5.4 Conservation of Momentum in Two Dimensions

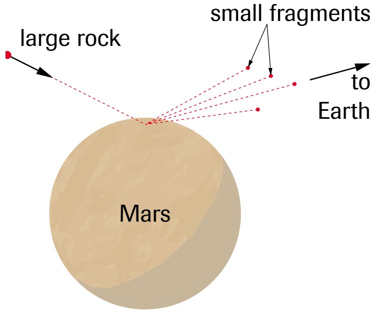

Figure 1
The two-dimensional nature of the collision between a rock and the surface of Mars

How could a chunk of rock from Antarctica provide a basis for research into the possibility that life once existed on Mars? The answer lies in a two-dimensional collision that occurred about 15 million years ago, when a large high-speed rock crashed into Mars at a glancing angle (**Figure 1**). Some of the kinetic energy of the rock was converted into thermal energy, melting some of the surface rock and trapping bubbles from the atmosphere in material that splashed off the surface, cooled, and flew off into space. Eventually a chunk of that surface material from Mars became the rock that landed in Antarctica. Researchers discovered that the rock's dissolved gases were the same as those identified by probes landing on Mars. Further research identified possible materials that may be associated with microscopic life forms.

In the previous section, we examined the conservation of momentum in one-dimensional collisions, such as the recoil of a raft when a person walks on it or the head-on collisions of automobiles. For momentum to be conserved, the net force on the system must be zero. The forces exerted by the objects on each other are equal in magnitude but opposite in direction (from Newton's third law) and add to zero. If other forces on the system also add to zero or are so small as to be negligible, then the net force on the system will be zero.

The same reasoning applies to two-dimensional situations, such as the collision between pucks on an air table with negligible friction (see Investigation 5.3.1). Since both the net force and momentum are vector quantities, when we say that momentum is conserved, we mean that both the magnitude and direction of the momentum vector do not change. Alternatively, in two dimensions, we can state that both the x- and y-components of the momentum do not change.

The law of conservation of momentum applies to any situation in which a system is subject to a net force of zero. It applies to collisions between all sorts of objects; it also applies to interactions that are not collisions, such as the ejection of gases from a rocket thruster to control the spacecraft's motion.

SAMPLE problem 1

A 38-kg child is standing on a 43-kg raft that is drifting with a velocity of 1.1 m/s [N] relative to the water. The child then walks on the raft with a net velocity of 0.71 m/s [E] relative to the water. Fluid friction between the raft and water is negligible. Determine the resulting velocity of the raft relative to the water.

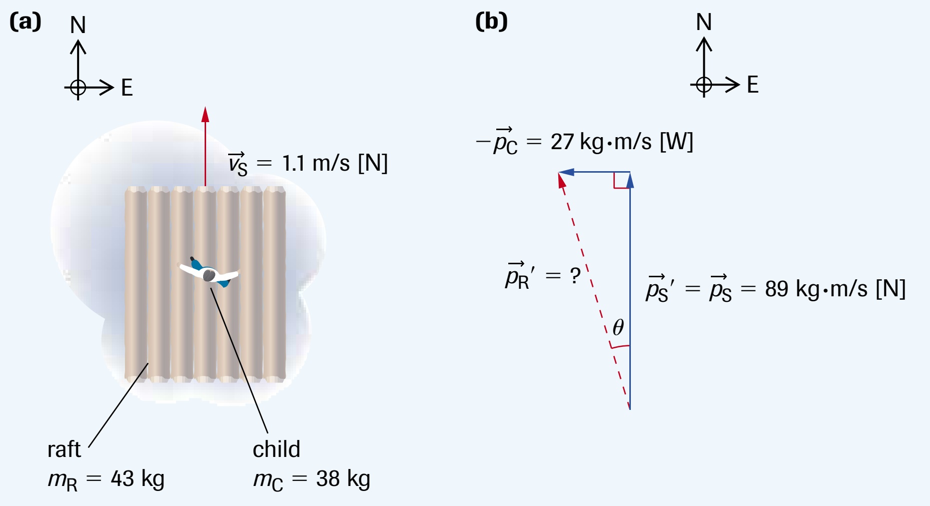

Figure 2
(a) The basic situation
(b) Determining the final momentum of the raft

Solution

Figure 2(a) shows the situation. Since there is no net force acting on the system, momentum is conserved. Thus,

$$\vec{p}_S = \vec{p}_S'$$

where the subscript S represents the system. Finding the initial momentum of the system:

