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Sec. 6.1 - 6.2 - Gravity and Kepler's Laws

Learning Goal: By the end of today I will be able to use Kepler's three laws of planetary motion.

Review $1\ N = 1\ kg*m/s^2$

$$\frac{m}{s^2} = \frac{N}{kg}$$

Gravitational Field strength and acceleration due to gravity are the same thing

Therefore 9.8 m/s^2 is the same as 9.8 N/kg (at Earth's surface).

One describes an acceleration, the other describes an attractive force proportional to the amount of mass an object has.

$$F_G = \frac{Gm_em}{r^2} \qquad F_g = m \cdot g$$

$$F_G = F_g$$

$G = 6.67\text{x}10^{-11}\ N*m^2/kg^2$

$G = 6.67408 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

$$\frac{Gm_em}{r^2} = m \cdot g$$

Field Strength $$\frac{Gm_e}{r^2} = g \qquad \frac{m}{s^2} = \frac{N}{kg}$$

m_e can be the mass of any planet

Table 1 Magnitude of the Gravitational Field Strength of Planets Relative to Earth's Value (g = 9.80 N/kg)

Planet	Surface Gravity (Earth = 1.00)
Mercury	0.375
Venus	0.898
Earth	1.00
Mars	0.375
Jupiter	2.53
Saturn	1.06
Uranus	0.914
Neptune	1.14
Pluto	0.067

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How to draw an ellipse

- cardboard
- two pins/tacks/tape
- ~20cm of string tied in a loop

Task -draw line down the center of you paper (longest direction)

1. Place your two pins at the extreme ends of the loop of string, move them in about 1cm - hold the tacks in place and trace out the shape.

   What shape is traced out by your pencil?

   Mark a point on the shape, measure the distance from the point to each one of the pins, add it together. Choose another point on the shape, measure the distance to the pins, and add together.

   What do you notice?

2. Move the two pins closer together (2-4cm increments), repeat part #1

   Check that the distance from the pins to a point on the curve pattern exits for these new shapes.

3. When the two pins are right on top of each other, what shape is created? Does the measure of the distance from the pins to the point still exist?

An *ellipse* is defined as a closed curve such that the sum of the distances from any point P to two other fixed points, the *foci* F_1 and F_2, is a constant: $PF_1 + PF_2$ equals a constant. **Figure 2** shows an ellipse. A circle is a special case of an ellipse for which the two foci are at the same position. Ellipses have different elongations as indicated by a quantity called *eccentricity* (e), shown in the diagram to be $e = \frac{c}{a}$; for a circle, $e = 0$ and for a long, thin ellipse $e \rightarrow 1$.

For this activity, each group of three or four students needs a pencil, a ruler, a piece of string tied to create a loop of length 40 cm, two tacks, and a piece of cardboard at least 40 cm by 40 cm. With the foci located 10 cm apart, attach the ends of the string to the tacks, hold the string taut with the pencil held against it, and draw an ellipse around the tacks. Note that the sum of the distances from any point on the curve to the two foci (tacks) is a constant (the length of that part of the string). On the reverse side of the cardboard, draw a second ellipse with a distance of 15 cm between the foci.

(a) Label the major and minor axes for each ellipse. Compare the eccentricities of your two ellipses.

(b) Planets travel in ellipses. What must be at one focus of the ellipse of each planet? What is at one focus of the elliptical orbit of the Moon?

Store the cardboard with the ellipses in a safe place so that you can use it for further study in this chapter.

Exercise care with the tacks, and remove them after you have drawn the ellipses.

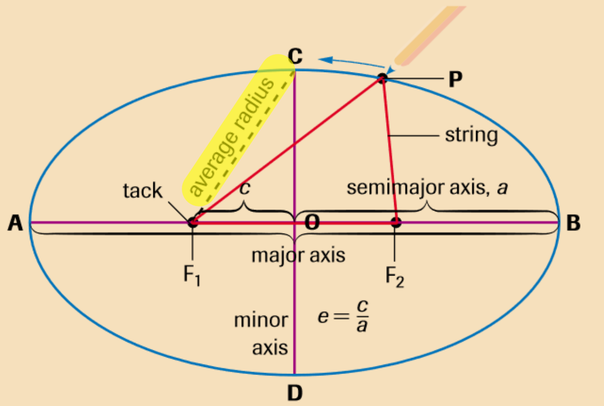

Figure 2
An ellipse. The line AB is the *major axis* of the ellipse; CD is the *minor axis*. The distance AO or OB is the length a of the *semimajor axis*. The eccentricity is defined as $\frac{c}{a}$, where c is the distance OF_1 or OF_2 from a focus to the centre of the ellipse.

Review - orbital velocity

To maintain a circular orbit around Earth, we found in sec. 3.4 that the following was true.

$$v = \sqrt{\frac{Gm_e}{r}}$$

$G = 6.67 \times 10^{-11}$ N*m^2/kg^2

m_e is the mass of Earth

Generalize by removing Earth mass

$$v = \sqrt{\frac{GM}{r}}$$

M is the mass of a planet/object

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Example

Determine the speeds of Venus and Earth around the Sun. Values are given, but can be found in appendix C of the textbook.

