5.1 Momentum and Impulse

Figure 1
The start zone of a bobsleigh race is about 15 m long. The racers must exert as large a force as possible on the bobsleigh over that distance to increase its velocity and give it as much momentum as possible.

In bobsleigh racing (**Figure 1**), also called bobsled racing, team members push as hard as they can on the bobsleigh in the start zone, then jump in to race it at the fastest speed possible down an icy, curved course over 1.2 km long. The two-person bobsleigh has a mass of about 220 kg and requires a large force to achieve a high velocity. When the competitors jump into the bobsleigh, the total mass can be as high as 390 kg.

The quantities of mass and velocity combine to give an object **linear momentum**. The linear momentum of a moving object is the product of the object's mass and (instantaneous) velocity:

$$\vec{p} = m\vec{v}$$

where $\vec{p}$ is the linear momentum of the object in kilogram metres per second, m is its mass in kilograms, and $\vec{v}$ is its velocity in metres per second. The direction of the linear momentum is the same as the direction of the velocity.

Linear momentum depends on both the mass and the velocity of an object. For any given mass, the linear momentum is directly proportional to the velocity, and for any given velocity, the linear momentum is directly proportional to the mass. A truck has more linear momentum than a car travelling at the same speed, but a fast-moving car may have the same linear momentum as a slow-moving truck.

Linear momentum is a vector quantity, being the product of a scalar (mass) and a vector (velocity). We will often consider the components of linear momentum:

$$p_x = mv_x$$
$$p_y = mv_y$$

linear momentum ($\vec{p}$) the product of the mass of a moving object and its velocity; a vector quantity

In most of our discussions, we will omit the word "linear" to describe the type of momentum that involves a mass moving linearly. Another type of momentum, called *angular momentum*, is possessed by a rotating object, such as a spinning figure skater. (Angular momentum is not presented in detail in this text.)

SAMPLE problem 1

Determine the momentum of a Pacific leatherback turtle of mass 8.6×10^2 kg, swimming at a velocity of 1.3 m/s [forward]. (The Pacific leatherback turtle is the world's largest species of turtle.)

Solution

$m = 8.6 \times 10^2$ kg
$\vec{v} = 1.3$ m/s [forward]
$\vec{p} = ?$

$$\vec{p} = m\vec{v}$$
$$= (8.6 \times 10^2 \text{ kg})(1.3 \text{ m/s [forward]})$$
$$\vec{p} = 1.1 \times 10^3 \text{ kg·m/s [forward]}$$

The momentum of the turtle is 1.1×10^3 kg·m/s [forward].

Practice

Understanding Concepts

1. Calculate the momentum of each of the following:
   (a) a 7.0×10^3-kg African elephant running at 7.9 m/s [E]
   (b) a 19-kg mute swan flying at 26 m/s [S]
   (c) an electron of mass 9.1×10^{-31} kg moving at 1.0×10^7 m/s [forward]
2. A personal watercraft and its rider have a combined mass of 405 kg, and a momentum of 5.02×10^3 kg·m/s [W]. Determine the velocity of the craft.
3. A bullet travelling at 9.0×10^2 m/s [W] has a momentum of 4.5 kg·m/s [W]. What is its mass?
4. (a) By estimating your top running speed, estimate the magnitude of your momentum when you are running at this speed.
   (b) How fast would a typical compact car have to be moving to reach the same momentum? State your assumptions and show your calculations.

Answers

1. (a) 5.5×10^4 kg·m/s [E]
   (b) 4.9×10^2 kg·m/s [S]
   (c) 9.1×10^{-24} kg·m/s [forward]
2. 12.4 m/s [W]
3. 5.0×10^{-3} kg, or 5.0 g

Impulse and Change in Momentum

Consider the factors that cause the momentum of the bobsleigh shown in **Figure 1** to build in the start zone from zero to the maximum possible value. The force applied by the team is an obvious factor: the greater the force applied, the greater is the final momentum. The other factor is the time interval over which the force is applied: the greater the time interval, the greater is the final momentum. To analyze these relationships, we refer to Newton's second law of motion.

