Conservation of Momentum in One Dimension 5.2

Imagine standing at rest on skates on essentially frictionless ice, and throwing a basketball forward (**Figure 1**). As the ball moves in one direction, you move in the opposite direction at a much lower speed. This situation can be explained using Newton's third law: you exert a forward force on the ball as it exerts a backward force on you. We can also explain the situation using the *law of conservation of linear momentum*.

Figure 1
As a person on skates throws a basketball forward, the force exerted by the skater on the ball is equal in magnitude, but opposite in direction to the force of the ball on the skater. Friction between the skates and the ice is assumed to be negligible. Thus, the net force on the system is zero and the linear momentum of this system is conserved.

Law of Conservation of Linear Momentum

If the net force acting on a system of interacting objects is zero, then the linear momentum of the system before the interaction equals the linear momentum of the system after the interaction.

The system comprised of you (on skates) and the basketball has zero momentum before you throw the ball. Thus, the momentum of the system after the ball is thrown must also equal zero. Therefore, the momentum of the ball forward must be balanced by your momentum in the opposite direction.

During the seventeenth century, scientists including Newton discovered that the total momentum of colliding objects before and after a collision remains constant. To analyze the conditions under which this is true, we can study a simple collision between two gliders on a frictionless air track. It is useful to imagine a boundary around the gliders to focus our attention on the two-glider system. This system and its boundary are shown in **Figure 2(a)**.

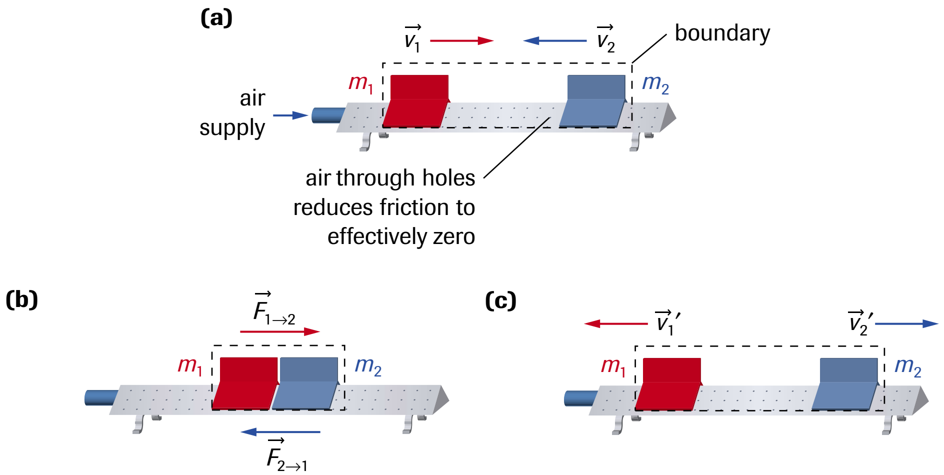

Figure 2
(a) Two gliders on an air track will soon collide.
(b) The collision is in progress.
(c) After colliding the gliders move apart.

Before the two gliders collide, the total momentum of the system is the vector sum of the momentum of each glider:

$$\vec{p}_{\text{total}} = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $\vec{p}_{\text{total}}$ is the total momentum, m_1 is the mass and $\vec{v}_1$ is the velocity of one glider, and m_2 is the mass and $\vec{v}_2$ is the velocity of the other glider.

When the two gliders collide, each exerts a force on the other as in **Figure 2(b)**. According to Newton's third law of motion, the two forces are equal in magnitude, but opposite in direction:

$$\vec{F}_{2\rightarrow1} = -\vec{F}_{1\rightarrow2}$$

where $\vec{F}_{2\rightarrow1}$ is the force glider 2 exerts on glider 1, and $\vec{F}_{1\rightarrow2}$ is the force glider 1 exerts on glider 2. Thus, the net force acting on the two-glider system is zero:

$$\vec{F}_{2\rightarrow1} + \vec{F}_{1\rightarrow2} = 0$$

Note that the vertical forces—gravity and the upward force exerted by the air—also add to zero. Therefore, the net force on the system is zero.

