The Law of Conservation of Energy 4.4

What does a computer have in common with a moving truck? What does the production of starlight have in common with a waterfall? In each case, energy is converted from one form into another form, while following an extremely important law of nature—the *law of conservation of energy*.

Law of Conservation of Energy

For an isolated system, energy can be converted into different forms, but cannot be created or destroyed.

The law of conservation of energy is an example of a conservation law that applies to an **isolated system**, which is a system of particles that is completely isolated from outside influences. As the particles of an isolated system move about and interact with one another, the total energy of the system remains constant with no energy flowing into or out of the system. An example of an isolated system is the system of your calculator sliding across your desk after your hand has stopped pushing it. The kinetic energy of the calculator is converted into other forms of energy, mainly thermal energy, and a small amount of sound energy as the particles of the desk and calculator rub against each other.

As far as we know, the law of conservation of energy cannot be violated. It is one of the fundamental principles in operation in the universe. When applied in physics, the law provides a very useful tool for analyzing a variety of problems.

As an example, consider a 50.0-kg swimmer falling (from rest) from a 3.00-m diving board under negligible air resistance. At the diving board, the swimmer has zero kinetic energy and maximum gravitational potential energy relative to the water surface below. Together, the kinetic energy and gravitational potential energy make up the swimmer's *mechanical energy*. When the swimmer is falling, the gravitational potential energy is converted into kinetic energy, but the total amount of mechanical energy remains constant. **Table 1** gives sample values for this example.

Table 1 Various Energies of a Falling Swimmer

Height (m)	Gravitational Potential Energy E_g (J)	Kinetic Energy E_K (J)	Total Mechanical Energy $E_T = E_g + E_K$ (J)
3.00	1.47×10^3	0.00	1.47×10^3
2.00	9.80×10^2	4.90×10^2	1.47×10^3
1.00	4.90×10^2	9.80×10^2	1.47×10^3
0.00	0.00	1.47×10^3	1.47×10^3

So far, the only forms of energy we have discussed in detail are gravitational potential energy and kinetic energy. The sum of these two energies is called the *total mechanical energy* E_T, where $E_T = E_g + E_K$. We will extend this equation in Section 4.5 to include other forms of energy, such as elastic potential energy.

DID YOU KNOW?

Energy Conservation

Do not confuse the expressions "conservation of energy" and "energy conservation." Conservation of energy is a law of nature. Energy conservation, which refers to the wise use of energy resources, is something that we should all practise.

isolated system a system of particles that is completely isolated from outside influences

LEARNING TIP

Closed and Open Systems

An isolated system is a *closed* system. The opposite of a closed system is an *open* system, which either gains or loses energy to an outside system. In the situations analyzed in this text, we will look only at closed (or isolated) systems using the law of conservation of energy.

SAMPLE problem 1

A basketball player makes a free-throw shot at the basket. The basketball leaves the player's hand at a speed of 7.2 m/s from a height of 2.21 m above the floor. Determine the speed of the basketball as it goes through the hoop, 3.05 m above the floor. **Figure 1** shows the situation.

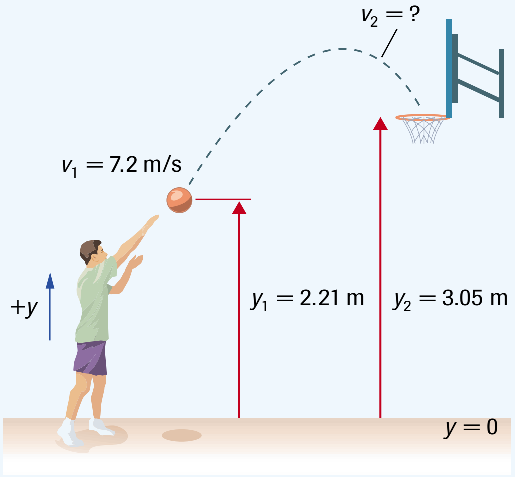

Figure 1

LEARNING TIP

Choosing Reference Levels
In Sample Problem 1, the floor was used as the reference level. A different reference level, such as the release position, would yield the same final answer. When you solve a problem involving gravitational potential energy, remember to choose a reference level and use that level throughout the problem.