Newton's second law states that an object acted upon by an external net force accelerates in the direction of the net force; the relationship between the object's mass, acceleration, and the net force acting on it is expressed by the equation $\sum\vec{F} = m\vec{a}$. We can derive an equation to express an object's change in momentum in terms of the net force (assumed to be constant) and the time interval starting with this equation:

$$\sum\vec{F} = m\vec{a}$$

$$\sum\vec{F} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right)$$

$$\sum\vec{F}\Delta t = m(\vec{v}_f - \vec{v}_i)$$

$$\sum\vec{F}\Delta t = \vec{p}_f - \vec{p}_i$$

Since the vector subtraction $\vec{p}_f - \vec{p}_i = \Delta\vec{p}$, we have:

$$\sum\vec{F}\Delta t = \Delta\vec{p}$$

The product $\sum\vec{F}\Delta t$ is called the **impulse**, which is *equal to the change in momentum*. The SI unit of impulse is the newton second (N·s), and the direction of the impulse is the same as the direction of the change in momentum.

impulse the product $\sum\vec{F}\Delta t$, equal to the object's change in momentum

You can apply the concepts implied in the equation $\sum\vec{F}\Delta t = \Delta\vec{p}$ in your daily activities. For example, catching a ball with your bare hands will hurt depending on the force $\sum\vec{F}$ of the ball. Since the ball always approaches you at the same speed, its change in momentum as you stop it is always the same: $\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) = m(0 - \vec{v}_i)$. Thus, if you allow your hands to move with the ball as you catch it, Δt will be larger, $\sum\vec{F}$ will be smaller, and your hands will hurt less.

DID YOU KNOW?

High-Speed Particles

The familiar form of Newton's second law of motion, $\sum\vec{F} = m\vec{a}$, does not apply to tiny particles, such as electrons, when their speeds approach the speed of light. However, the more general form, $\sum\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$, does apply even at these high speeds.

LEARNING TIP

Finding the Areas on Graphs

Estimating the average force on a force-time graph involving a nonconstant force is one way of determining the impulse (i.e., the area) on an object. Another way is to draw a grid system on the graph as in **Figure 2(c)** and count the small rectangles of known area. A third way involves applying integral calculus, a topic left for more advanced physics courses.

The relationship between impulse and change in momentum can be stated for components:

$$\sum F_x \Delta t = \Delta p_x$$
$$\sum F_y \Delta t = \Delta p_y$$

The form of Newton's second law of motion that we are familiar with is $\sum\vec{F} = m\vec{a}$, but by rearranging $\sum\vec{F}\Delta t = \Delta\vec{p}$, we can express Newton's second law as

$$\sum\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

This equation indicates that *the net force on an object equals the rate of change of the object's momentum.* This form of the second law is actually more general than $\sum\vec{F} = m\vec{a}$ because it lets us handle situations in which the mass changes. In fact, it is the way in which Newton originally stated his second law. In terms of *x*- and *y*-components, we can write this equation as

$$\sum F_x = \frac{\Delta p_x}{\Delta t} \quad \text{and} \quad \sum F_y = \frac{\Delta p_y}{\Delta t}$$

In deriving the equation $\sum\vec{F}\Delta t = \Delta\vec{p}$, we assumed that the acceleration of the object is constant, and thus the net force on the object is constant. However, in many situations, the force applied to an object changes nonlinearly during its time of application. The equation $\sum\vec{F}\Delta t = \Delta\vec{p}$ still applies, provided the net force $\sum\vec{F}$ is equal to the *average force* acting on the object over the time interval Δt.

To understand the term "average force," consider **Figure 2(a)**, which shows the typical shape of a graph of the magnitude of the force as a function of time during a collision or other interaction of time interval Δt. The graph could represent, for example, the force acting on a soccer ball as a player kicks it. The area between the curved line and the time axis represents the impulse given to the object. (You can verify this by considering that the unit of the area, the newton second (N·s), represents impulse.) The *average force* is the constant force that would yield the same impulse as the changing force does in the same time interval. On a force-time graph, the average force is the constant force, shown as the straight line in **Figure 2(b)**, that would yield the same area as the curved line in the same time interval.