The forces exerted by the gliders on each other cause each glider to accelerate according to Newton's second law of motion, $\sum\vec{F} = m\vec{a}$. Starting with the equation involving the forces we have:

$$\vec{F}_{2\rightarrow1} = -\vec{F}_{1\rightarrow2}$$
$$m_1\vec{a}_1 = -m_2\vec{a}_2$$
$$m_1\frac{\Delta\vec{v}_1}{\Delta t_1} = -m_2\frac{\Delta\vec{v}_2}{\Delta t_2}$$

We know that $\Delta t_1 = \Delta t_2$ because the force $\vec{F}_{1\rightarrow2}$ acts only as long as the force $\vec{F}_{2\rightarrow1}$ acts; that is, $\vec{F}_{1\rightarrow2}$ and $\vec{F}_{2\rightarrow1}$ act only as long as the gliders are in contact with each other. Thus,

$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$

This equation summarizes the law of conservation of (linear) momentum for two colliding objects. It states that *during an interaction between two objects on which the total net force is zero, the change in momentum of object 1* ($\Delta\vec{p}_1$) *is equal in magnitude but opposite in direction to the change in momentum of object 2* ($\Delta\vec{p}_2$). Thus,

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

LEARNING TIP

Interactions and Collisions

A collision between two or more objects in a system can also be called an *interaction*. However, there are interactions that are not collisions, although they obey the law of conservation of momentum. Examples include a person throwing a ball, an octopus ejecting water in one direction to propel itself in the opposite direction, and fireworks exploding into several pieces.

Let us now consider the glider system before and after the collision (**Figure 2(c)**). We will use the prime symbol (′) to represent the final velocities:

$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$
$$m_1(\vec{v}_1' - \vec{v}_1) = -m_2(\vec{v}_2' - \vec{v}_2)$$
$$m_1\vec{v}_1' - m_1\vec{v}_1 = -m_2\vec{v}_2' + m_2\vec{v}_2$$
$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

This equation represents another way of summarizing the law of conservation of (linear) momentum. It states that *the total momentum of the system before the collision equals the total momentum of the system after the collision.* Thus,

$$\vec{p}_{\text{system}} = \vec{p}'_{\text{system}}$$

It is important to remember that momentum is a vector quantity; thus, any additions or subtractions in these conservation of momentum equations are vector additions or vector subtractions. These equations also apply to the conservation of momentum in two dimensions, which we will discuss in Section 5.4. Note that the equations written for components are:

$$m_1\Delta v_{1x} = -m_2\Delta v_{2x}$$
$$m_1\Delta v_{1y} = -m_2\Delta v_{2y}$$
$$m_1v_{1x} + m_2v_{2x} = m_1v_{1x}' + m_2v_{2x}'$$
$$m_1v_{1y} + m_2v_{2y} = m_1v_{1y}' + m_2v_{2y}'$$

LEARNING TIP

Systems with More than Two Objects

The equation that relates the total momentum of the system before and after a collision can be applied to interactions involving more than two objects. For example, if three figure skaters, initially at rest in a huddle, push away from each other, the initial momentum of the system is zero and the vector sum of the skaters' momentums after the interaction is also zero.

SAMPLE problem 1

A vacationer of mass 75 kg is standing on a stationary raft of mass 55 kg. The vacationer then walks toward one end of the raft at a speed of 2.3 m/s relative to the water. What are the magnitude and direction of the resulting velocity of the raft relative to the water? Neglect fluid friction between the raft and the water.

Solution

In problems involving conservation of momentum, it is useful to draw diagrams that show the initial and final situations (**Figure 3**).

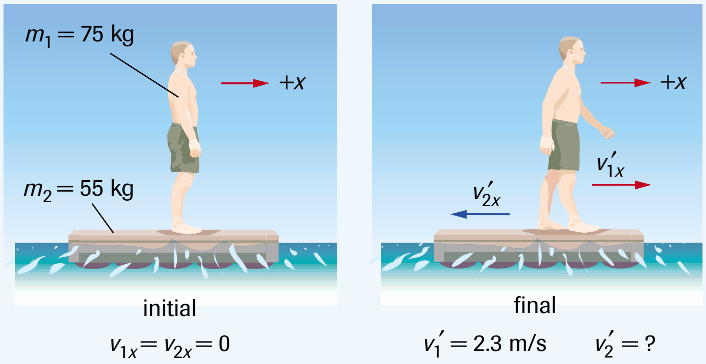

Figure 3
The initial and final situations

A coordinate system is then chosen; we will arbitrarily select the $+x$ direction to lie in the direction of the vacationer's final velocity. Since there is no net force acting on the system of the raft and person, we can apply the law of conservation of momentum:

$$m_1v_{1x} + m_2v_{2x} = m_1v'_{1x} + m_2v'_{2x}$$

where the subscript 1 refers to the person and the subscript 2 refers to the raft. Since this is a one-dimensional problem, we can omit the x subscripts:

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

Note that the v's in the equation represent velocity components and can have positive, zero, or negative values. They do not represent the magnitudes of the velocities, which must be nonnegative.

