Sec. 5.2 - Conservation of Momentum

Learning Goal: By the end of today, I will be able to conserve momentum in a ONE dimensional problem.

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Law of Conservation of Linear Momentum

If the net force acting on a system of interacting objects is zero, then the linear momentum of the system before the interaction equals the linear momentum of the system after the interaction.

Example

If I roll one ball towards another ball ...

(a) I remove my hand from the ball before impact - net force is zero, conservation is applied

(b) I keep my hand on the ball - net force is not zero, conservation is not conserved

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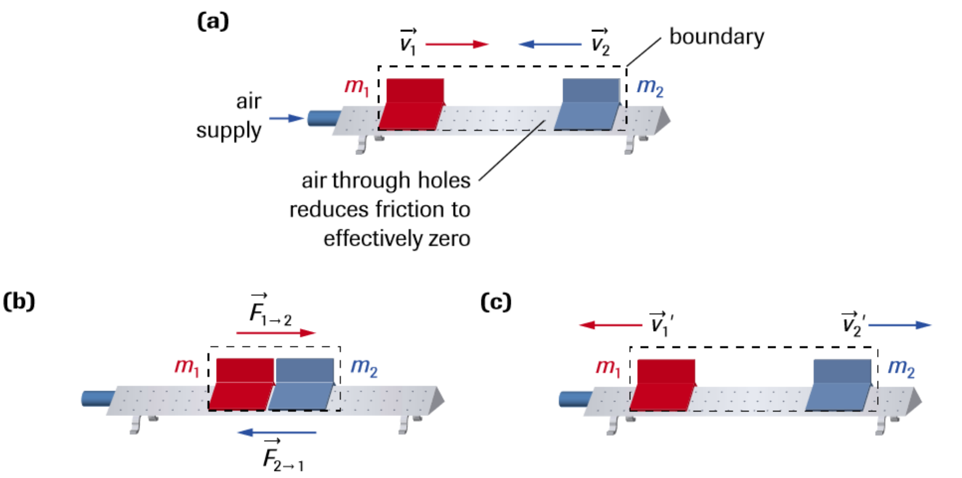
Frictionless Sliders

Each slider has momentum at the beginning of the interaction.



When they collide, they experience a force of equal magnitude, but opposite direction (Newton 3rd). We will assume there are no deformations, so the momentum in the system has to stay the same, before and after the collision.

$$\bar{\rho}_{total} = m_1\bar{v}_1 + m_2\bar{v}_2$$

3 minutes - read proof on page 240

$$\vec{F}_{2\to 1} = -\vec{F}_{1\to 2}$$

where $\vec{F}_{2\to 1}$ is the force glider 2 exerts on glider 1, and $\vec{F}_{1\to 2}$ is the force glider 1 exerts on glider 2. Thus, the net force acting on the two-glider system is zero:

$$\vec{F}_{2\to 1} + \vec{F}_{1\to 2} = 0$$

Note that the vertical forces—gravity and the upward force exerted by the air—also add to zero. Therefore, the net force on the system is zero.

The forces exerted by the gliders on each other cause each glider to accelerate according to Newton's second law of motion, $\sum\vec{F} = m\vec{a}$. Starting with the equation involving the forces we have:

$$\begin{aligned} \vec{F}_{2\to 1} &= -\vec{F}_{1\to 2} \\ m_1\vec{a}_1 &= -m_2\vec{a}_2 \\ m_1\frac{\Delta\vec{v}_1}{\Delta t_1} &= -m_2\frac{\Delta\vec{v}_2}{\Delta t_2} \end{aligned}$$

We know that $\Delta t_1 = \Delta t_2$ because the force $\vec{F}_{1\to 2}$ acts only as long as the force $\vec{F}_{2\to 1}$ acts; that is, $\vec{F}_{1\to 2}$ and $\vec{F}_{2\to 1}$ act only as long as the gliders are in contact with each other. Thus,

$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$

This equation summarizes the law of conservation of (linear) momentum for two colliding objects. It states that *during an interaction between two objects on which the total net force is zero, the change in momentum of object 1 ($\Delta\vec{p}_1$) is equal in magnitude but opposite in direction to the change in momentum of object 2 ($\Delta\vec{p}_2$)*. Thus,

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

Let us now consider the glider system before and after the collision (**Figure 2(c)**). We will use the prime symbol (′) to represent the final velocities:

$$\begin{aligned} m_1\Delta\vec{v}_1 &= -m_2\Delta\vec{v}_2 \\ m_1(\vec{v}_1' - \vec{v}_1) &= -m_2(\vec{v}_2' - \vec{v}_2) \\ m_1\vec{v}_1' - m_1\vec{v}_1 &= -m_2\vec{v}_2' + m_2\vec{v}_2 \\ m_1\vec{v}_1 + m_2\vec{v}_2 &= m_1\vec{v}_1' + m_2\vec{v}_2' \end{aligned}$$