Solution

According to the law of conservation of energy, the total energy of the basketball is constant as it travels through the air. Using the subscript 1 for the release position and 2 for the position where the basketball goes through the hoop, and taking the heights y_1 and y_2 relative to the floor,

$v_1 = 7.2$ m/s $\qquad g = 9.80$ m/s^2

$y_1 = 2.21$ m $\qquad v_2 = ?$

$y_2 = 3.05$ m

Applying the law of conservation of energy:

$$E_{T1} = E_{T2}$$

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$$

$$mv_1^2 + 2mgy_1 = mv_2^2 + 2mgy_2$$

$$v_1^2 + 2gy_1 = v_2^2 + 2gy_2$$

$$v_2^2 = v_1^2 + 2gy_1 - 2gy_2$$

$$v_2^2 = v_1^2 + 2g(y_1 - y_2)$$

$$v_2 = \pm\sqrt{v_1^2 + 2g(y_1 - y_2)}$$

$$= \pm\sqrt{(7.2\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(2.21\text{ m} - 3.05\text{ m})}$$

$$v_2 = \pm 5.9\text{ m/s}$$

Only the positive square root applies since speed is always greater than zero. The speed of the basketball through the hoop is therefore 5.9 m/s. It is logical that this speed is less than the release speed because the ball is at a higher level; the gravitational potential energy of the ball is greater while its kinetic energy (and thus its speed) must be less.

The direction of the initial velocity and the parabolic path that the basketball follows are not important in solving Sample Problem 1. The kinetic and potential energies are scalar quantities, and do not involve a direction.

In general, we consider the work done by the net force as the amount of energy transformed from one form into another. The forms of energy involved depend on the nature of the net force. However, if the work done by the net force is positive, then the kinetic energy increases. Similarly, if the work done by the net force is negative, the kinetic energy decreases.

The law of conservation of energy can be applied to many practical situations. A common example is found in the operation of a "gravity clock," like the one in **Figure 2**. To explore this application and others, perform Activity 4.4.1 in the Lab Activities section at the end of this chapter.

ACTIVITY 4.4.1

Applying the Law of Conservation of Energy (p. 220)

Gravity clocks tend to be used more for decoration than for accurate timekeeping. They do, however, provide a good example of the application of the law of conservation of energy. Describe what you think are the energy transformations responsible for the operation of a gravity clock.

Figure 2
A gravity clock

Practice

Understanding Concepts

1. A ball has an initial speed of 16 m/s. After a single external force acts on the ball, its speed is 11 m/s. Has the force done positive or negative work on the ball? Explain.
2. Two snowboarders start from rest at the same elevation at the top of a straight slope. They take different routes to the bottom, but end at the same lower elevation. If the energy lost due to friction and air resistance is identical for both snowboarders, how do their final speeds compare? Would your answer be different if the slope has dips and rises, instead of being straight?
3. Apply energy concepts to determine the maximum speed reached by a roller coaster at the bottom of the first hill, if the vertical drop from the top of the hill is 59.4 m, and the speed at the top of the hill is approximately zero. Friction and air resistance are negligible. Express your answer in metres per second and kilometres per hour.
4. A skier, moving at 9.7 m/s across the top of a mogul (a large bump), becomes airborne, and moves as a projectile under negligible air resistance. The skier lands on the downward side of the hill at an elevation 4.2 m below the top of the mogul. Use energy concepts to determine the skier's speed upon touching the hillside.
5. The highest waterfall in Canada is the Della Falls in British Columbia, with a change in elevation of 4.4×10^2 m. When the water has fallen 12% of its way to the bottom, its speed is 33 m/s. Neglecting air resistance and fluid friction, determine the speed of the water at the top of the waterfall.
6. A cyclist reaches the bottom of a gradual hill with a speed of 9.7 m/s—a speed great enough to coast up and over the next hill, 4.7 m high, without pedalling. Friction and air resistance are negligible. Find the speed at which the cyclist crests the hill.
7. A simple pendulum, 85.5 cm long, is held at rest so that its amplitude will be 24.5 cm as illustrated in **Figure 3**. Neglecting friction and air resistance, use energy concepts to determine the maximum speed of the pendulum bob after release.