In this problem, $v_1 = v_2 = 0$, since the vacationer and the raft are initially stationary. Therefore, we can write:

$$0 = m_1v'_1 + m_2v'_2$$

$$v'_2 = \frac{-m_1v'_1}{m_2}$$

$$= \frac{-(75\text{ kg})(2.3\text{ m/s})}{55\text{ kg}}$$

$$v'_2 = -3.1\text{ m/s}$$

The final velocity of the raft is 3.1 m/s in the opposite direction to the vacationer's velocity (as the negative sign indicates).

Conservation of momentum can be applied to many collisions. For example, in the collision of two billiard balls on a table (**Figure 4**), the force exerted on the first ball by the second is equal in magnitude, but opposite in direction to the force exerted on the second ball by the first. The net force on the system is zero and, therefore, the momentum of the system is conserved. (In this case, we can ignore any friction because it is very small compared to the forces exerted by the billiard balls on each other. Vertically there is no acceleration and, thus, there is no net vertical force.)

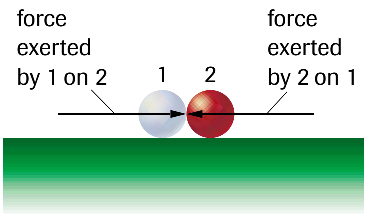

Figure 4
Two billiard balls colliding: the momentum of the system is conserved.

DID YOU KNOW ?

Rocket Propulsion

An important application of conservation of momentum is rocket propulsion, both on Earth and in the "vacuum" of outer space. As the rocket thruster exerts an action force on the hot gases ejected backward, the gases exert a reaction force equal in magnitude on the spacecraft, causing it to accelerate forward. Since the spacecraft and ejected gases form an isolated system, the change in momentum of the gases is equal in magnitude to the change in momentum of the spacecraft. As the thruster continues to fire its engines, its fuel supply becomes smaller, so there is a decrease in the mass of the spacecraft and remaining fuel. Although the conservation of momentum applies, the mathematical analysis of this changing-mass system is left for calculus-based physics texts.

SAMPLE problem 2

During a football game, a fullback of mass 108 kg, running at a speed of 9.1 m/s, is tackled head-on by a defensive back of mass 91 kg, running at a speed of 6.3 m/s. What is the speed of this pair just after the collision?

Solution

Figure 5 shows the initial and final diagrams. We choose the $+x$-axis as the direction of the initial velocity of the fullback. Therefore, the initial velocity of the defensive back is negative.

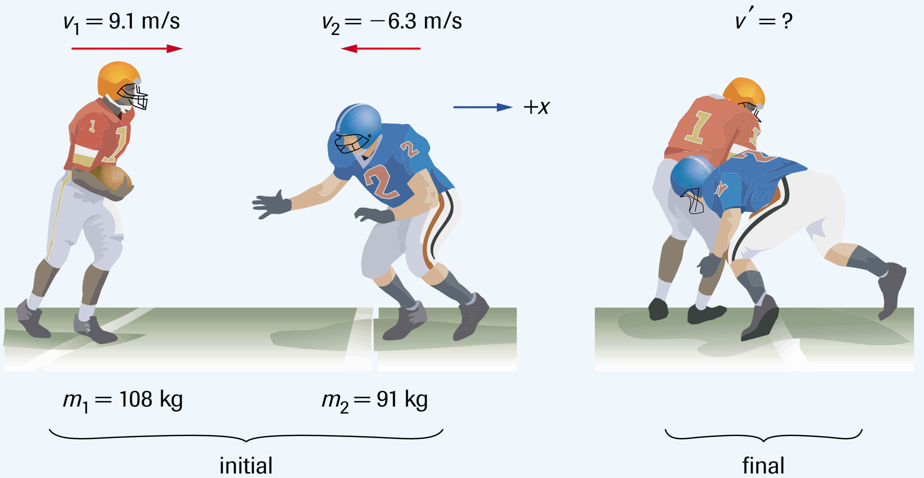

Figure 5
The initial and final situations

During the collision, there is no net force on the two-player system. (The horizontal force exerted between the players is much larger than friction, which can therefore be neglected. In the vertical direction, there is no acceleration because there is no vertical net force.) Therefore, the momentum of this system is conserved. Thus,