This equation represents another way of summarizing the law of conservation of (linear) momentum. It states that *the total momentum of the system before the collision equals the total momentum of the system after the collision*. Thus,

$$\vec{p}_{\text{system}} = \vec{p}'_{\text{system}}$$

It is important to remember that momentum is a vector quantity; thus, any additions or subtractions in these conservation of momentum equations are vector additions or vector subtractions. These equations also apply to the conservation of momentum in two dimensions, which we will discuss in Section 5.4. Note that the equations written for components are:

$$\begin{aligned} m_1\Delta v_{1x} &= -m_2\Delta v_{2x} \\ m_1\Delta v_{1y} &= -m_2\Delta v_{2y} \\ m_1v_{1x} + m_2v_{2x} &= m_1v_{1x}' + m_2v_{2x}' \\ m_1v_{1y} + m_2v_{2y} &= m_1v_{1y}' + m_2v_{2y}' \end{aligned}$$

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Ideal situation - external net force is zero

Conservation of Momentum Before and After

Before After

$$m_1\bar{v}_1 + m_2\bar{v}_2 = m_1\bar{v}_1' + m_2\bar{v}_2'$$

Non-Ideal situation

Before After

$$m_1\bar{v}_1 + m_2\bar{v}_2 + J = m_1\bar{v}_1' + m_2\bar{v}_2'$$

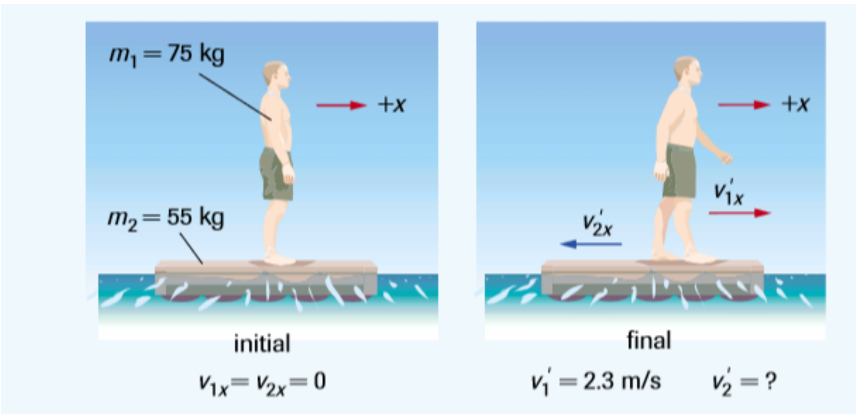
J is impulse and represents momentum transfer in/out of the system

Example

A vacationer of mass 75 kg is standing on a stationary raft of mass 55 kg. The vacationer then walks toward one end of the raft at a speed of 2.3 m/s relative to the water. What are the magnitude and direction of the resulting velocity of the raft relative to the water? Neglect fluid friction between the raft and the water.

Not walking - no velocities
- no momentum

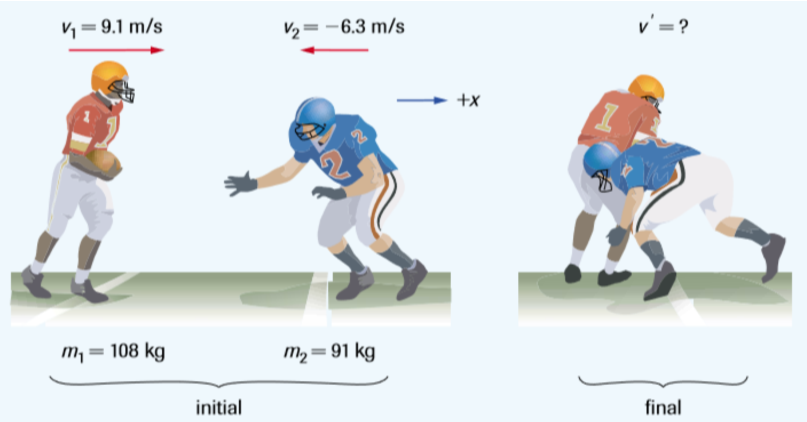
Walking - has velocity, and momentum



$$m_1\bar{v}_1 + m_2\bar{v}_2 = m_1\bar{v}'_1 + m_2\bar{v}'_2$$

Example

During a football game, a fullback of mass 108 kg, running at a speed of 9.1 m/s, is tackled head-on by a defensive back of mass 91kg, running at a speed of 6.3 m/s. What is the speed of this pair just after the collision?



$$m_1\bar{v}_1 + m_2\bar{v}_2 = m_1\bar{v}_1' + m_2\bar{v}_2'$$

(after collision, masses are stuck together travelling the same velocity)

major assumption - "sticky" problems

SUMMARY Conservation of Momentum in One Dimension

- The law of conservation of linear momentum states that if the net force acting on a system is zero, then the momentum of the system is conserved.
- During an interaction between two objects in a system on which the total net force is zero, the change in momentum of one object is equal in magnitude, but opposite in direction, to the change in momentum of the other object.
- For any collision involving a system on which the total net force is zero, the total momentum before the collision equals the total momentum after the collision.

Homework

Read page 239-245

page 243 #3, 5, 7, 9a-c

page 245 #5, 6, 7, 10