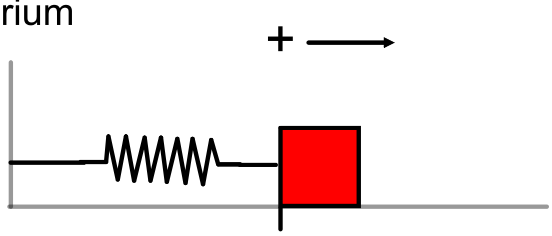
Simple Harmonic Motion (SHM)

Simple harmonic motion (SHM) is periodic vibratory motion in which the force (and the acceleration) is directly proportional to the displacement.

Learning Goal: By the end of today, I will understand the aforementioned relationships.

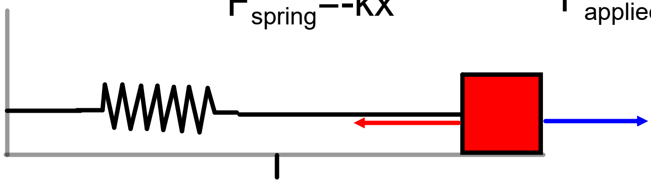
Horizontal Model

Equilibrium



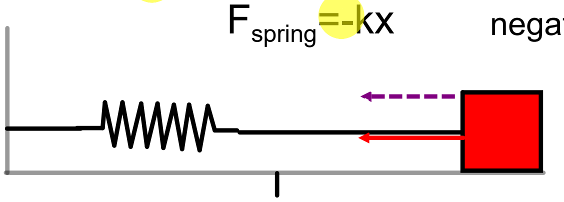
Held in place

$F_{spring}=-kx$ $F_{applied}=kx$



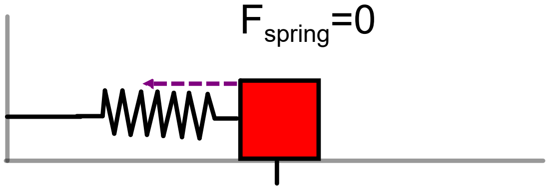
Released - Tension End (+x)

$F_{spring}=-kx$ negative force = to the left



At x=0

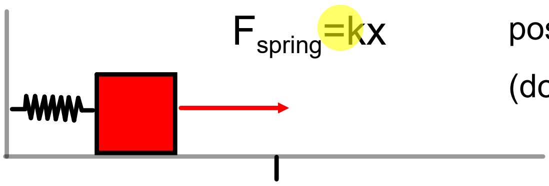
$F_{spring}=0$



Stopped - Compression End (-x)

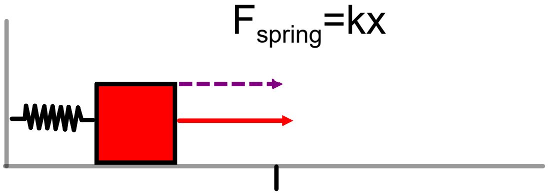
$F_{spring}=kx$ positive force = to the right

(double negatives from k and x)



Compression End (-x)

$F_{spring}=kx$



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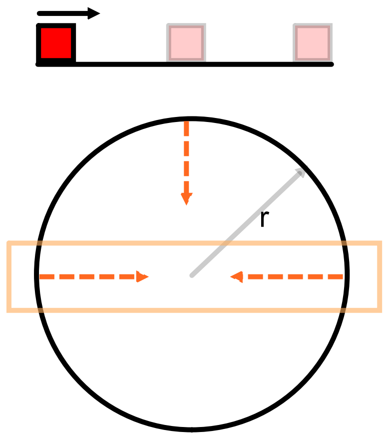
Put a piece of tape on a wheel and view it sideways and you see the same motion as the horizontal mass.

We might recognize this as periodic motion from math class (sine and cosine), but we can also make a connection to circular motion from our last unit.

The connection we are going to make happens through the Period (time for one cycle).