Applying Inquiry Skills

8. A 5.00-kg rock is released from rest from a height of 8.00 m above the ground. Prepare a data table to indicate the kinetic energy, the gravitational potential energy, and the total mechanical energy of the rock at heights of 8.00 m, 6.00 m, 4.00 m, 2.00 m, and 0.00 m. Plot a single graph showing all three energies as a function of the height above the ground. (If possible, use a spreadsheet program to generate the graph.)

Making Connections

9. A wrecking ball used to knock over a wall provides an example of the law of conservation of energy. Describe how a wrecking ball is used and list the energy transformations that occur in its use.

Answers

3. 34.1 m/s; 123 km/h
4. 13 m/s
5. 5.0 m/s
6. 1.4 m/s
7. 0.838 m/s

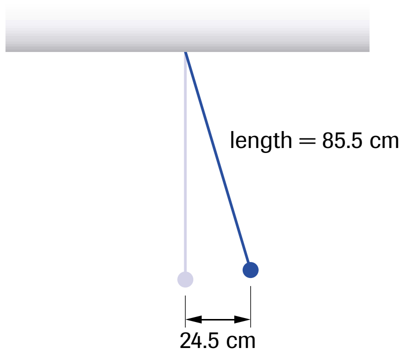

Figure 3
For question 7

Other Forms of Energy

Kinetic energy and gravitational potential energy are just two of the many forms of energy. **Table 2** lists several other forms of energy.

As the law of conservation of energy states, energy can be changed from one form to another. However, the efficiency of conversion is often not 100% because of friction. Friction causes kinetic energy to transform into **thermal energy**, or internal energy, which is associated with the motion of atoms and molecules. For example, picture a curling rock sliding along the ice in a straight line toward its target, covering a horizontal distance Δd (**Figure 4**). After the rock has left the player's hand, the only force that does work on the rock is the force of kinetic friction, $\vec{F}_K$. (Gravity and the normal force are both perpendicular to the displacement, and do no work on the rock.) Since the kinetic friction is in the opposite direction to the displacement, the angle θ between this force and the displacement is 180°. Since $\cos 180° = -1$, the work done by the kinetic friction is

$$W = (F_K \cos \theta)\Delta d$$
$$W = -F_K \Delta d$$

thermal energy (E_{th}) internal energy associated with the motion of atoms and molecules

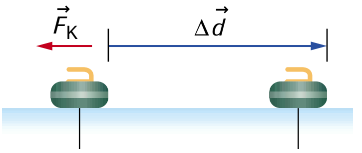

Figure 4
As the speed of the curling rock along the ice decreases, kinetic energy is transformed into thermal energy.

Table 2 Common Forms of Energy

Form of Energy	Comment
electromagnetic	• carried by travelling oscillations called electromagnetic waves • includes light energy, radio waves, microwaves, infrared waves, ultraviolet waves, X rays, and gamma rays • travels in a vacuum at 3.00×10^8 m/s, the speed of light
electrical	• results from the passage of electrons, for example, along wires in appliances in your home
electric potential	• associated with electric force • changes as charges are moved
gravitational potential	• associated with the gravitational force • changes as masses are moved relative to each other
chemical potential	• stored in the chemical bonds that hold the atoms of molecules together
nuclear potential	• the stored energy in the nucleus of an atom • converts into other forms by rearranging the particles inside a nucleus, by fusing nuclei together (fusion), or by breaking nuclei apart (fission)
sound	• carried by longitudinal waves from molecule to molecule
elastic potential	• stored in objects that are stretched or compressed
thermal	• associated with the motion of atoms and molecules • for a monatomic gas such as helium, it is the total kinetic energy of all the atoms • for more complicated molecules and for atoms in solids, it is partly kinetic energy and partly electric potential energy • differs from heat, which is the transfer of energy due to a difference in temperatures

