# Sec. 3.1 - Uniform Circular Motion

Learning Goal: By the end of today, I will be able to recognize the key elements in uniform circular motion (tangential velocity, tangential acceleration, radial or centripetal acceleration).

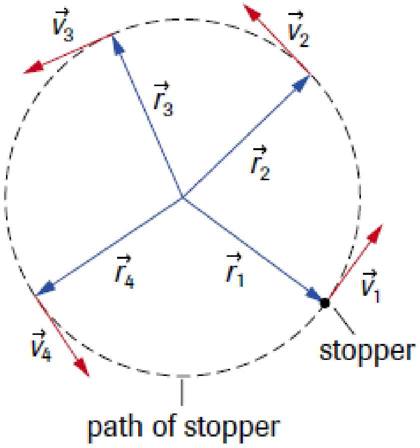



**Figure 2**
In uniform circular motion, the speed of the object remains constant, but the velocity vector changes because its direction changes. The radius of the path also remains constant. Notice that the instantaneous position vector (also called the radius vector) is perpendicular to the velocity vector and the velocity vectors are tangent to the circle.

## Centripetal Acceleration

The "lim" or limit is a concept that tells us to make the time interval between calculations very small, approaching zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

Reminder: $$v_1 + \Delta v = v_2$$

$$\Delta v = v_2 - v_1$$

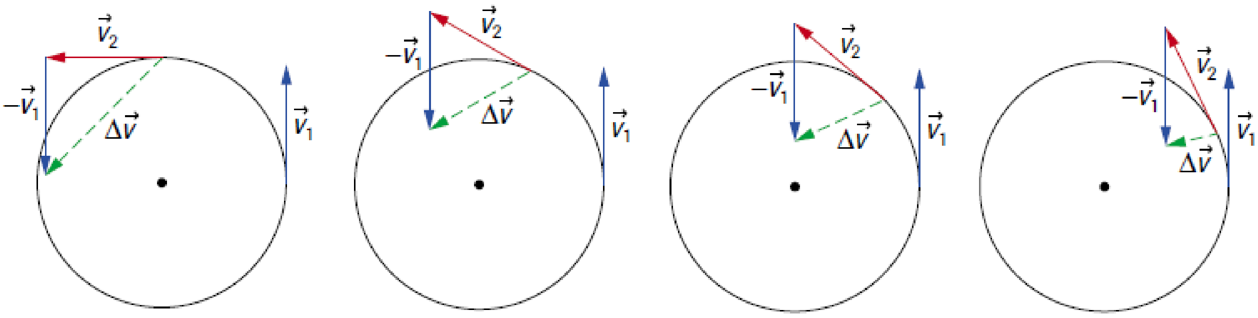
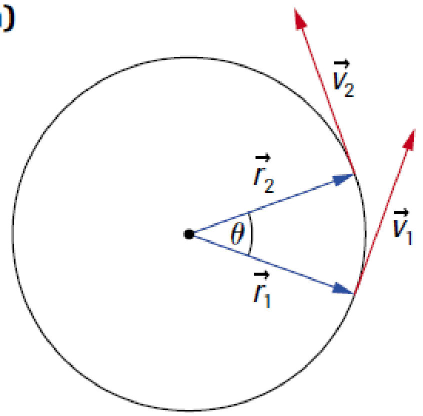



**Figure 3**
As the time interval between $\vec{v}_1$ and $\vec{v}_2$ is made shorter and shorter, $\Delta\vec{v}$ comes closer and closer to pointing toward the centre of the circle. In the diagram on the far right, $\Delta t$ is very small and the $\Delta\vec{v}$ vector is nearly perpendicular to the instantaneous velocity vector $\vec{v}_2$.

Notice: delta v is getting smaller, and is starting to align with the radius, as $v_1$ and $v_2$ get closer together.

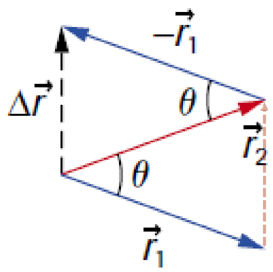
## Geometry Elements

(a)




- position and velocity vectors with a very small angle in between them

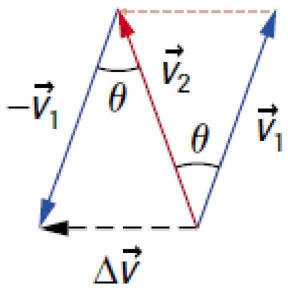
(b)




$$r_1 + \Delta r = r_2$$
$$\Delta r = r_2 - r_1$$

(change in position)

