Gravitational Potential Energy at Earth's Surface 4.3

A roller coaster, like the one in **Figure 1**, is called a "gravity ride" for a very good reason. Work is done on the coaster to raise it to the top of the first hill, which is the highest position in the ride. Once the coaster leaves that position, the only work that keeps the coaster moving is the work done by gravity. At the highest position, the coaster has the maximum *potential* to develop kinetic energy. The coaster has **gravitational potential energy** due to its elevation above Earth's surface.

gravitational potential energy (E_g) the energy due to elevation above Earth's surface

To analyze gravitational potential energy mathematically, consider a situation in which a box of groceries is raised by an applied force to rollers at a higher level (**Figure 2**). Because gravity acts vertically, we will use Δy rather than Δd for the magnitude of the displacement. The force applied to the box to raise it is in the same direction as the displacement and has a magnitude equal to mg. The work done by the force on the box is

$$W = (F\cos\theta)\Delta y$$
$$= mg(\cos 0°)\Delta y$$
$$W = mg\Delta y$$

At the top of the rollers (the higher position), the box of groceries has gravitational potential energy relative to all lower positions. In other words, gravitational potential energy is a relative quantity in which the height of an object above some reference level must be known. Thus, the work done in increasing the elevation of an object is equal to the change in the gravitational potential energy:

$$\Delta E_g = mg\Delta y$$

where ΔE_g is the change in gravitational potential energy, in joules; m is the mass, in kilograms; g is the magnitude of the gravitational field constant in newtons per kilogram or metres per second squared; and Δy is the vertical component of the displacement, in metres.

There are a few important points to remember in using this equation:

- The equation determines the change in gravitational potential energy and does *not* determine an absolute value of gravitational potential energy. In practical problems involving the equation, Earth's surface is often used as a reference level of zero gravitational potential energy, although any other convenient arbitrary level may be chosen.
- The value of Δy is the vertical displacement of the object. This means that the horizontal path an object follows in changing its vertical height is not significant.
- The equation may only be used when Δy is small enough that g does not vary appreciably over Δy.
- Values of Δy (and ΔE_g) are positive if the displacement is upward, and negative if the displacement is downward.

Many of the situations in which this equation is applied involve objects that are thrown or lifted up away from Earth, or dropped toward Earth. In such cases, kinetic energy is converted into gravitational potential energy as the object moves upward, and gravitational potential energy is converted into kinetic energy after the object is released and falls downward. In both cases, if friction is negligible, the sum total of kinetic energy and gravitational potential energy remains constant.

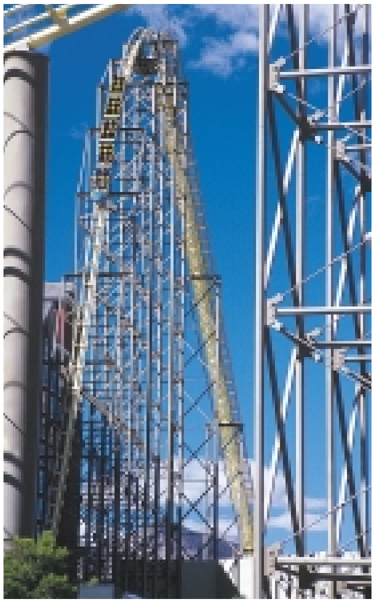

Figure 1
In any "gravity ride," such as this roller coaster, the first hill is the highest. Can you explain why?

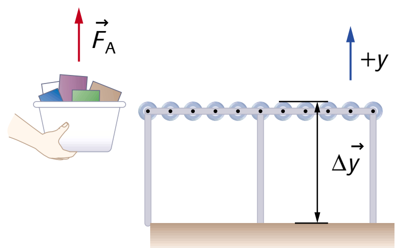

Figure 2
Work is done on the box of groceries to raise it from the floor to the top of the rollers.