$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

$M_S = 1.99 \times 10^{30}\ \text{kg}$

$r_V = 1.08 \times 10^{11}\ \text{m}$

$r_E = 1.49 \times 10^{11}\ \text{m}$

$v_V = ?$

$v_E = ?$

Venus

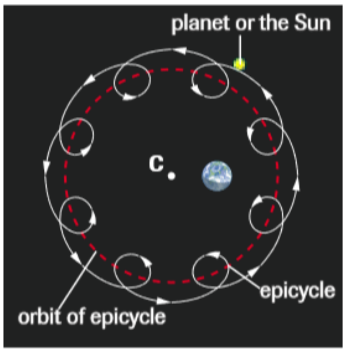
$$v_V = \sqrt{\frac{GM_S}{r_V}}$$

Earth

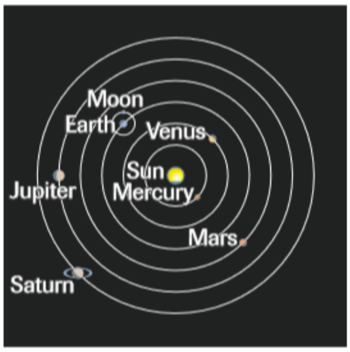
$$v_E = \sqrt{\frac{GM_S}{r_E}}$$

A brief history of planetary observations

- Earth was thought to be the center of the universe, and all planets and the sun revolved around it - geocentric



- Copernicus (1473-1543) published a book (1543) supporting a heliocentric model (sun centered) which lead to some big trouble with the church
- first believed that everything was circular in nature - proved incorrect by data collection



- Tycho Brahe (1546 - 1630) spent 25 years collecting data
- hired mathematician Johannes Kepler (1571-1630) to analyze data
- Kepler gets the credit

Kepler's Laws

Kepler's First Law of Planetary Motion

Each planet moves around the Sun in an orbit that is an ellipse, with the Sun at one focus of the ellipse.

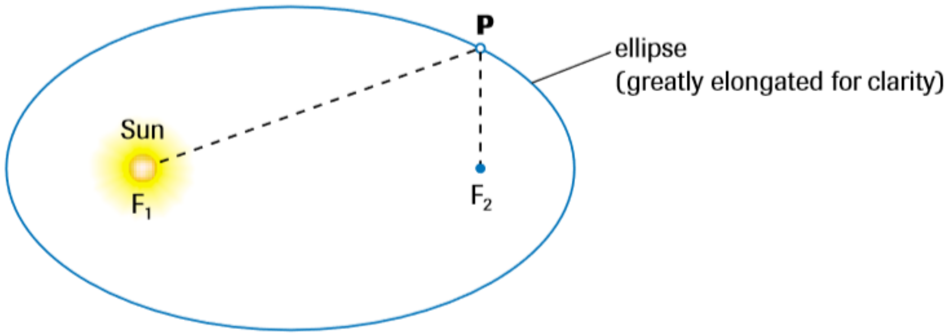
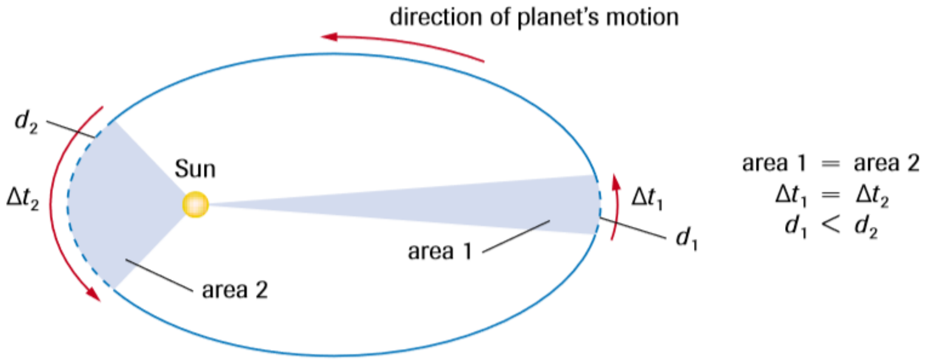


Figure 5
The orbit of a planet is an ellipse with the Sun at one focus. Based on the definition of an ellipse, for any point P, the distance $PF_1 + PF_2$ is a constant.

Most orbits are very close to circles, Earth is about 3% variation.

Kepler's Second Law of Planetary Motion

The straight line joining a planet and the Sun sweeps out equal areas in space in equal intervals of time.



Where would the planet appear to move fastest?

Kepler's Third Law of Planetary Motion
The cube of the average radius r of a planet's orbit is directly proportional to the square of the period T of the planet's orbit.

Writing Kepler's third law mathematically, we have:

$$r^3 \propto T^2$$

$$r^3 = C_S T^2$$

or $$C_S = \frac{r^3}{T^2}$$

where C_S is the constant of proportionality for the Sun for Kepler's third law. In SI units, C_S is stated in metres cubed per seconds squared (m^3/s^2).

The sun has its own constant C_s value, so does Earth C_E

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SUMMARY Orbits and Kepler's Laws

- The orbits of planets are most easily approximated as circles even though they are ellipses.
- Kepler's first law of planetary motion states that each planet moves around the Sun in an orbit that is an ellipse, with the Sun at one focus of the ellipse.
- Kepler's second law of planetary motion states that the straight line joining a planet and the Sun sweeps out equal areas in space in equal intervals of time.
- Kepler's third law of planetary motion states that the cube of the average radius r of a planet's orbit is directly proportional to the square of the period T of the planet's orbit.

Homework

Read 274 - 277

page 276 #3, 4, 6

page 277 #7

Read 278 - 284

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