5.3 Elastic and Inelastic Collisions

Figure 1
A hockey helmet is designed to spread the force and energy of a collision over as large an area as possible.

If you have inspected the structure of a safety helmet, such as a hockey or bicycle helmet, you may have noticed that the inside is made of relatively soft material designed to fit tightly around the head (**Figure 1**). A hockey helmet protects a player's head during a collision, whether with another player, the ice, or the goal post. For example, if a player without a helmet trips and slides head-first into a goal post, the contact with the post affects only a small area of the head, which must absorb a considerable amount of the player's kinetic energy. This of course is extremely dangerous. With a properly-fitted helmet, on the other hand, the force of the impact is spread out over a much larger surface area, so that any one spot would have to absorb only a fraction of the energy absorbed without the helmet. (The padding in the helmet also increases the time interval of the collision, reducing the force applied to the helmet as the collision causes the player to come to a stop.) In this section, we will explore the relationship of energy to various types of collisions.

Experiments in which different sets of balls are thrown toward each other so that they collide head-on can be used to illustrate different types of collisions (**Figure 2**). The experimental observations vary greatly depending on the type of balls selected. When two superballs collide, they bounce off each other at high speed; tennis balls bounce off each other with moderate speed; and putty balls (of similar mass) stick together and have negligible speed after the collision. In each collision, momentum is conserved. To understand the differences between the collisions, we must consider the kinetic energy of each system.

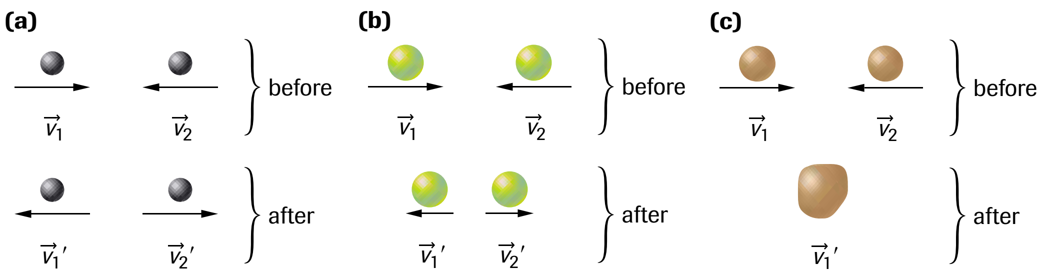

Figure 2
(a) Collision of two superballs
(b) Collision of two tennis balls
(c) Collision of two balls made of soft putty

In the superball collision, the total kinetic energy of the system before the collision is equal to the total kinetic energy of the system after the collision. This type of collision is called an **elastic collision**. For a system undergoing an elastic collision,

elastic collision a collision in which the total kinetic energy after the collision equals the total kinetic energy before the collision

$$E_K{}' = E_K$$
$$\vec{p}\,' = \vec{p}$$

where the prime symbol represents the final condition of the system.

When the tennis balls collide, the total kinetic energy of the system after the collision is not equal to the total kinetic energy of the system before the collision. This is an **inelastic collision**. For a system undergoing an inelastic collision,

inelastic collision a collision in which the total kinetic energy after the collision is different from the total kinetic energy before the collision

$$E_K{}' \neq E_K$$
$$\vec{p}\,' = \vec{p}$$

When two objects stick together during a collision, as is the case with the putty balls, we have a **completely inelastic collision**. The decrease in total kinetic energy in a completely inelastic collision is the maximum possible. For a system undergoing a completely inelastic collision,

$$E_K' < E_K$$
$$\vec{p}\,' = \vec{p}$$

Note that in a completely inelastic collision, the objects stick together and thus have the same final velocity.

It is important to realize that we compare the kinetic energies of the colliding objects *before* and *after* the collision, not during the collision. Consider two gliders of equal mass with springs attached approaching each other at the same speed (**Figure 3**). Just before the gliders collide, their speeds and kinetic energies are at a maximum, but in the middle of the collision, their speeds and kinetic energies are zero. The kinetic energy is transformed into elastic potential energy stored in the springs. This potential energy is at a maximum when the kinetic energy is at a minimum. After the elastic collision, when the springs no longer touch, the elastic potential energy drops back to zero and the kinetic energies return to their original values.