$m_S = 38 \text{ kg} + 43 \text{ kg} = 81 \text{ kg}$
$\vec{v}_S = 1.1 \text{ m/s [N]}$
$\vec{p}_S = ?$

$$\begin{aligned}\vec{p}_S &= m_S\vec{v}_S \\ &= (81 \text{ kg})(1.1 \text{ m/s [N]}) \\ \vec{p}_S &= 89 \text{ kg}\cdot\text{m/s [N]}\end{aligned}$$

The final momentum of the system is equal to the vector addition of the child (indicated by subscript C) and the raft (indicated by subscript R):

$$\vec{p}_S' = \vec{p}_C' + \vec{p}_R'$$

Determine $\vec{p}_C'$:

$m_C = 38 \text{ kg}$
$\vec{v}_C' = 0.71 \text{ m/s [E]}$
$\vec{p}_C' = ?$

$$\begin{aligned}\vec{p}_C' &= m_C\vec{v}_C' \\ &= (38 \text{ kg})(0.71 \text{ m/s [E]}) \\ \vec{p}_C' &= 27 \text{ kg}\cdot\text{m/s [E]}\end{aligned}$$

Since $\vec{p}_S = \vec{p}_S'$, we can now solve for $\vec{p}_R'$:

$$\vec{p}_R' = \vec{p}_S' - \vec{p}_C'$$
$$\vec{p}_R' = \vec{p}_S' + (-\vec{p}_C')$$

Figure 2(b) shows the vector subtraction. Using the law of Pythagoras, we find:

$$\begin{aligned}|\vec{p}_R'|^2 &= |\vec{p}_S'|^2 + |\vec{p}_C'|^2 \\ |\vec{p}_R'| &= \sqrt{(89 \text{ kg}\cdot\text{m/s})^2 + (27 \text{ kg}\cdot\text{m/s})^2} \\ |\vec{p}_R'| &= 93 \text{ kg}\cdot\text{m/s}\end{aligned}$$

The angle θ can now be found:

$$\begin{aligned}\theta &= \tan^{-1}\frac{27 \text{ kg}\cdot\text{m/s}}{89 \text{ kg}\cdot\text{m/s}} \\ \theta &= 17^\circ\end{aligned}$$

Thus, the direction of the raft's final momentum and final velocity is 17° W of N.

Finally, we solve for the final velocity of the raft:

$$\begin{aligned}\vec{p}_R' &= m_R\vec{v}_R' \\ \vec{v}_R' &= \frac{\vec{p}_R'}{m_R} \\ &= \frac{93 \text{ kg}\cdot\text{m/s [17° W of N]}}{43 \text{ kg}} \\ \vec{v}_R' &= 2.2 \text{ m/s [17° W of N]}\end{aligned}$$

The resulting velocity of the raft relative to the water is 2.2 m/s [17° W of N].

DID YOU KNOW?

Bike Helmets

Studies have shown that wearing a bike helmet reduces the risk of death or injury in an accident by more than 80%.

SAMPLE problem 2

In a game of marbles, a collision occurs between two marbles of equal mass m. One marble is initially at rest; after the collision, the marble acquires a velocity of 1.10 m/s at an angle of $\theta = 40.0°$ from the original direction of motion of the other marble, which has a speed of 1.36 m/s after the collision. What is the initial speed of the moving marble?