Since F_K and Δd are both positive (being the magnitudes of vectors), the work done by friction is negative. This means that friction is removing kinetic energy from the rock. However, because energy is always conserved, the kinetic energy is transformed into another form, in this case into thermal energy of the rock and the ice. As the rock slides along the ice, the atoms in the rock and the ice vibrate with increased energy. Both the rock and the ice warm up, with a small portion of the ice developing a thin layer of melted water.

Whenever kinetic friction does negative work to slow an object down, the magnitude of the work equals the thermal energy produced. Since kinetic friction is always opposite to the direction of the displacement, the work done by friction can be written as

$$W = -F_K \Delta d$$

Note that the magnitude of the work done by friction is $F_K \Delta d$. Thus, we can write

$$E_{th} = F_K \Delta d$$

where E_{th} is the thermal energy produced by kinetic friction, in joules; F_K is the magnitude of the kinetic friction, in newtons; and Δd is the magnitude of the displacement, in metres.

SAMPLE problem 2

After leaving a player's hand, a 19.9-kg curling rock slides in a straight line for 28.8 m, experiencing friction with a coefficient of kinetic friction of 0.105. The situation is shown in **Figure 5**.

(a) How much thermal energy is produced during the slide?

(b) Determine, using energy conservation, the rock's speed just as it left the player's hand.

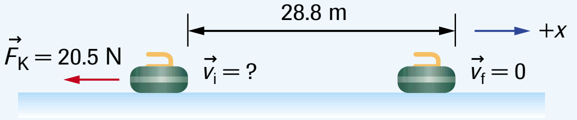

Figure 5
Using components to illustrate the rock's motion

Solution

(a) $\mu_K = 0.105$ $\quad F_K = \mu_K F_N$

$m = 19.9$ kg $\quad F_N = mg$

$\Delta d = 28.8$ m $\quad E_{th} = ?$

$$\begin{aligned} E_{th} &= F_K \Delta d \\ &= \mu_K F_N \Delta d \\ &= \mu_K mg \Delta d \\ &= (0.105)(19.9 \text{ kg})(9.80 \text{ N/kg})(28.8 \text{ m}) \\ E_{th} &= 5.90 \times 10^2 \text{ J} \end{aligned}$$

The thermal energy produced is 5.90×10^2 J.

(b) According to the law of conservation of energy, the initial kinetic energy of the rock must equal the thermal energy produced during the slide, because there is no kinetic energy remaining at the end of the slide. (Gravitational potential energy is not considered because the ice surface is level.)

$E_{th} = 5.90 \times 10^2$ J

$v_i = ?$

$$E_{Ki} = E_{th}$$

$$\frac{mv_i^2}{2} = E_{th}$$

$$v_i^2 = \frac{2E_{th}}{m}$$

$$v_i = \pm\sqrt{\frac{2(5.90 \times 10^2\ \text{J})}{19.9\ \text{kg}}}$$

$$v_i = \pm 7.70\ \text{m/s}$$

We choose the positive root because speed is always positive. Thus, the rock's initial speed is 7.70 m/s.

Practice

Understanding Concepts

10. (a) You push this book across a horizontal desk at a constant velocity. You are supplying some energy to the book. Into what form(s) does this energy go?

(b) You push the same book with a larger force so that the book accelerates. Into what form(s) does the energy now go?

11. A force of kinetic friction, of magnitude 67 N, acts on a box as it slides across the floor. The magnitude of the box's displacement is 3.5 m.

(a) What is the work done by friction on the box?

(b) How much thermal energy is produced?

12. A plate produces 0.620 J of thermal energy as it slides across a table. The kinetic friction force acting on the plate has a magnitude of 0.83 N. How far does the plate slide?