We are also only going to focus on one direction of motion, in this case it is the "x" direction.



$$a_c = \frac{4\pi^2 r}{T^2}$$

$$T^2 = \frac{4\pi^2 r}{a_c}$$

$$T = \sqrt{\frac{4\pi^2 r}{a_c}}$$

Note: where does the centripetal acceleration affect the horizontal motion of the most?

a = big	a = 0	a = big
v = 0	v = big	v = 0
big spring force	no spring force	big spring force

$$T = 2\pi\sqrt{\frac{r}{a_c}}$$

Period for circular motion (isolated)

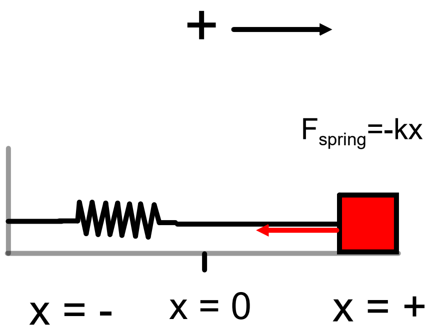
$$\boxed{T = 2\pi\sqrt{\frac{A}{a_c}}}$$

equation #1

radius = Amplitude

Mar 18-1:26 PM

For the horizontal mass, we are going to link two already developed equations:



Hooke's Law

$$F_x = -kx$$

Newton's Law

$$F_x = ma_x$$

$$F_x = F_x$$

$$-kx = ma_x$$

$$\frac{-k}{m}x = a_x$$

Acceleration and displacement are proportional (and linear).

Consider.. (i) as x gets larger and positive, a_x gets...

(ii) as x gets closer to zero, a_x gets...

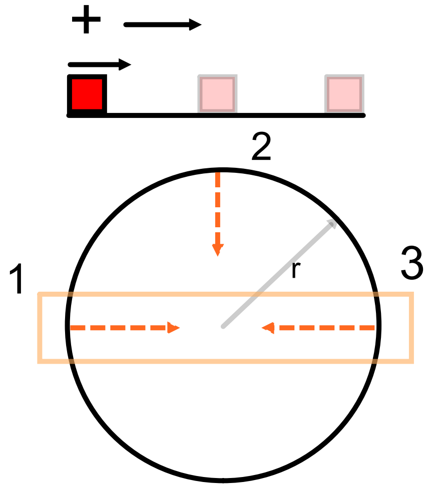
Rearranging

always positive output → $$\frac{-x}{a_x} = \frac{m}{k}$$ ← constant

tricky thought

Tricky Connection con't

$$\frac{-x}{a_x} = \frac{m}{k}$$



Where "A" is the max amplitude

At 1

-x and $+a_x$

or

r=A and $+a_c$

same as A

$$\frac{-(-x)}{a_x} = \frac{m}{k}$$

At 2

x=0 and a_x=0

or

r =0 and a_c=0

(Horizontal components only)

$$\frac{-x}{a_x} = \frac{m}{k}$$

$$-x = \frac{m}{k} a_x$$

$$0 = \frac{m}{k} 0$$

At 3

+x and $-a_x$

or

r=A and $-a_c$

same as A

$$\frac{-x}{(-a_x)} = \frac{m}{k}$$

general case

$$\frac{-x}{a_x} = \frac{A}{a_c}$$

unique case at ends

equation #2

Last connection

$$\frac{-x}{a_x} = \frac{A}{a_c}$$

(always positive)

$$T = 2\pi\sqrt{\frac{A}{a_c}}$$

$$T = 2\pi\sqrt{\frac{x}{a_x}}$$

but $\frac{x}{a_x} = \frac{m}{k}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

This works for both vertical and horizontal systems.

T is in seconds

m is in kg

k is in N/m

We can determine the period of a SHM system by knowing the mass and the spring constant k......WOW!

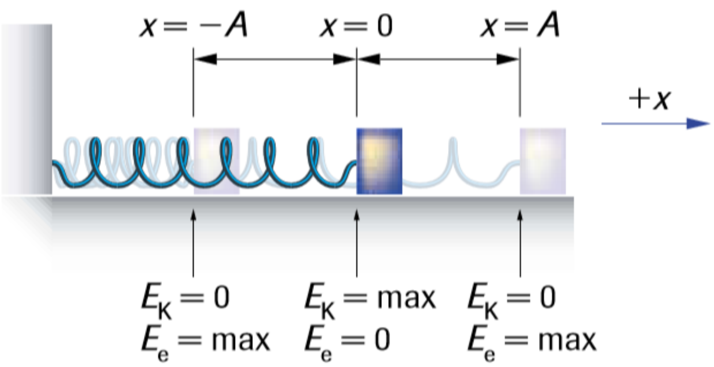
What factor DOES NOT affect period?