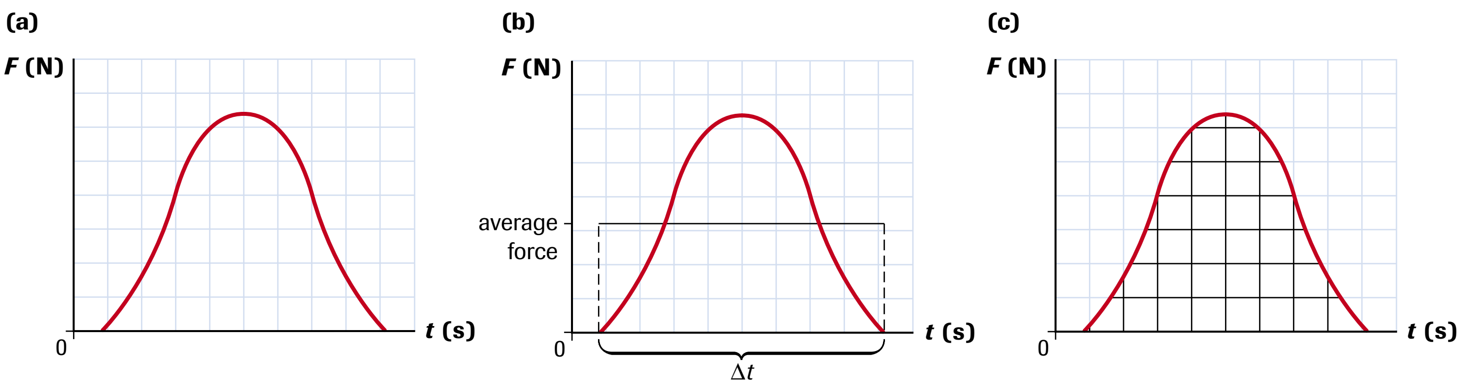

Figure 2

(a) The magnitude of the force acting on an object during a typical collision
(b) The average force, acting over Δt, gives the same area as the area under the curve in (a).
(c) Estimating the area under the curve by counting squares of known area on a superimposed grid

SAMPLE problem 2

A baseball of mass 0.152 kg, travelling horizontally at 37.5 m/s [E], collides with a baseball bat. The collision lasts for 1.15 ms. Immediately after the collision, the baseball travels horizontally at 49.5 m/s [W] (**Figure 3**).

(a) Determine the initial momentum of the baseball.

(b) What is the average force applied by the bat to the baseball?

(c) Determine the ratio of the magnitude of this force to the magnitude of the force of gravity on the baseball.

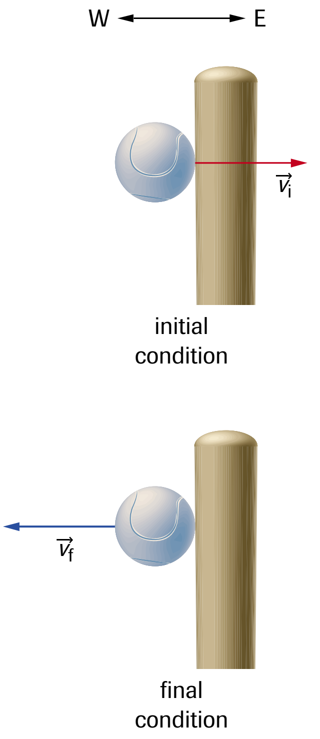

Figure 3
The situation for Sample Problem 2

Solution

(a) $m = 0.152$ kg

$\vec{v}_i = 37.5$ m/s [E]

$\vec{p}_i = ?$

$$\vec{p}_i = m\vec{v}_i$$
$$= (0.152 \text{ kg})(37.5 \text{ m/s [E]})$$
$$\vec{p}_i = 5.70 \text{ kg·m/s [E]}$$

The initial momentum is 5.70 kg·m/s [E].