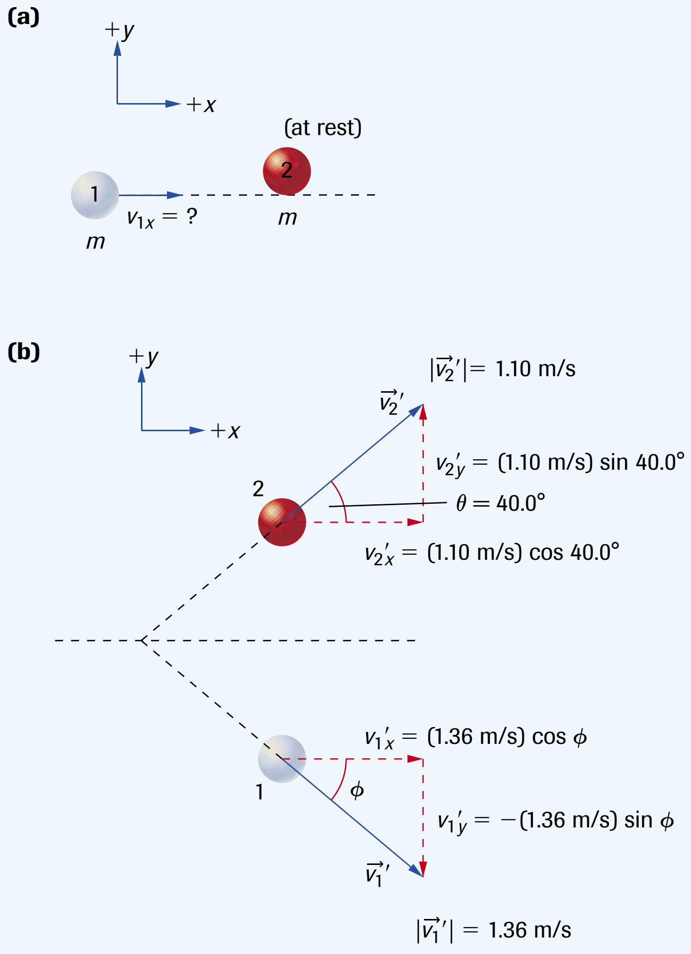

Figure 3
(a) Initial situation
(b) Final situation

Solution

Figure 3(a) shows the initial situation. Since the momentum is conserved,

$$\vec{p} = \vec{p}'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

Since $m_1 = m_2$ and $\vec{v}_2 = 0$, we can simplify:

$$\vec{v}_1 = \vec{v}_1' + \vec{v}_2'$$

The components used to perform this vector addition are shown in **Figure 3(b)**, with the chosen directions of $+x$ rightward and $+y$ upward. Since we do not know the direction of $\vec{v}_1'$, we use components to analyze the situation and solve for the angle ϕ. Applying conservation of momentum to the y-components:

$$v_{1y} = v_{1y}' + v_{2y}'$$
$$0 = -1.36 \text{ m/s} \sin \phi + 1.10 \text{ m/s} \sin \theta$$
$$\sin \phi = \frac{1.10 \text{ m/s} \sin \theta}{1.36 \text{ m/s}}$$
$$\sin \phi = \frac{1.10 \text{ m/s} \sin 40.0°}{1.36 \text{ m/s}}$$
$$\phi = 31.3°$$

Applying conservation of momentum to the x-components:

$$v_{1x} = v_{1x}' + v_{2x}'$$
$$= 1.36 \text{ m/s} \cos \phi + 1.10 \text{ m/s} \cos \theta$$
$$= 1.36 \text{ m/s} \cos 31.3° + 1.10 \text{ m/s} \cos 40.0°$$
$$v_{1x} = 2.00 \text{ m/s}$$

The initial speed of the moving marble is 2.00 m/s.

Practice

Understanding Concepts

1. Bowling involves numerous collisions that are essentially two-dimensional. Copy the 5-pin setup in **Figure 4** and complete the diagram to show where a bowling ball could be aimed to cause a "strike" (i.e., a hit in which all the pins are knocked down).

2. A 52-kg student is standing on a 26-kg cart that is free to move in any direction. Initially, the cart is moving with a velocity of 1.2 m/s [S] relative to the floor. The student then walks on the cart and has a net velocity of 1.0 m/s [W] relative to the floor.
(a) Use a vector scale diagram to determine the approximate final velocity of the cart.
(b) Use components to determine the approximate final velocity of the cart.