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

where the subscript 1 refers to the fullback and the subscript 2 refers to the defensive back. Remember that v represents a velocity component, not a velocity magnitude.

Since the two players have the same final velocity:

$$v_1' = v_2' = v'$$

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$$

$$v' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$= \frac{(108 \text{ kg})(9.1 \text{ m/s}) + (91 \text{ kg})(-6.3 \text{ m/s})}{(108 \text{ kg} + 91 \text{ kg})}$$

$$v' = +2.1 \text{ m/s}$$

The final velocity of the players is 2.1 m/s in the direction of the initial velocity of the fullback (as the positive sign indicates).

It is a misconception to think that momentum is conserved in *all* collisions. There are many collisions in which the net force on the colliding objects is not zero and, therefore, momentum is not conserved. For example, if a person jumps from a ladder to a wooden deck, the momentum of the person–deck system is not conserved because there is a large normal force exerted by the deck supports and the ground during the collision. In other words, the deck is not free to move, so $\Delta\vec{p}_{jumper}$ does not equal $-\Delta\vec{p}_{deck}$. However, if we change the boundary of the system to include Earth, momentum would be conserved because $\Delta\vec{p}_{jumper} = -\Delta\vec{p}_{Earth}$. Since the mass of Earth is very large compared to the mass of the person, Earth's change in velocity when the person lands is, of course, too small to measure. The person–Earth system is isolated, but the person–deck system is not.

Experiments can be performed to determine if momentum is conserved in a variety of collisions. However, much more can be learned about the collisions if energy is also analyzed. For greater accuracy, these experiments should involve collisions of objects that are isolated from external forces, as presented in Investigation 5.2.1 in the Lab Activities section at the end of this chapter.

INVESTIGATION 5.2.1

Analyzing One-Dimensional Collisions (p. 260)

Different types of apparatus, including gliders on air tracks with motion and force sensors to collect data, can be used to create and analyze one-dimensional collisions. How do you think the challenge of isolating two colliding objects will affect the degree to which momentum and energy are conserved?

Practice

Understanding Concepts

1. What condition(s) must be met for the total momentum of a system to be conserved?
2. State whether you agree or disagree with the following statement: "The law of conservation of momentum for a system on which the net force is zero is equivalent to Newton's first law of motion." Give a reason for your answer.
3. You drop a 59.8-g hairbrush toward Earth (mass 5.98×10^{24} kg).
   (a) What is the direction of the gravitational force exerted by Earth on the hairbrush?
   (b) What is the direction of the gravitational force exerted by the hairbrush on Earth?
   (c) How do the forces in (a) and (b) compare in magnitude?
   (d) What is the net force on the system consisting of Earth and the hairbrush?
   (e) What can you conclude about the momentum of this system?
   (f) If we consider Earth and the hairbrush to be initially stationary, how does Earth move as the hairbrush falls down?
   (g) If the hairbrush reaches a speed of 10 m/s when it hits Earth (initially stationary), what is Earth's speed at this time?
4. A 45-kg student stands on a stationary 33-kg raft. The student then walks with a velocity of 1.9 m/s [E] relative to the water. What is the resulting velocity of the raft, relative to the water, if fluid friction is negligible.
5. Two ice skaters, initially stationary, push each other so that they move in opposite directions. One skater of mass of 56.9 kg has a speed of 3.28 m/s. What is the mass of the other skater if her speed is 3.69 m/s? Neglect friction.
6. A stationary 35-kg artillery shell accidentally explodes, sending two fragments of mass 11 kg and 24 kg in opposite directions. The speed of the 11-kg fragment is 95 m/s. What is the speed of the other fragment?
7. A railway car of mass 1.37×10^4 kg, rolling at 20.0 km/h [N], collides with another railway car of mass 1.12×10^4 kg, also initially rolling north, but moving more slowly. After the collision, the coupled cars have a velocity of 18.3 km/h [N]. What is the initial velocity of the second car?
8. A 0.045-kg golf ball is hit with a driver. The head of the driver has a mass of 0.15 kg, and travels at a speed of 56 m/s before the collision. The ball has a speed of 67 m/s as it leaves the clubface. What is the speed of the head of the driver immediately after the collision?