LEARNING TIP

Inelastic Collisions

In most inelastic collisions, for example, between two tennis balls, the total final kinetic energy of the system is less than the total initial kinetic energy of the system. However, in some inelastic collisions, such as a collision that initiates an explosion, kinetic energy is produced, giving a total final kinetic energy of the system greater than the initial kinetic energy of the system.

completely inelastic collision a collision in which there is a maximum decrease in kinetic energy after the collision since the objects stick together and move at the same velocity

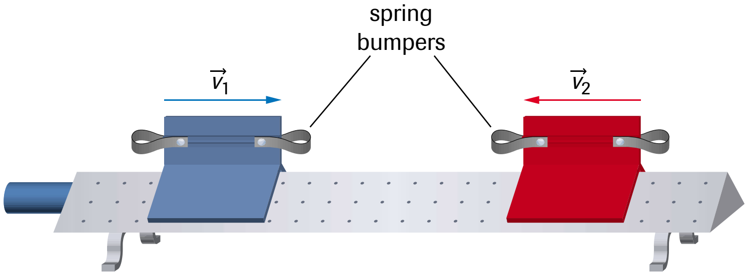

Figure 3
When gliders with springs collide, the duration of the collision is greater than without the springs, making it easier to observe what happens to the energies of the gliders as they collide.

Figure 4 is a graph of the mechanical energy of the system of gliders and springs. At any instant in this elastic collision, the sum of the elastic potential energy and the kinetic energy of the system remains constant, even though the kinetic energy does not return to its initial value until after the collision is complete.

In practice, it is almost impossible to have a truly elastic collision between two macroscopic objects, such as gliders with springs or superballs. There is always some small amount of kinetic energy transformed into other forms. For instance, when superballs collide, thermal energy and sound energy are produced. However, in this text we will treat certain collisions between macroscopic objects as being elastic, and we will ignore the small amount of kinetic energy that is lost. Collisions involving molecules, atoms, and subatomic particles, on the other hand, can be perfectly elastic.

After an inelastic collision or a completely inelastic collision, the total final kinetic energy of the system is not equal to the initial kinetic energy of the system. Usually the final kinetic energy is less than the initial kinetic energy unless the collision is explosive. Since energy is conserved, the lost kinetic energy must be transformed into other forms of energy. For example, when two putty balls collide, the putty balls become warmer because the kinetic energy has been transformed into thermal energy. Depending on the properties of the colliding objects, the kinetic energy could be transformed into sound energy, elastic potential energy, thermal energy, or another form of energy.

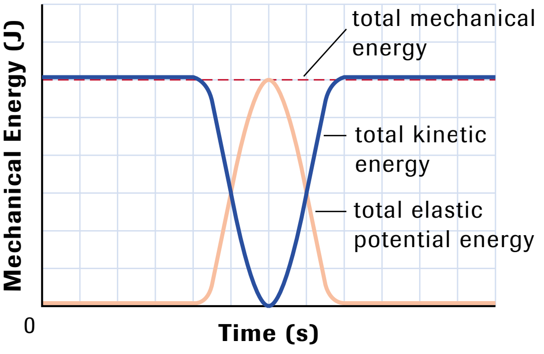

Figure 4
Mechanical energy in an elastic collision as a function of time

In the early history of the solar system, there were many completely inelastic collisions between relatively large objects, such as the Moon and chunks of rock (**Figure 5**). Such collisions produced the craters on the Moon's surface. Similar collisions created craters on Earth, although most have been destroyed in erosion by rain and wind. However, some relatively recent craters still exist, such as the famous Barringer Crater in Arizona, which is believed to be about 50 000 years old (**Figure 6**). This crater, with a diameter of 1.6 km and a depth of 180 m, resulted when a large meteorite collided with Earth.

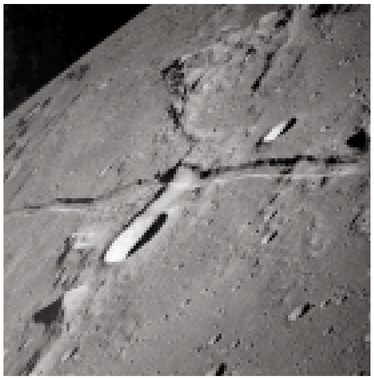

Figure 5
Craters on the Moon are the result of completely inelastic collisions with large rocks.