(c)




$$v_1 + \Delta v = v_2$$
$$\Delta v = v_2 - v_1$$

(change in velocity

Key elements

- $r_1$ and $r_2$ have the same magnitude

- $v_1$ and $v_2$ have the same magnitude

- both position and velocity triangles are isosceles, and similar (90° angle, SAS)

- $\Delta r = r_2 - r_1$

- $\Delta v = v_2 - v_1$

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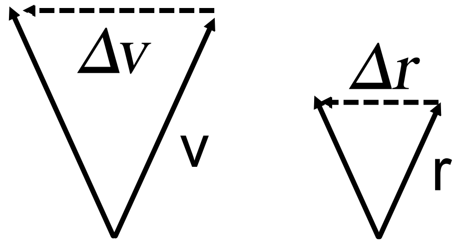
From Similarity, we can state




$$\frac{|\Delta\vec{v}|}{|\vec{v}|} = \frac{|\Delta\vec{r}|}{|\vec{r}|} \quad \text{where} \quad |\vec{v}| = |\vec{v}_1| = |\vec{v}_2| \text{ and } |\vec{r}| = |\vec{r}_1| = |\vec{r}_2|$$

or $$|\Delta\vec{v}| = \frac{|\vec{v}| \times |\Delta\vec{r}|}{|\vec{r}|}$$

items that are easy to measure or determine

Now, the magnitude of the centripetal acceleration $\vec{a}_c$ is

$$|\vec{a}_c| = \lim_{\Delta t \to 0} \frac{|\Delta\vec{v}|}{\Delta t}$$

We can divide both sides of the $|\Delta\vec{v}|$ equation by $\Delta t$ to obtain

$$\frac{|\Delta\vec{v}|}{\Delta t} = \frac{|\vec{v}|}{|\vec{r}|} \times \frac{|\Delta\vec{r}|}{\Delta t}$$

Therefore, $$|\vec{a}_c| = \lim_{\Delta t \to 0} \left( \frac{|\vec{v}|}{|\vec{r}|} \times \frac{|\Delta\vec{r}|}{\Delta t} \right)$$

Now the magnitude of the instantaneous velocity is

$$|\vec{v}| = \lim_{\Delta t \to 0} \frac{|\Delta\vec{r}|}{\Delta t}$$

Therefore, $$|\vec{a}_c| = \frac{|\vec{v}|}{|\vec{r}|} \times |\vec{v}|$$

Hence, the magnitude of the centripetal acceleration is

$$a_c = \frac{v^2}{r}$$

# Example

A child on a merry-go-round is 4.4 m from the centre of the ride, travelling at a constant speed of 1.8 m/s.

Determine the magnitude of the child’s centripetal acceleration.

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## Wave Mechanics Refresh

Frequency is the number of cycles per second

Period is the time (or length) of one complete cycle

$$f = \frac{1}{T} \qquad v = f\lambda$$

In one complete circular revolution, an object travels the circumference of the circle.

The velocity of the object is given by $v = \frac{d}{t}$

or $v = \frac{2\pi r}{T}$

This gives us another version of our centripetal acceleration formula.

$$a_c = \frac{(2\pi r)^2}{T^2 r}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

If we introduce f = 1/T then we get another equation.

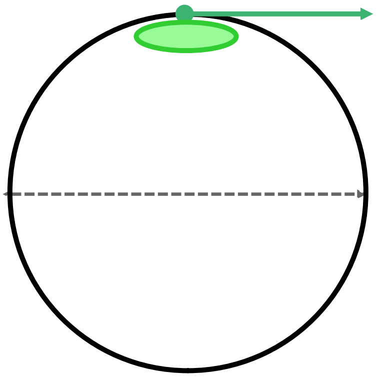
$$a_c = 4\pi^2 r f^2$$

## Summary

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

# Example

Find the magnitude and direction of the centripetal acceleration of a piece of lettuce on the inside of a rotating salad spinner. The spinner has a diameter of 19.4 cm and is rotating at 780 rpm (revolutions perminute). The rotation is clockwise as viewed from above. At the instant of inspection, the lettuce is moving eastward.



$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

# Example

Determine the frequency and period of rotation of an electric fan if a spot at the end of one fan blade is 15 cm from the centre and has a centripetal acceleration of magnitude $2.37 \times 10^3$ m/s$^2$.

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

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## SUMMARY Uniform Circular Motion

- Uniform circular motion is motion at a constant speed in a circle or part of a circle with a constant radius.
- Centripetal acceleration is the acceleration toward the centre of the circular path of an object travelling in a circle or part of a circle.
- Vector subtractions of position and velocity vectors can be used to derive the equations for centripetal acceleration.

Homework

Read 122 - 127

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