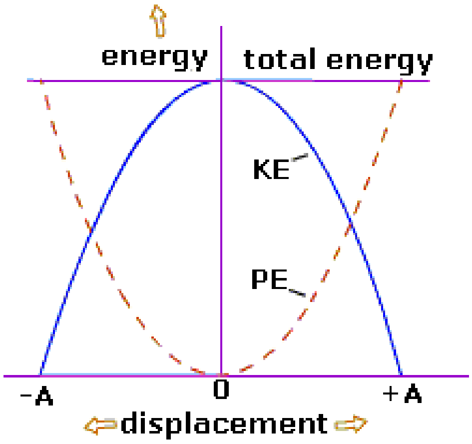
Mar 18-2:06 PM

Example

A 0.45-kg mass is attached to a spring with a force constant of 1.4×10^2 N/m. The mass-spring system is placed horizontally, with the mass resting on a surface that has negligible friction. The mass is displaced 15 cm, and is then released. Determine the period and frequency of the SHM.

Energy in SHM





elastic energy

$$E_e = \frac{1}{2}kx^2$$

$$E_{emax} = \frac{1}{2}kA^2$$

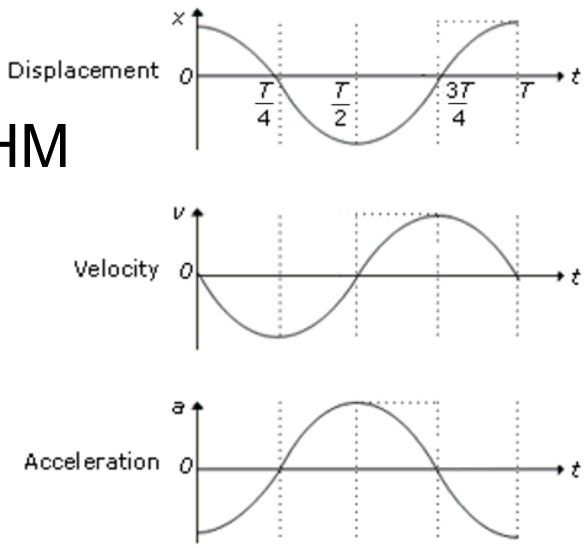
kinetic energy

$$E_k = \frac{1}{2}mv^2$$

Conservation of Energy

$$E_T = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

Motion Graphs for SHM



Example

A 55-g box is attached to a horizontal spring of force constant 24 N/m. The spring is then compressed to a position A = 8.6 cm to the left of the equilibrium position. The box is released and undergoes SHM.

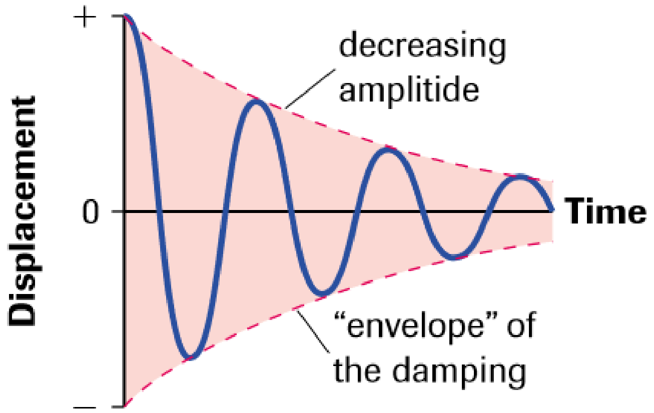
(a) What is the speed of the box when it is at position x = 5.1cm from the equilibrium position?

(b) What is the maximum speed of the box?

Damped Harmonic Motion

Damped harmonic motion is periodic or repeated motion in which the amplitude of vibration and thus the energy decrease with time.

- scale
- car suspension system
- compound bow
- earthquake design



Beginning Final

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + E_{lost}$$

SUMMARY Elastic Potential Energy and Simple Harmonic Motion

- Hooke's law for an ideal spring states that the magnitude of the force exerted by or applied to a spring is directly proportional to the displacement the spring has moved from equilibrium.
- The constant of proportionality k in Hooke's law is the force constant of the spring, measured in newtons per metre.
- Elastic potential energy is the energy stored in objects that are stretched, compressed, twisted, or bent.
- The elastic potential energy stored in a spring is proportional to the force constant of the spring and to the square of the stretch or compression.
- Simple harmonic motion (SHM) is periodic vibratory motion such that the force (and thus the acceleration) is directly proportional to the displacement.
- A reference circle can be used to derive equations for the period and frequency of SHM.
- The law of conservation of mechanical energy can be applied to a mass–spring system and includes elastic potential energy, kinetic energy, and, in the case of vertical systems, gravitational potential energy.
- Damped harmonic motion is periodic motion in which the amplitude of vibration and the energy decrease with time.

Homework

Read page 212-219

page 214 #16 - 19

page 217 #23-26

More practice if you need it.

page 219 #6, 8, 10, 12, 15 Homework