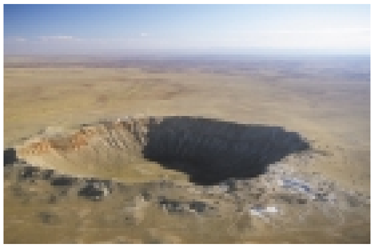

Figure 6
Analyzing impact craters, such as the Barringer Crater, helps scientists understand more about the history of the solar system. The meteorite that created this crater was about 45 m in diameter, but it was travelling at about 65 000 km/h (relative to Earth) when it collided.

Figure 7
Newton's cradle

TRY THIS activity — Newton's Cradle

Figure 7 shows a Newton's cradle. Each sphere has the same mass *m*. If the raised sphere is released, it will move, and just before it collides with the stationary spheres, its momentum has a magnitude of *mv*.

(a) Is it true that momentum can be conserved, no matter how many spheres fly outward after the initial collision? Why or why not?
(b) Is it true that kinetic energy can be conserved, no matter how many spheres fly outward after the initial collision? Why or why not?
(c) Based on your calculations, predict what will happen when a single sphere hits the stationary spheres. Test your prediction by trying the demonstration.
(d) Predict what will happen when two spheres, and also when three spheres, strike the stationary spheres. Test your predictions.

Practice

Understanding Concepts

1. In a completely inelastic collision between two objects, under what condition(s) will all of the original kinetic energy be transformed into other forms of energy?
2. In a certain collision between two cars, the cars end up sticking together. Can we conclude that the collision is completely inelastic? Explain.
3. Use physics principles to explain why head-on vehicle collisions are usually more dangerous than other types of collisions.

Applying Inquiry Skills

4. Describe how you would use the elasticity of a ball when you squeeze it to predict how well it will bounce off a hard floor. Test your answer experimentally.
5. Draw a graph similar to the one in **Figure 4**, illustrating mechanical energy as a function of time for
   (a) an inelastic collision
   (b) a completely inelastic collision

Making Connections

6. A safety helmet spreads the force of an impact over as large an area as possible; the soft interior also changes the time interval of a collision.
   (a) Why is the impact force reduced for a helmet with a soft interior versus a hard interior?
   (b) How is safety reduced if the helmet does not fit properly?
   (c) Once a helmet has been involved in a collision, it should be replaced. Why?
7. If you were designing a passenger train, would you favour a design with a rigid frame or a flexible frame? Why?

Solving Collision Problems

In solving problems involving collisions, it is important to distinguish between elastic, inelastic, and completely inelastic collisions. For all collisions involving two objects on which the net force is zero, momentum is conserved:

$$mv_1 + mv_2 = mv_1' + mv_2'$$

where m_1 and m_2 are the masses of the colliding objects, v_1 and v_2 are the velocities before the collision, and v_1' and v_2' are the velocities after the collision. (Remember that vector notation is omitted because the collisions are in one dimension.) If the collision is inelastic, this is the only equation that can be used. In the case of a completely inelastic collision, the objects stick together and their final velocities are equal $v_1' = v_2'$.

For an elastic collision (which will be stated clearly in the problem), the total kinetic energy before the collision equals the total kinetic energy after the collision:

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$$

We can combine this equation with the equation for conservation of momentum to solve problems that involve elastic collisions.

LEARNING TIP

Solving Simultaneous Equations

Whenever a problem involves an elastic collision, the chances are great that there will be two unknowns. To solve for two unknowns, you need to set up two simultaneous equations (one involving momentum conservation and the other involving kinetic energy conservation), and simplify them.

In problems involving inelastic and completely inelastic collisions, there will usually be one or two unknowns, but the conservation of kinetic energy does not apply. You must solve the problem by applying the equation for the conservation of momentum and then work out the kinetic energies if needed.

SAMPLE problem 1

A billiard ball with mass *m* and initial speed v_1, undergoes a head-on elastic collision with another billiard ball, initially stationary, with the same mass *m*. What are the final speeds of the two balls?

