
  - Where is this point located? Are there any other such points? (*Hint:* Apply the quadratic formula after setting up the related equations.)

- What is the physical meaning of the root of the quadratic equation whose value exceeds the Earth-Moon distance? (An FBD of the object in this circumstance will enhance your answer.)

### Applying Inquiry Skills

- Using **Figure 6**, you can illustrate what happens to the magnitude of the gravitational force of attraction on an object as it recedes from Earth. Make a larger version of the graph and complete it for the force of gravity acting on you as you move from the surface of Earth to 7.0 Earth radii from the centre of Earth.

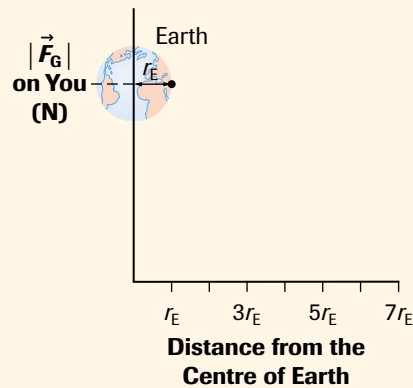


Figure 6

### Making Connections

- A geosynchronous satellite must remain at the same location above Earth's equator as it orbits Earth.
  - What period of revolution must a geosynchronous satellite have?
  - Set up an equation to express the distance of the satellite from the centre of Earth in terms of the universal gravitation constant, the mass of Earth, and the period of revolution around Earth.
  - Determine the value of the distance required in (b). (Refer to Appendix C for data.)
  - Why must the satellite remain in a fixed location (relative to an observer on Earth's surface)?
  - Research the implications of having too many geosynchronous satellites in the space available above the equator. Summarize your findings in a brief report.



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