(b) $m = 0.152$ kg

$\Delta t = 1.15 \text{ ms} = 1.15 \times 10^{-3}$ s

$\vec{v}_i = 37.5$ m/s [E]

$\vec{v}_f = 49.5$ m/s [W]

$\sum\vec{F} = ?$

$$\sum\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$
$$= \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$
$$= \frac{(0.152 \text{ kg})(49.5 \text{ m/s [W]} - 37.5 \text{ m/s [E]})}{1.15 \times 10^{-3} \text{ s}}$$
$$= \frac{(0.152 \text{ kg})(49.5 \text{ m/s [W]} + 37.5 \text{ m/s [W]})}{1.15 \times 10^{-3} \text{ s}}$$
$$\sum\vec{F} = 1.15 \times 10^4 \text{ N [W]}$$

The average force is 1.15×10^4 N [W].

(c) Determine the magnitude of the force of gravity on the ball:

$$F_g = mg$$
$$= (0.152 \text{ kg})(9.80 \text{ N/kg})$$
$$F_g = 1.49 \text{ N}$$

We can now calculate the required ratio:

$$\frac{1.15 \times 10^4 \text{ N}}{1.49 \text{ N}} = 7.72 \times 10^3$$

The ratio of forces is $7.72 \times 10^3 : 1$.

In Sample Problem 2(c), the magnitude of the average force exerted by the bat on the baseball is almost 8000 times larger than the magnitude of the force of gravity. In general, the forces between objects involved in collisions tend to be much larger than other forces, such as gravity. We can therefore usually ignore other forces when analyzing collisions.

To analyze the effect on impulse of a "follow-through" in sports activities, we will look at a specific example of a typical collision between a tennis ball and a racket.

DID YOU KNOW?

Realistic Values

Numerical values used in questions and sample problems in this text are all realistic. For example, the official mass of a tennis ball ranges from 56.7 g to 58.5 g, so the 57-g ball described in Sample Problem 3 is a realistic value.

SAMPLE problem 3

A 57-g tennis ball is thrown upward and then struck just as it comes to rest at the top of its motion. The racket exerts an average horizontal force of magnitude 4.2×10^2 N on the tennis ball.

(a) Determine the speed of the ball after the collision if the average force is exerted on the ball for 4.5 ms.

(b) Repeat the calculation, assuming a time interval of 5.3 ms.

(c) Explain the meaning and advantage of follow-through in this example.

Solution

(a) Since this is a one-dimensional problem, we can use components.

$m = 57\text{ g} = 0.057\text{ kg}$ $\quad v_{ix} = 0$

$\sum F_x = 4.2 \times 10^2\text{ N}$ $\quad v_{fx} = ?$

$\Delta t = 4.5\text{ ms} = 4.5 \times 10^{-3}\text{ s}$

$$\sum F_x \Delta t = \Delta p_x$$
$$\sum F_x \Delta t = m(v_{fx} - v_{ix})$$
$$\sum F_x \Delta t = mv_{fx} - mv_{ix}$$
$$mv_{fx} = \sum F_x \Delta t + mv_{ix}$$
$$v_{fx} = \frac{\sum F_x \Delta t}{m} + v_{ix}$$
$$= \frac{(4.2 \times 10^2\text{ N})(4.5 \times 10^{-3}\text{ s})}{0.057\text{ kg}} + 0$$
$$v_{fx} = 33\text{ m/s}$$

The speed of the tennis ball after the collision is 33 m/s.

(b) $m = 57\text{ g} = 0.057\text{ kg}$ $\quad v_{ix} = 0$

$\sum F_x = 4.2 \times 10^2\text{ N}$ $\quad v_{fx} = ?$

$\Delta t = 5.3\text{ ms} = 5.3 \times 10^{-3}\text{ s}$

We use the same equation for v_{fx} as was derived in part (a):

$$v_{fx} = \frac{\sum F_x \Delta t}{m} + v_{ix}$$
$$= \frac{(4.2 \times 10^2\text{ N})(5.3 \times 10^{-3}\text{ s})}{0.057\text{ kg}} + 0$$
$$v_{fx} = 39\text{ m/s}$$

The speed of the tennis ball after the collision is 39 m/s.

