

## Sec. 4.1 - 4.2 - Work and Kinetic Energy

Learning Goal: By the end of today I will be able to calculate mechanical work done by an applied force, and I will be able to calculate the kinetic energy of a moving object.

## Section 5.1 - Work

"Work" is another word that has very different meanings in the science world and the everyday world.

"Mechanical" Work is done by applying a force over a distance.

For example, pushing a heavy object across a floor required mechanical work.

To calculate the concept we call WORK, we multiply the applied Force magnitude times the **displacement**.

$$\text{Work} = \text{Force} \times \text{Displacement} \quad \text{or} \quad W = F \Delta d$$

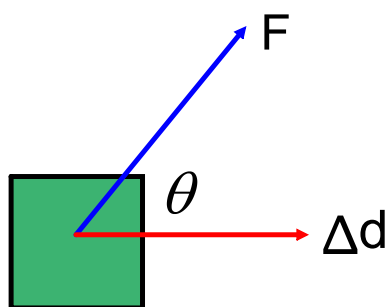
Work is a Scalar (magnitude, no direction) even though it has two vectors being multiplied together. (we'll return to this in Calculus)

The SI unit of work is a Newton-Meter (N·m) also called a Joule.

$$1 \text{ Joule} = 1 \text{ N}\cdot\text{m}$$

When the force and the displacement are NOT aligned, we need to use a revised Work equation that uses just the component of Force that is in the direction of the displacement. ( $\theta$  is the angle between the force and displacement)

$$W = F(\cos \theta) \cdot \Delta d$$



#### LEARNING TIP

##### Dot Products

The scalar product of two vectors is commonly indicated by placing a dot between the vector symbols (e.g.,  $W = \vec{F} \cdot \Delta \vec{d}$ ) and unit symbols (N·m). It is for this reason that a scalar product is sometimes called a dot product. A dot product may also be represented by using scalar symbols without the dot between them (as we do in this book),  $W = F\Delta d$ .

If you drag a 20 kg box, 5m across a floor, using a force of 50 N ( $30^\circ$ ) to the horizontal;

(a) How much work has the applied force done on a near frictionless surface?

(b) How much work has the Normal force done?

## Zero Work

Zero work is done when the angle between the applied net force and the displacement of the object is  $90^\circ$ .

Objects travelling at constant velocity can also experience zero work (but not always).

### Sample Problem 2

Ranbir wears his backpack as he walks forward in a straight hallway. He walks at a constant velocity of  $0.8 \text{ m/s}$  for a distance of  $12 \text{ m}$ . How much mechanical work does Ranbir do on his backpack?

Consider the system diagram shown in **Figure 5**. Ranbir walks at constant velocity. Thus, there is no acceleration in the direction of displacement and no applied force on the backpack in that direction. The only applied force on the backpack is the force that Ranbir's shoulders apply on the backpack ( $\vec{F}_a$ ) to oppose the force of gravity on the backpack ( $\vec{F}_g$ , the backpack's weight). However, neither the applied force nor the force of gravity does work on the backpack because both forces are perpendicular to the displacement. Therefore, Ranbir does no mechanical work at all on the backpack.

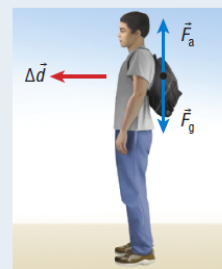
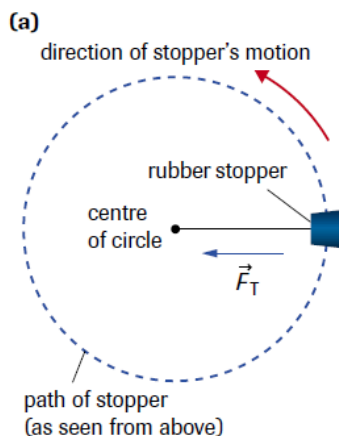


Figure 5

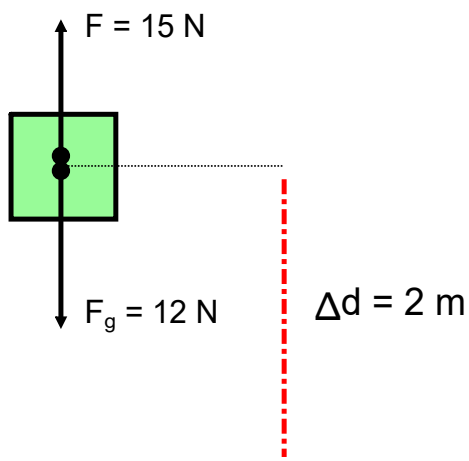


## Positive and Negative Work

Sometimes an object has more than one force acting on it.