13. A clerk pushes a filing cabinet of mass 22.0 kg across the floor by exerting a horizontal force of magnitude 98 N. The magnitude of the force of kinetic friction acting on the cabinet is 87 N. The cabinet starts from rest. Use the law of conservation of energy to determine the speed of the cabinet after it moves 1.2 m.

14. A pen of mass 0.057 kg slides across a horizontal desk. In sliding 25 cm, its speed decreases to 5.7 cm/s. The force of kinetic friction exerted on the pen by the desk has a magnitude of 0.15 N. Apply the law of conservation of energy to determine the initial speed of the pen.

Applying Inquiry Skills

15. (a) Describe how you would use the motion of a simple pendulum to verify the law of conservation of energy.

(b) Describe sources of random and systematic error in your experiment.

Making Connections

16. Most of the thermal energy associated with operating a vehicle is produced by the combustion of fuel, although some is also produced by friction. Describe how the circulation systems in a car (the oil circulation system and the water-cooling system) relate to the thermal energy produced in a moving car.

Answers

11. (a) -2.3×10^2 J

(b) 2.3×10^2 J

12. 0.75 m

13. 1.1 m/s

14. 1.1 m/s

SUMMARY The Law of Conservation of Energy

- The law of conservation of energy states that for an isolated system, energy can be converted into different forms, but cannot be created or destroyed.
- The work done on a moving object by kinetic friction results in the conversion of kinetic energy into thermal energy.
- The law of conservation of energy can be applied to solve a great variety of physics problems.

Section 4.4 Questions

Understanding Concepts

1. Why do roller-coaster rides always start by going uphill?
2. You crack an egg and drop the contents of mass 0.052 kg from rest 11 cm above a frying pan. Determine, relative to the frying pan ($y = 0$),
   (a) the initial gravitational potential energy of the egg's contents
   (b) the final gravitational potential energy of the egg's contents
   (c) the change in gravitational potential energy as the egg's contents fall
   (d) the kinetic energy and the speed of the egg's contents just before hitting the pan
3. A child throws a ball, which hits a vertical wall at a height of 1.2 m above the ball's release point. The speed of the ball at impact is 9.9 m/s. What was the initial speed of the ball? Use conservation of energy and ignore air resistance.
4. A river flows at a speed of 3.74 m/s upstream from a waterfall of vertical height 8.74 m. During each second, 7.12×10^4 kg of water pass over the waterfall.
   (a) What is the gravitational potential energy of this mass of water at the top of the waterfall, relative to the bottom?
   (b) If there is a complete conversion of gravitational potential energy into kinetic energy, what is the speed of the water at the bottom?
5. An acrobat, starting from rest, swings freely on a trapeze of length 3.7 m (**Figure 6**). If the initial angle of the trapeze is 48°, use the law of conservation of energy to determine
   (a) the acrobat's speed at the bottom of the swing
   (b) the maximum height, relative to the initial position, to which the acrobat can rise
6. A skier of mass 55.0 kg slides down a slope 11.7 m long, inclined at an angle ϕ to the horizontal. The magnitude of the kinetic friction is 41.5 N. The skier's initial speed is 65.7 cm/s and the speed at the bottom of the slope is 7.19 m/s. Determine the angle ϕ from the law of conservation of energy. Air resistance is negligible.
7. A skateboarder, initially at rest at the top edge of a vertical ramp half-pipe (**Figure 7**), travels down the pipe, reaching a speed of 6.8 m/s at the bottom of the pipe. Friction is negligible. Use the law of conservation of energy to find the radius of the half-pipe.