Applying Inquiry Skills

9. The graph in **Figure 6** shows velocity as a function of time for a system of two carts that undergo an experimental collision on a horizontal, frictionless surface. The mass of cart A is 0.40 kg. The mass of cart B is 0.80 kg.

Answers

3. (g) 1×10^{-25} m/s
4. 2.6 m/s [W]
5. 50.6 kg
6. 44 m/s
7. 16.2 km/h [N]
8. 36 m/s
9. (b) 0.24 kg·m/s [E]
   (c) 0.10 m/s [W]

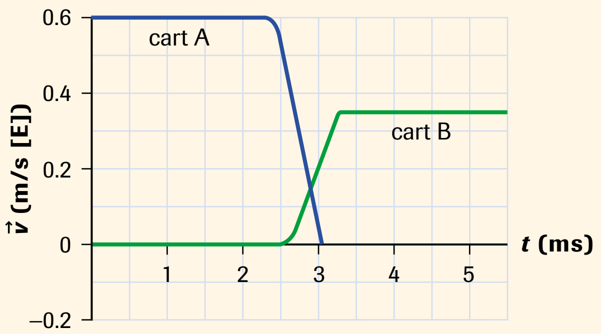

Figure 6
For question 9

(a) If you were conducting this experiment, describe what you would observe based on the graph.
(b) Determine the momentum of the system of carts before the collision.
(c) Assuming momentum is conserved, determine the velocity of cart A after the collision is complete.
(d) Copy the graph into your notebook, and complete the line for cart A. Superimpose lines that you think you would obtain in an experiment in which friction is not quite zero.

Making Connections

10. During a space walk, an astronaut becomes stranded a short distance from the spacecraft. Explain how the astronaut could solve the problem by applying conservation of momentum to return safely to the spacecraft. State any assumptions needed for your solution to work.

SUMMARY Conservation of Momentum in One Dimension

- The law of conservation of linear momentum states that if the net force acting on a system is zero, then the momentum of the system is conserved.
- During an interaction between two objects in a system on which the total net force is zero, the change in momentum of one object is equal in magnitude, but opposite in direction, to the change in momentum of the other object.
- For any collision involving a system on which the total net force is zero, the total momentum before the collision equals the total momentum after the collision.

Section 5.2 Questions

Understanding Concepts

1. A bowling ball (B) moving at high speed is about to collide with a single stationary pin (P) (**Figure 7**). The mass of the ball is more than four times greater than the mass of the pin. For the short time interval in which the collision occurs, state whether each of the following statements is true or false. If the statement is false, write a corrected version.
(a) The magnitude of the force exerted by B on P is greater than the magnitude of the force exerted by P on B.
(b) The magnitude of the change in velocity of B equals the magnitude of the change in velocity of P.

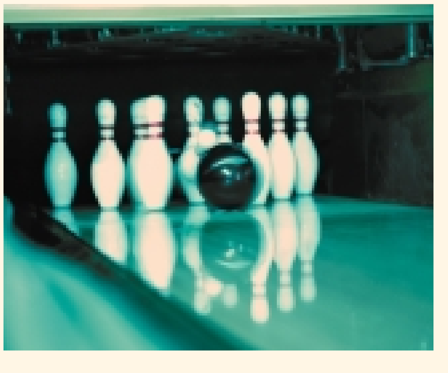

Figure 7
The bowling ball has a greater mass than the pin.

(c) The time interval of the collision for B equals the time interval of the collision for P.
(d) The magnitude of the change in momentum of B is less than the magnitude of the change in momentum of P.

2. Can individual objects in a system have nonzero momentum while the momentum of the entire system is zero? If "yes," give an example. If "no," explain why not.