Solution

Figure 8 shows the initial and final diagrams. We choose the $+x$-axis as the direction of motion of the initially moving ball (ball 1). Since the problem states that the collision is elastic, we know that the total initial kinetic energy equals the total final kinetic energy. Momentum is conserved in this collision. We can thus write two equations, one for kinetic energy and one for momentum:

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$$

$$mv_1 + mv_2 = mv_1' + mv_2'$$

where the subscript 1 refers to the initially moving ball and the subscript 2 refers to the initially stationary ball. Note that the *v*'s represent velocity components (not velocity magnitudes) and can be positive or negative. Since the masses are equal, they cancel:

$$v_1^2 + v_2^2 = v_1'^2 + v_2'^2$$

$$v_1 + v_2 = v_1' + v_2'$$

Since ball 2 is initially stationary, $v_2 = 0$, and we can write:

$$v_1^2 = v_1'^2 + v_2'^2$$

$$v_1 = v_1' + v_2'$$

We now have two equations and two unknowns, so we rearrange the latter equation to solve for v_1':

$$v_1' = v_1 - v_2'$$

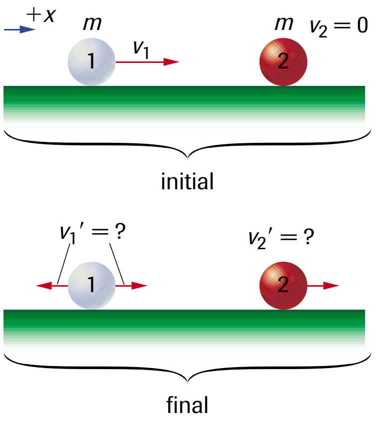

Figure 8
The situations before and after the collision for Sample Problem 1

Substituting for v_1':

$$\begin{aligned} v_1^2 &= (v_1 - v_2')^2 + v_2'^2 \\ &= v_1^2 - 2v_1v_2' + v_2'^2 + v_2'^2 \\ 0 &= -2v_1v_2' + 2v_2'^2 \\ 0 &= -v_2'(v_1 - v_2') \end{aligned}$$

Therefore, either $v_2' = 0$ (which is not an appropriate solution since it means that no collision occurred) or $v_1 - v_2' = 0$. Thus, we can conclude that $v_2' = v_1$. Substituting this value into the equation $v_1 = v_1' + v_2'$:

$$\begin{aligned} v_1 &= v_1' + v_1 \\ v_1' &= 0 \end{aligned}$$

Therefore, ball 1, which was initially moving, is at rest after the collision ($v_1' = 0$); ball 2, which was initially stationary, has the same speed after the collision that ball 1 had before the collision ($v_2' = v_1$). Note that this conclusion is not valid for all elastic collisions in which one object is initially stationary—the colliding objects must have the same mass.

SAMPLE problem 2

A child rolls a superball of mass 2.5×10^{-2} kg along a table at a speed of 2.3 m/s to collide head-on with a smaller stationary superball of mass 2.0×10^{-2} kg. The collision is elastic. Determine the velocity of each ball after the collision.

Solution

$m_1 = 2.5 \times 10^{-2}$ kg $\qquad v_1' = ?$

$m_2 = 2.0 \times 10^{-2}$ kg $\qquad v_2' = ?$

$v_1 = 2.3$ m/s

Figure 9 shows diagrams of the situation. We choose the $+x$ axis as the direction of the initial velocity of the larger ball. Since the collision is elastic, both kinetic energy and momentum are conserved:

$$\begin{aligned} \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 &= \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \\ m_1v_1 + m_2v_2 &= m_1v_1' + m_2v_2' \end{aligned}$$

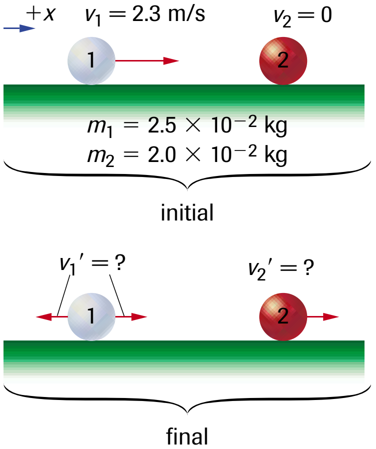

Figure 9
The situations before and after the collision for Sample Problem 2

where the subscript 1 refers to the larger ball and the subscript 2 refers to the smaller ball. Since ball 2 is initially stationary, $v_2 = 0$. Substituting into both equations and multiplying the kinetic energy equation by 2, we have:

$$\begin{aligned} m_1v_1^2 &= m_1v_1'^2 + m_2v_2'^2 \\ m_1v_1 &= m_1v_1' + m_2v_2' \end{aligned}$$