(c) The racket in (b) exerts the same average force as in (a) but over a longer time interval. The additional time interval of 0.8 ms is possible only if the player follows through in swinging the racket. The advantage of follow-through is that the final speed of the tennis ball after the collision is greater even though the average force on the ball is the same.

Practice

Understanding Concepts

5. Show that the units of impulse and change in momentum are equivalent.

6. A snowball of mass 65 g falls vertically toward the ground where it breaks apart and comes to rest. Its speed just before hitting the ground is 3.8 m/s. Determine
(a) the momentum of the snowball before hitting the ground
(b) the momentum of the snowball after hitting the ground
(c) the change in momentum

7. A truck's initial momentum is 5.8×10^4 kg·m/s [W]. An average force of 4.8×10^3 N [W] increases the truck's momentum for the next 3.5 s.
(a) What is the impulse on the truck over this time interval?
(b) What is the final momentum of the truck?

8. A 0.27-kg volleyball, with an initial velocity of 2.7 m/s horizontally, hits a net, stops, and then drops to the ground. The average force exerted on the volleyball by the net is 33 N [W]. How long, in milliseconds, is the ball in contact with the net?

9. In its approach to an airport runway, an airplane of mass 1.24×10^5 kg has a velocity of 75.5 m/s [11.1° below the horizontal]. Determine the horizontal and vertical components of its momentum.

Applying Inquiry Skills

10. Determine the impulse imparted during the interaction represented in each graph in **Figure 4**.

Making Connections

11. In boxing matches in the nineteenth century, boxers fought with bare hands. Today's boxers use padded gloves.
(a) How do gloves help protect a boxer's head (and brain) from injury?
(b) Boxers often "roll with the punch." Use physics principles to explain how this manoeuvre helps protect them.

Answers

6. (a) 0.25 kg·m/s [down]
(b) zero
(c) 0.25 kg·m/s [up]
7. (a) 1.7×10^4 N·s [W]
(b) 7.5×10^4 kg·m/s [W]
8. 22 ms
9. 9.19×10^6 kg·m/s; 1.80×10^6 kg·m/s
10. (a) 1.0 N·s [E]
(b) about 4.0 N·s [S]

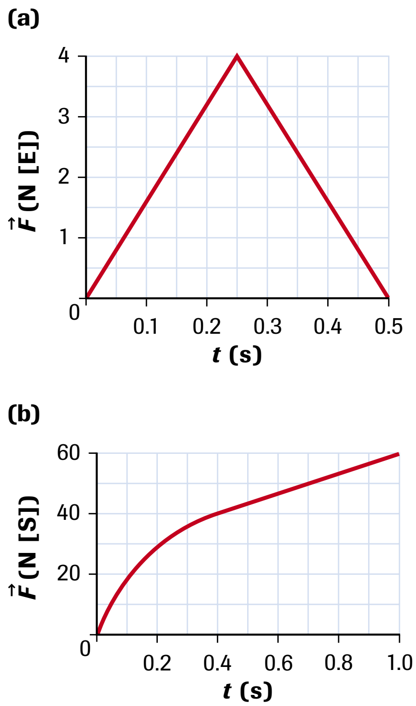

Figure 4
For question 10

SUMMARY Momentum and Impulse

- The linear momentum of an object is the product of the object's mass and velocity. It is a vector quantity whose SI base units are kg·m/s.
- The impulse given to an object is the product of the average net force acting on the object and the time interval over which that force acts. It is a vector quantity whose SI base units are N·s.
- The impulse given to an object equals the change in momentum experienced by the object.

Section 5.1 Questions

Understanding Concepts

1. State Newton's second law as it was originally proposed by Newton, as a relationship between momentum and force. Write an equation expressing this relationship.

2. What is the property of the forces for which the concept of impulse proves most helpful when analyzing the changes in motion that occur as a result of the force?

3. Determine the impulse exerted in each of the following cases:
(a) An average force of 24 N [E] is applied to a dynamics cart for 3.2 s.
(b) A hockey stick exerts an average force of 1.2×10^2 N [forward] on a puck during the 9.1 ms they are in contact.
(c) Earth pulls down on a 12-kg rock during the 3.0 s it takes the rock to fall from a cliff.
(d) A toy car crashes into a brick wall, experiencing the changing force shown on the force-time graph in **Figure 5**.