The act of picking up a book has an applied force [up] to lift the book and a downward force of gravity.

In cases such as this, the TOTAL work done is the algebraic sum of the work done by all of the forces acting on the object.



$$\begin{aligned} \text{Work} &= 15 \times 2 \\ &= 30 \text{ J} \end{aligned}$$

Work done by the applied force

$$\begin{aligned} \text{Work} &= -12 \times 2 \\ &= -24 \text{ J} \end{aligned}$$

Work done by the gravity force

$$\begin{aligned} \text{Total (Net) Work} &= 30 - 24 \\ &= 6 \text{ J} \end{aligned}$$

OR

$$\text{Work} = F_{\text{net}} \times \text{displacement}$$

Graphing Work Done - Positive and Negative Values - Force vs Displacement Graph

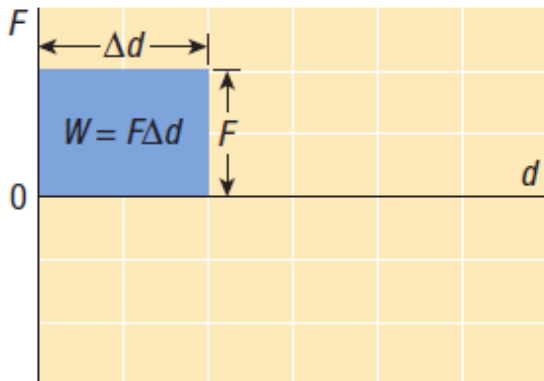


Figure 7  $F$ - $d$  graph for a constant force acting through a displacement

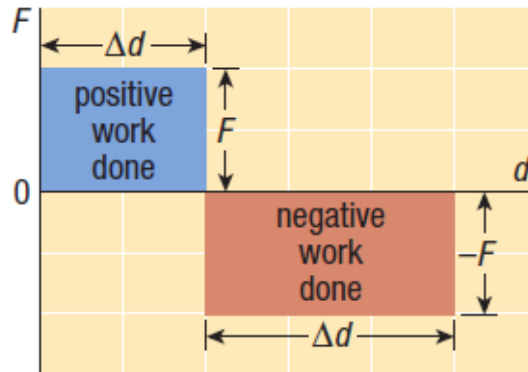


Figure 8  $F$ - $d$  graph representing positive and negative work done

The AREA of the bar graph is the amount of WORK done.  
 (Just like the area of a rectangle is given by base x height)

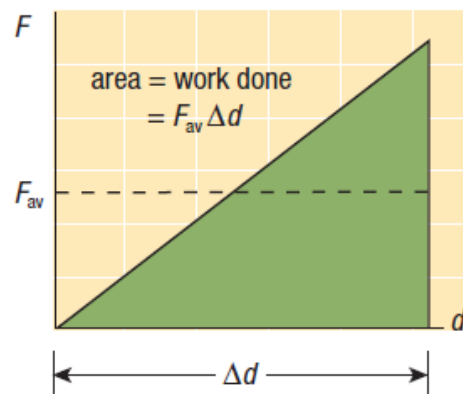


Figure 9  $F$ - $d$  graph representing a uniformly increasing force

The AREA under the Force - Time graph (triangle) is the amount of WORK done.

## Kinetic Energy - The Energy of Moving Things

We will combine several ideas of the past chapters to give us insight into what is Kinetic Energy.