3. In which of the following situations is the momentum conserved for the system of objects A and B?
(a) A vacationer A stands in a stationary raft B; the vacationer then walks in the raft. (Neglect fluid friction.)
(b) A freely rolling railway car A strikes a stationary railway car B.
(c) A veggie burger A is dropped vertically into a frying pan B and comes to rest.

4. In 1920, a prominent newspaper wrote the following about Robert Goddard, a pioneer of rocket ship development: "Professor Goddard does not know the relationship of action to reaction, and of the need to have something better than a vacuum against which to react. Of course, he only seems to lack the knowledge ladled out daily in high schools." Explain why the newspaper was wrong (as it itself admitted years later).

5. A 57-kg factory worker takes a ride on a large, freely rolling 27-kg cart. The worker initially stands still on the cart, and they both move at a speed of 3.2 m/s relative to the floor. The worker then walks on the cart in the same direction as the cart is moving. The worker's speed is now 3.8 m/s relative to the floor. What are the magnitude and direction of the final velocity of the cart?

6. A hiker of mass 65 kg is standing on a stationary raft of mass 35 kg. He is carrying a 19-kg backpack, which he throws horizontally. The resulting velocity of the hiker and the raft is 1.1 m/s [S] relative to the water. What is the velocity with which the hiker threw the backpack, relative to the water?

7. Two automobiles collide. One automobile of mass 1.13×10^3 kg is initially travelling at 25.7 m/s [E]. The other automobile of mass 1.25×10^3 kg has an initial velocity of 13.8 m/s [W]. The vehicles become attached during the collision. What is their common velocity immediately after the collision?

8. (a) Determine the magnitude and direction of the change in momentum for each automobile in question 7.
(b) How are these two quantities related?
(c) What is the total change in momentum of the two-automobile system?

9. A stationary quarterback is tackled by an 89-kg linebacker travelling with an initial speed of 5.2 m/s. As the two players move together after the collision, they have a speed of 2.7 m/s. What is the mass of the quarterback?

10. Two balls roll directly toward each other. The 0.25-kg ball has a speed of 1.7 m/s; the 0.18-kg ball has a speed of 2.5 m/s. After the collision, the 0.25-kg ball has reversed its direction and has a speed of 0.10 m/s. What is the magnitude and direction of the velocity of the 0.18-kg ball after the collision?

Applying Inquiry Skills

11. You are given two dynamics carts of masses m_1 and m_2, with nearly frictionless wheels. The carts are touching each other and are initially at rest. Cart 1 has an internal spring mechanism that is initially compressed (**Figure 8**). The spring is suddenly released, driving the carts apart. Describe an experimental procedure that you could use to test conservation of momentum for this "exploding" system. Include a list of apparatus you would need, safety precautions you would follow, and measurements you would take.

Making Connections

12. On a two-lane highway where the posted speed limit is 80 km/h, a car of mass m_C and an SUV of mass m_S have a head-on collision. The collision analyst observes that both vehicles came to a stop at the location of the initial impact. Researching the mass of the vehicles, the analyst found that $m_S = 2m_C$. Both drivers survived the collision and each claimed to be travelling at the legal speed limit when the collision occurred.
(a) It is obvious that the analyst cannot believe both drivers. Use numerical data to explain why.
(b) If both vehicles had been travelling at the legal speed limit before the collision, how would the accident scene have been different? (Assume that the collision was still head-on.)

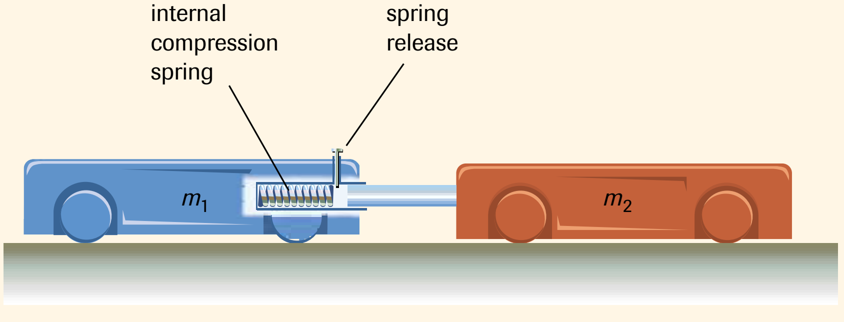

Figure 8
For question 11