We can rearrange the second equation to solve for v_1' in terms of v_2':

$$\begin{aligned} v_1' &= v_1 - \frac{m_2}{m_1}v_2' \\ &= 2.3 \text{ m/s} - \left(\frac{2.0 \times 10^{-2} \text{ kg}}{2.5 \times 10^{-2} \text{ kg}}\right)v_2' \\ v_1' &= 2.3 \text{ m/s} - 0.80\, v_2' \end{aligned}$$

which we can then substitute into the first equation and solve for v_2'. However, before substituting for v_1', we can substitute known numbers into the first equation:

$$m_1v_1^2 = m_1v_1'^2 + m_2v_2'^2$$
$$(2.5 \times 10^{-2} \text{ kg})(2.3 \text{ m/s})^2 = (2.5 \times 10^{-2} \text{ kg})v_1'^2 + (2.0 \times 10^{-2} \text{ kg})v_2'^2$$

Multiplying by 10^2 to eliminate each 10^{-2}, we have:

$$(2.5\text{ kg})(2.3\text{ m/s})^2 = (2.5\text{ kg})v_1'^2 + (2.0\text{ kg})v_2'^2$$
$$13.2\text{ kg}\cdot\text{m}^2/\text{s}^2 = (2.5\text{ kg})v_1'^2 + (2.0\text{ kg})v_2'^2$$

Now, substituting the expression for v_1':

$$13.2\text{ kg}\cdot\text{m}^2/\text{s}^2 = (2.5\text{ kg})(2.3\text{ m/s} - 0.80v_2')^2 + (2.0\text{ kg})v_2'^2$$
$$13.2\text{ kg}\cdot\text{m}^2/\text{s}^2 = (2.5\text{ kg})(5.29\text{ m}^2/\text{s}^2 - 3.68\text{ m/s}v_2' + 0.64v_2'^2) + (2.0\text{ kg})v_2'^2$$
$$13.2\text{ kg}\cdot\text{m}^2/\text{s}^2 = 13.2\text{ kg}\cdot\text{m}^2/\text{s}^2 - 9.2\text{ kg}\cdot\text{m/s}\ v_2' + 1.6\text{ kg}\ v_2'^2 + 2.0\text{ kg}\ v_2'^2$$
$$0 = -9.2\text{ kg}\cdot\text{m/s}\ v_2' + 3.6\text{ kg}\ v_2'^2$$
$$0 = (-9.2\text{ kg}\cdot\text{m/s} + 3.6\text{ kg}\ v_2')v_2'$$

Thus, $0 = -9.2\text{ kg}\cdot\text{m/s} + 3.6\text{ kg}\ v_2'$ or $v_2' = 0$

Since $v_2' = 0$ corresponds to no collision, we must have:

$$0 = -9.2\text{ kg}\cdot\text{m/s} + 3.6\text{ kg}\ v_2'$$
$$v_2' = +2.6\text{ m/s}$$

We can now substitute this value for v_2' to solve for v_1':

$$v_1' = 2.3\text{ m/s} - 0.80v_2'$$
$$= 2.3\text{ m/s} - 0.80(2.6\text{ m/s})$$
$$v_1' = +0.3\text{ m/s}$$

Thus, after the collision, both balls are moving in the $+x$ direction (the same direction as the larger ball was originally moving). The speeds are 2.6 m/s and 0.3 m/s for the smaller and larger balls, respectively.

The collisions analyzed so far in this chapter have been one-dimensional. Most collisions, however, involve two- or three-dimensional situations. By performing an investigation to study two-dimensional collisions, you will find the theory and mathematical analysis of such collisions much easier to understand. To explore two-dimensional collisions further, perform Investigation 5.3.1 in the Lab Activities section at the end of this chapter.

INVESTIGATION 5.3.1

Analyzing Two-Dimensional Collisions (p. 262)

There are various ways of creating collisions in the laboratory in which the objects collide with a crisp, clear bang, or the objects stick together and have a common final velocity. What problems would you anticipate having to overcome in analyzing two-dimensional collisions between pucks on a horizontal air table?