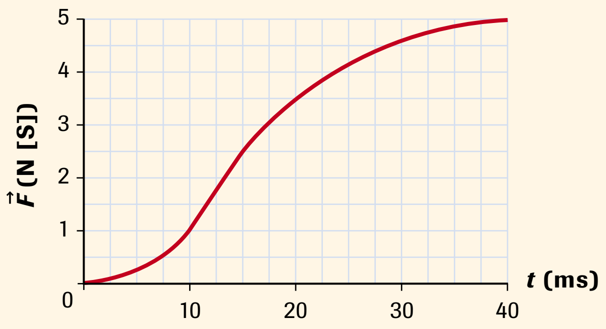

Figure 5

4. What velocity will a 41-kg child sitting on a 21-kg wagon acquire if pushed from rest by an average force of 75 N [W] for 2.0 s?

5. What average force will stop a 1.1×10^3-kg car in 1.5 s, if the car is initially moving at 22 m/s [E]?

6. A billiard ball of mass 0.17 kg rolls toward the right-hand cushion of a billiard table at 2.1 m/s, then rebounds straight back at 1.8 m/s.
(a) What is the change in momentum of the ball as a result of hitting the cushion?
(b) What impulse does the cushion give to the ball?

7. A 0.16-kg hockey puck is sliding along a smooth, flat section of ice at 18 m/s when it encounters some snow. After 2.5 s of sliding through the snow, it returns to smooth ice, continuing at a speed of 11 m/s.
(a) What is the change in the momentum of the puck?
(b) What impulse does the snow exert on the puck?
(c) What average frictional force does the snow exert on the puck?

8. A frictionless disk of mass 0.50 kg is moving in a straight line across an air table at a speed of 2.4 m/s [E] when it bumps into an elastic band stretched between two fixed posts. The elastic band exerts an average opposing force of 1.4 N [W] on the disk for 1.5 s. What is the final velocity of the disk?

9. A 2.0-kg skateboard is rolling across a smooth, flat floor when a child kicks it, causing it to speed up to 4.5 m/s [N], in 0.50 s, without changing direction. The average force exerted by the child on the skateboard is 6.0 N [N]. What is the initial velocity of the skateboard?

10. A 0.61-kg basketball is thrown vertically downward. Just before it hits the floor, its speed is 9.6 m/s. It then rebounds upward with a speed of 8.5 m/s just as it leaves the floor. The basketball is in contact with the floor for 6.5 ms.
(a) Determine the basketball's change in momentum.
(b) Determine the average force exerted on the basketball by the floor. (Apply an equation involving the change in momentum.)

11. Explain why the force of gravity can be ignored during the collision between the basketball and the floor in question 10.

Applying Inquiry Skills

12. Describe an experiment you would perform to determine which of four or five athletes have good follow-through when they are striking a ball (or a puck) with a tennis racket, golf club, baseball bat, or hockey stick. Assume that you are allowed to use sophisticated apparatus, such as a digital camera or a video camera, and a stroboscopic light source. Draw sketches to compare photographic images of a good follow-through with a weak follow-through.

Making Connections

13. Experience has taught you that when you land upright on the ground after a jump, you feel less pain if you bend your knees as you are landing than if you land with your knees locked. Explain why bending your knees helps alleviate pain and injury.

14. A collision analyst applies physics principles to reconstruct what happened at automobile crashes. Assume that during a head-on crash between two cars, the cars came to a stop, and a large component of a plastic bumper broke off one car and skidded along the highway shoulder leaving skid marks.
(a) What information could the analyst gather from the bumper component and from the skid marks it produced? What measurements would be needed to estimate the speed of the cars just before the collision, and how would they be used?
(b) Find out more about how traffic collision analysts apply physics and math in their investigations using the Internet or other suitable publications. Report your findings in a paper, highlighting connections with momentum.

GO www.science.nelson.com