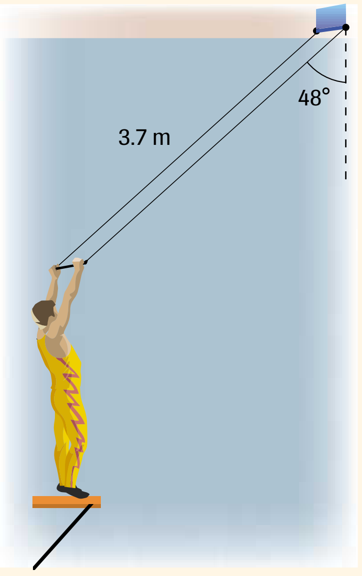

Figure 6
For question 5

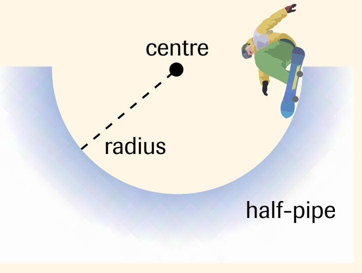

Figure 7
For question 7

8. A 55-g chalkboard eraser slides along the ledge at the bottom of the chalkboard. Its initial speed is 1.9 m/s. After sliding for 54 cm, it comes to rest.
   (a) Calculate, from the law of conservation of energy, the coefficient of kinetic friction between the eraser and the ledge.
   (b) Repeat the calculation using kinematics and force principles.
   (c) Into what form is the kinetic energy converted?

9. Brakes are applied to a car initially travelling at 85 km/h. The wheels lock and the car skids for 47 m before coming to a halt. The magnitude of the kinetic friction between the skidding car and the road is 7.4×10^3 N.
   (a) How much thermal energy is produced during the skid?
   (b) In what form was this thermal energy before the skid?
   (c) Use the law of conservation of energy to determine the mass of the car.
   (d) Determine the coefficient of kinetic friction between the tires and the road.
10. A box of apples of mass 22 kg slides 2.5 m down a ramp inclined at 44° to the horizontal. The force of friction on the box has a magnitude of 79 N. The box starts from rest.
   (a) Determine the work done by friction.
   (b) Determine the box's final kinetic energy. (*Hint:* Use the law of conservation of energy.)
   (c) Find the thermal energy produced.

Applying Inquiry Skills

11. The backward-looping roller coaster in **Figure 8** accelerates from the top of the lift by the station, travels through several vertical loops, and then moves part way up a second lift. At this stage, the coaster must be pulled up to the top of the second lift, where it is released to accelerate backward, returning the riders to the first lift and the station.
   (a) What physics principle(s) would you apply to determine the amount of thermal energy produced as the coaster travels from the top of the first lift to the position where it starts to be pulled up the second lift?
   (b) List the equations you would need to determine the amount of energy converted into thermal energy for the situation in (a).
   (c) Describe how you would conduct measurements from the ground outside the ride to determine an estimate of the thermal energy mentioned in (a).

Making Connections

12. In medieval times, the weapon of choice for military engineers was a *trebuchet*, a mechanical device that converted gravitational potential energy of a large rock into kinetic energy. **Figure 9** shows the design of a gravity-powered trebuchet used to hurl projectiles at castle walls. Find out more about this device, and explain why it is a good illustration of the law of conservation of energy.

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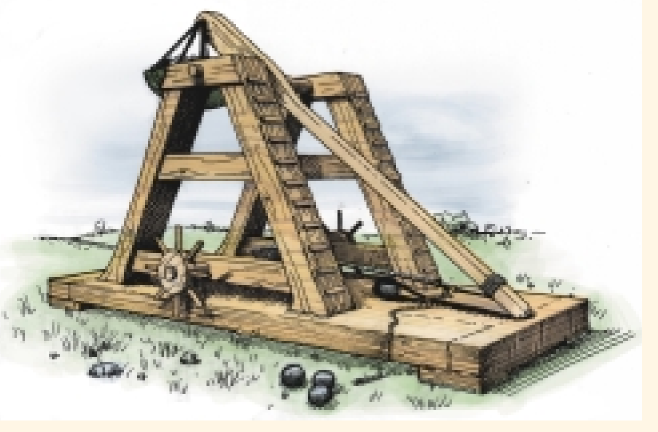

Figure 9
A medieval trebuchet

Figure 8
A reversing ride is easy to analyze to determine energy losses due to friction.