Practice

Understanding Concepts

8. A small truck and a large truck have the same kinetic energies. Which truck has the greater momentum? Justify your answer.
9. (a) Can an object have kinetic energy, but no momentum? Can an object have momentum, but no kinetic energy? Explain.
   (b) Repeat (a) for an isolated system of two interacting objects.
10. During a friendly snowball fight, two snowballs, each of mass 0.15 kg, collide in mid-air in a completely inelastic collision. Just before the collision, both balls are travelling horizontally, one ball with a velocity of 22 m/s [N] and the other 22 m/s [S]. What is the velocity of each ball after the collision?
11. A proton travelling with an initial speed of 815 m/s collides head-on with a stationary proton in an elastic collision. What is the velocity of each proton after the collision? Show your work.

Answers

10. 0 m/s
11. 0 m/s; 815 m/s in the direction of initial velocity

Answers

12. 85 km/h [N]
13. 4.1×10^6 J; 4.0×10^6 J; 1×10^5 J
14. 5.15×10^2 m/s
16. (b) $|\vec{p}_T{}'| = |\vec{p}_R| + |\vec{p}_R{}'|$; $|\vec{p}_T{}'| = |\vec{p}_L| - |\vec{p}_L{}'|$

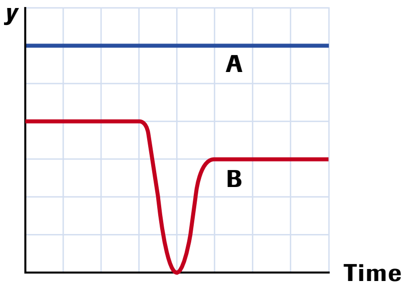

Figure 10
For question 15

12. A truck of mass 1.3×10^4 kg, travelling at 9.0×10^1 km/h [N], collides with a car of mass 1.1×10^3 kg, travelling at 3.0×10^1 km/h [N]. If the collision is completely inelastic, what are the magnitude and direction of the velocity of the vehicles immediately after the collision?

13. Calculate the total kinetic energy before and after the collision described in question 12. Determine the decrease in kinetic energy during the collision.

14. A nitrogen molecule of mass 4.65×10^{-26} kg in the air undergoes a head-on elastic collision with a stationary oxygen molecule of mass 5.31×10^{-26} kg. After the collision, the nitrogen molecule has reversed its direction and has a speed of 34.1 m/s, while the oxygen molecule is travelling at 4.81×10^2 m/s in the original direction of the nitrogen molecule. What is the initial speed of the nitrogen molecule?

Applying Inquiry Skills

15. An experiment is performed in which two low-friction carts on an air track approach each other and collide. The motions of the carts are monitored by sensors connected to a computer that generates the graphs in **Figure 10**.
(a) One line on the graph represents the total momentum of the two-cart system, and the other represents the total kinetic energy. Which line is which? How can you tell?
(b) What type of collision occurred in the experiment? How can you tell?

Making Connections

16. In some situations, riot police use rubber bullets to control demonstrators. In designing these bullets, tests are carried out in labs to compare the collisions involving rubber bullets and lead bullets striking a target.
(a) In these tests, one type of bullet has an elastic collision with the test target, while the other type has an almost completely inelastic collision with the test target. Which bullet has the elastic collision and which has the almost completely inelastic collision?
(b) Using the subscripts R for the rubber bullet, L for the lead bullet, and T for the target, develop equations for the magnitude of the momentum transferred to the target after being struck by the rubber bullet and the lead bullet. Assume both bullets to have the same masses and initial speeds. Express your answers in terms of the magnitudes of the initial momentum of the bullet and the final momentum of the bullet. Which bullet transfers the larger magnitude of momentum to the target?
(c) Explain why rubber bullets are preferred in riot control.

SUMMARY *Elastic and Inelastic Collisions*

- In all elastic, inelastic, and completely inelastic collisions involving an isolated system, the momentum is conserved.
- In an elastic collision, the total kinetic energy after the collision equals the total kinetic energy before the collision.
- In an inelastic collision, the total kinetic energy after the collision is different from the total kinetic energy before the collision.
- In a completely inelastic collision, the objects stick together and move with the same velocity, and the decrease in total kinetic energy is at a maximum.
- Elastic collisions can be analyzed by applying both the conservation of kinetic energy and the conservation of momentum simultaneously.