3. Two automobiles collide at an intersection. One car of mass 1.4×10^3 kg is travelling at 45 km/h [S]; the other car of mass 1.3×10^3 kg is travelling at 39 km/h [E]. If the cars have a completely inelastic collision, what is their velocity just after the collision?

4. Two balls of equal mass m undergo a collision. One ball is initially stationary. After the collision, the velocities of the balls make angles of 31.1° and 48.9° relative to the original direction of motion of the moving ball.
(a) Draw a diagram showing the initial and final situations. If you are uncertain about the final directions of motion, remember that momentum is conserved.
(b) If the initial speed of the moving ball is 2.25 m/s, what are the speeds of the balls after the collision?
(c) Repeat (b) using a vector scale diagram.
(d) Is this collision elastic? Justify your answer.

5. A nucleus, initially at rest, decays radioactively, leaving a residual nucleus. In the process, it emits two particles horizontally: an electron with momentum 9.0×10^{-21} kg·m/s [E] and a neutrino with momentum 4.8×10^{-21} kg·m/s [S].
(a) In what direction does the residual nucleus move?
(b) What is the magnitude of its momentum?
(c) If the mass of the residual nucleus is 3.6×10^{-25} kg, what is its recoil velocity?

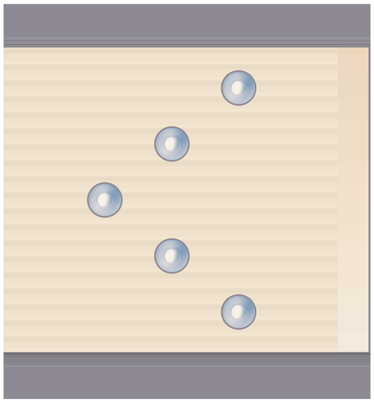

Figure 4
A 5-pin bowling setup is much easier to analyze than a 10-pin setup! (for question 1)

Answers

2. 4.1 m/s [61° S of E]
3. 3.0×10^1 km/h [51° S of E]
4. (b) 1.18 m/s at 48.9°; 1.72 m/s at 31.1°
   (d) no
5. (a) 28° N of W
   (b) 1.0×10^{-20} kg·m/s
   (c) 2.8×10^4 m/s [28° N of W]

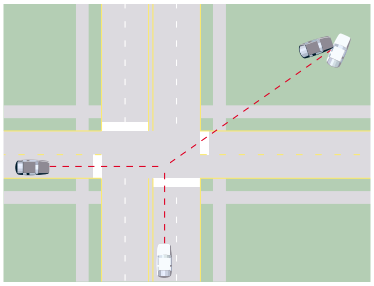

Figure 5
For question 6

Applying Inquiry Skills

6. The police report of an accident between two identical cars at an icy intersection contains the diagram shown in **Figure 5**.
   (a) Which car was travelling faster at the moment of impact? How can you tell?
   (b) What measurements could be made directly on the diagram to help an investigator determine the details of the collision?

Making Connections

7. Choose a sport or recreational activity in which participants wear protective equipment.
   (a) Describe the design and function of the protective equipment.
   (b) Based on the scientific concepts and principles you have studied thus far, explain how the equipment accomplishes its intended functions.
   (c) Using the Internet or other appropriate publications, research your chosen protective equipment. Use what you discover to enhance your answer in (b).

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SUMMARY Conservation of Momentum in Two Dimensions

- Collisions in two dimensions are analyzed using the same principles as collisions in one dimension: conservation of momentum for all collisions for which the net force on the system is zero, and both conservation of momentum and conservation of kinetic energy if the collision is elastic.

Section 5.4 Questions

Understanding Concepts

1. **Figure 6** shows an arrangement of billiard balls, all of equal mass. The balls travel in straight lines and do not spin. Draw a similar, but larger, diagram in your notebook and show the approximate direction that ball 1 must travel to get ball 3 into the end pocket if
   (a) ball 1 collides with ball 2 (in a combination shot)
   (b) ball 1 undergoes a single reflection off the side of the table and then collides with ball 3
2. A neutron of mass 1.7×10^{-27} kg, travelling at 2.7 km/s, hits a stationary lithium nucleus of mass 1.2×10^{-26} kg. After the collision, the velocity of the lithium nucleus is 0.40 km/s at 54° to the original direction of motion of the neutron. If the speed of the neutron after the collision is 2.5 km/s, in what direction is the neutron now travelling?
3. Two ice skaters undergo a collision, after which their arms are intertwined and they have a common velocity of 0.85 m/s [27° S of E]. Before the collision, one skater of mass 71 kg had a velocity of 2.3 m/s [12° N of E], while the other skater had a velocity of 1.9 m/s [52° S of W]. What is the mass of the second skater?