Work Equation - measured in N m or Joules

Force Equation - measured in Newtons (N)

$$W_{Net} = F_{Net} (\cos \theta) \Delta d \quad F_{Net} = m \cdot a$$



$$W_{Net} = m \cdot a (\cos \theta) \Delta d$$

$$W_{Net} = m (\cos \theta) \cdot a \Delta d$$



$$W_{Net} = m (\cos \theta) \cdot \frac{V_f^2 - V_i^2}{2}$$

$$W_{Net} = m \cdot \frac{V_f^2 - V_i^2}{2}$$

$$W_{Net} = \frac{mV_f^2}{2} - \frac{mV_i^2}{2}$$

$$W_{Net} = \frac{mV_f^2}{2}$$

Kinematic Equation

$$V_f^2 = V_i^2 + 2a \cdot \Delta d$$

$$\frac{V_f^2 - V_i^2}{2} = a \cdot \Delta d$$

**Assumption #1** - Force (and therefore acceleration) are in the same direction as displacement,  $\theta = 0$ ,  $\cos \theta = \cos 0 = 1$

**Assumption #2** - object is starting from Rest, therefore  $V_i = 0$



Almost there....

$$W_{Net} = \frac{mV_f^2}{2}$$



$$E_K = \frac{mv^2}{2}$$

Analyze from a unit perspective

$$\begin{aligned} &kg \cdot \left(\frac{m}{s}\right)^2 \\ &= \frac{kg \cdot m^2}{s^2} \\ &= \frac{kg \cdot m}{s^2} \cdot m \\ &= N \cdot m \\ &= \text{Joule} \end{aligned}$$

This is the formula for the Energy of an object, it is measured in Joules or N·m .

## The Relationship between Mechanical Work and Kinetic Energy

You can observe the relationship between mechanical work and kinetic energy by analyzing the mechanical work and kinetic energy equations. Since  $E_k = \frac{mv^2}{2}$ , the equation  $W_{\text{net}} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}$  may be written as  $W_{\text{net}} = E_{k_f} - E_{k_i}$  or  $W_{\text{net}} = \Delta E_k$ , where  $E_{k_f}$  is the final kinetic energy and  $E_{k_i}$  is the initial kinetic energy of the object. In words, this equation tells us that the total mechanical work,  $W$ , that increases the speed of an object is equal to the change in the object's kinetic energy,  $E_{k_f} - E_{k_i}$ . In other words, work is a change in energy. This relationship between kinetic energy and mechanical work is known as the **work-energy principle**.

**work-energy principle** the net amount of mechanical work done on an object equals the object's change in kinetic energy

$$W_{\text{net}} = E_{k2} - E_{k1}$$

Total work done

$$W_{\text{Net}} = \frac{mV_f^2}{2} - \frac{mV_i^2}{2}$$

Also,  $W_{\text{net}} = F_{\text{net}} \times d$

**Practice**

1. A 1300 kg car starts from rest at a stoplight and accelerates to a speed of 14 m/s over a displacement of 82 m. **T/I**
  - (a) Calculate the net work done on the car.
  - (b) Calculate the net force acting on the car.
2. A 52 kg ice hockey player moving at 11 m/s slows down and stops over a displacement of 8.0 m. **T/I C**
  - (a) Calculate the net force on the skater.
  - (b) Give two reasons why you can predict that the net work on the skater must be negative.

**SUMMARY*****Work Done by a Constant Force***

- Work is the energy transferred to an object when a force  $\vec{F}$ , acting on the object, moves it through a distance  $\Delta d$ .
- The SI unit of work is the joule (J).
- If the force causing an object to undergo a displacement is at an angle to the displacement, only the component of the force in the direction of the displacement does work on the object.
- Under certain conditions, zero work is done on an object even if the object experiences an applied force or is in motion.

**SUMMARY*****Kinetic Energy and the Work-Energy Theorem***

- Kinetic energy  $E_K$  is energy of motion. It is a scalar quantity, measured in joules (J).
- The work-energy theorem states that the total work done on an object equals the change in the object's kinetic energy, provided there is no change in any other form of energy.

Homework (these are short questions)

4.1

Read 178 - 183

page 181 #4, 6

page 183 #5, 7

4.2

Read 184 - 188

page 186 #2, 4, 5

page 188 #6, 7