Section 5.3 Questions

Understanding Concepts

1. A moving object collides with a stationary object.
   (a) Is it possible for both objects to be at rest after the collision? If "yes," give an example. If "no," explain why not.
   (b) Is it possible for only one object to be at rest after the collision? If "yes," give an example. If "no," explain why not.
2. A wet snowball of mass m, travelling at a speed v, strikes a tree. It sticks to the tree and stops. Does this example violate the law of conservation of momentum? Explain.
3. Two particles have the same kinetic energies. Are their momentums necessarily equal? Explain.
4. A 22-g superball rolls with a speed of 3.5 m/s toward another stationary 27-g superball. The balls have a head-on elastic collision. What are the magnitude and direction of the velocity of each ball after the collision?
5. An object of mass m has an elastic collision with another object initially at rest, and continues to move in the original direction but with one-third its original speed. What is the mass of the other object in terms of m?
6. A 66-kg skier, initially at rest, slides down a hill 25 m high, then has a completely inelastic collision with a stationary 72-kg skier. Friction is negligible. What is the speed of each skier immediately after the collision?

Applying Inquiry Skills

7. **Figure 11(a)** shows a ballistics pendulum used to determine speeds of bullets before the advent of modern electronic timing. A bullet is shot horizontally into a block of wood suspended by two strings. The bullet remains embedded in the wood, and the wood and bullet together swing upward.
   (a) Explain why the *horizontal* momentum of the bullet-wood system is conserved during the collision, even though the strings exert tension forces on the wood.
   (b) If the bullet and wood have masses of m and M, respectively, and the bullet has an initial speed of v, derive an algebraic expression for the speed of the bullet and wood immediately after the collision, before they swing upward, in terms of m, M, and v.
   (c) As the bullet and wood swing up, what law of nature can be used to relate the maximum vertical height to the speed just after the collision?
   (d) Use your answers to (b) and (c) to derive an expression for the maximum vertical height h in terms of m, M, v, and g.
   (e) Rearrange your expression in (d) so that if h is a known quantity, v can be calculated.
   (f) If a bullet of mass 8.7 g hits a block of wood of mass 5.212 kg, and the bullet and wood swing up to a maximum height of 6.2 cm, what is the initial speed of the bullet?
   (g) **Figure 11(b)** shows a modern ballistics pendulum used for student experimentation. Describe some of the sources of random and systematic error that should be minimized in determining the speed of the ball fired from the spring-loaded triggering mechanism.

Making Connections

8. Decades ago, cars were designed to be as rigid as possible. Modern cars, however, are designed with "crumple zones" that collapse upon impact. Explain the advantage of this design.
9. Chunks of material from space, both large and small, collide with Earth. Research the sizes of these materials and the frequencies of the collisions, as well as some of the well-known collisions scientists have studied. Some famous impact sites are Sudbury, Ontario; Chicxulub, Mexico; and Tunguska, Russia. Write a brief report summarizing what you discover.

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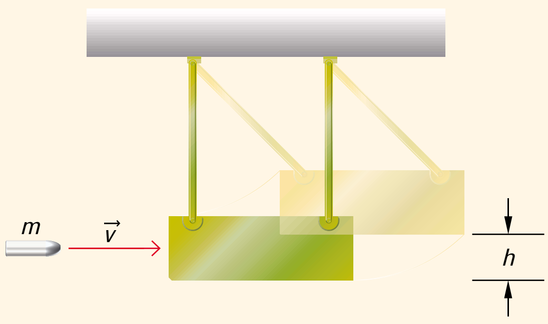
(a)



(b)

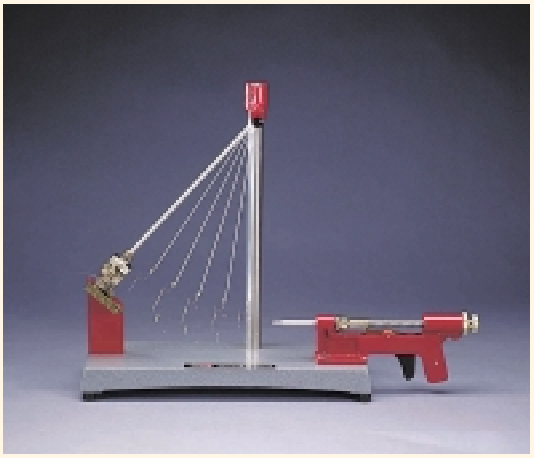


Figure 11
Ballistics pendulums (for question 7)