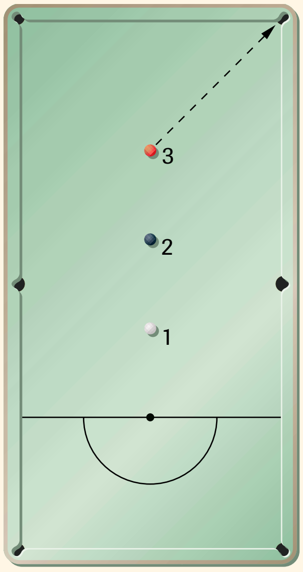

Figure 6
For question 1

4. A steel ball of mass 0.50 kg, moving with a velocity of 2.0 m/s [E], strikes a second ball of mass 0.30 kg, initially at rest. The collision is a glancing one, causing the moving ball to have a velocity of 1.5 m/s [30° N of E] after the collision. Determine the velocity of the second ball after the collision.

Applying Inquiry Skills

5. **Figure 7** shows the results of a collision between two pucks on a nearly frictionless air table. The mass of puck A is 0.32 kg, and the dots were produced by a sparking device every 0.50 s.
   (a) Trace the diagram onto a separate piece of paper and determine the mass of puck B. (*Hint:* Determine which equation applies, and then draw the vectors on your diagram.)
   (b) Determine the amount of kinetic energy lost in the collision.
   (c) Name the type of collision that occurred.
   (d) Identify the most likely sources of error in determining the mass of puck B.

Making Connections

6. Today's consumers are well aware that safety features are important in automobiles. For an automobile of your choice, analyze the design, the operation of the vehicle in a collision or other emergency, and the economic and social costs and benefits of its safety features. Use the following questions as a guideline:
   (a) What social and economic issues do you think are important in automobile safety, from an individual point of view, as well as society's point of view?
   (b) For the automobile you have chosen to analyze, what safety features do you think are essential?
   (c) What safety features are lacking that you think would be beneficial to the driver and passengers?
   (d) Considering your answers in (a), (b), and (c), perform a cost-benefit analysis of developing safety devices in automobiles. Write concluding remarks.

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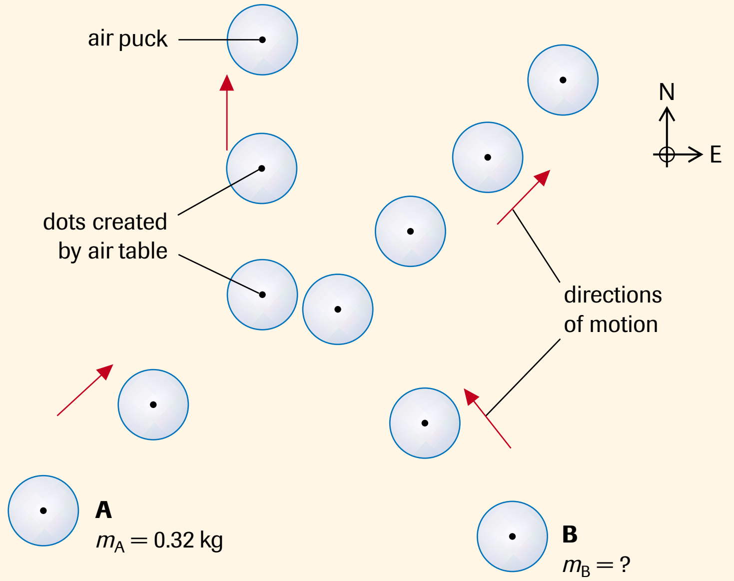

Figure 7
For question 5