When an object has gravitational potential energy relative to a lower position and the object is released, the force of gravity does work on the object, giving it kinetic energy, as specified by the work-energy theorem. For example, if you hold a basketball at shoulder height and then drop it, the ball has an initial velocity of zero, but a gravitational potential energy relative to the floor. When you release the ball, the force of gravity does work on the ball and the gravitational potential energy changes into kinetic energy. As a roller coaster leaves the highest hill, gravity does work on the coaster and gravitational potential energy changes into kinetic energy. The kinetic energy provides the coaster with enough speed to get up the next hill.

SAMPLE problem

A diver, of mass 57.8 kg, climbs up a diving-board ladder and then walks to the edge of the board. He then steps off the board and falls vertically from rest to the water 3.00 m below. The situation is shown in **Figure 3**. Determine the diver's gravitational potential energy at the edge of the diving board, relative to the water.

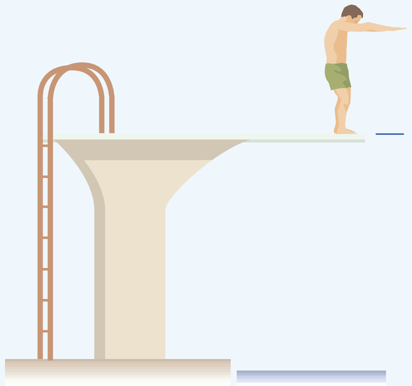

Figure 3

Solution

The $+y$ direction is upward. The reference position ($y = 0$) is the level of the water.

$m = 57.8$ kg

$g = 9.80$ m/s^2

$\Delta y = 3.00$ m

$\Delta E_g = ?$

$$\Delta E_g = mg\Delta y$$
$$= (57.8 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m})$$
$$\Delta E_g = 1.70 \times 10^3 \text{ J}$$

The diver's gravitational potential energy relative to the water is 1.70×10^3 J.

LEARNING TIP

Choosing the Reference Position

Gravitational potential energy is a relative quantity—it is always measured with respect to some arbitrary position where $y = 0$. In many cases, you will have to choose the reference position. It is usually easiest to choose $y = 0$ to be the lowest possible position. This ensures that all y values and all gravitational potential energies are positive (or zero), and therefore easy to work with. Often it is the change in gravitational potential energy, which does not depend on the choice of $y = 0$, that is important. Strictly speaking, y is the height of the centre of mass of the object.

Notice that in Sample Problem 1, the diver would have the same gravitational potential energy relative to the water surface no matter where he stood along the horizontal diving board. Furthermore, his gravitational potential energy relative to the water surface does not depend on the path he took to reach the higher level. If he had been lifted by a crane and placed on the diving board, instead of climbing the ladder, his gravitational potential energy relative to the water surface would still be 1.70×10^3 J.

Practice

Understanding Concepts

1. You lower your pen vertically by 25 cm, then you raise it vertically by 25 cm. During this motion, is the total work done by gravity positive, negative, or zero? Explain your answer.
2. What, relative to the ground, is the gravitational potential energy of a 62.5-kg visitor standing at the lookout level of Toronto's CN Tower, 346 m above the ground?
3. A 58.2-g tennis ball is dropped from rest vertically downward from a height of 1.55 m above the court surface.
   (a) Determine the gravitational potential energy of the ball relative to the court surface before it is dropped, and as it strikes the court surface.
   (b) How much work has the force of gravity done on the ball at the instant the ball strikes the court surface?
   (c) Relate the work done in (b) to the change in the kinetic energy from the release point to the court surface.
4. A 68.5-kg skier rides a 2.56-km ski lift from the base of a mountain to the top. The lift is at an angle of 13.9° to the horizontal. Determine the skier's gravitational potential energy at the top of the mountain relative to the base of the mountain.
5. A high jumper clears the pole at a height of 2.36 m, then falls safely back to the ground. The change in the jumper's gravitational potential energy from the pole to the ground is -1.65×10^3 J. Determine the jumper's mass.
6. On your desk you have *N* identical coins, each with a mass *m*. You stack the coins into a vertical pile to height *y*.
   (a) Approximately how much work, in terms of *m*, *g*, and *y*, must you do on the last coin to raise it from the desk to the top of the pile?
   (b) Approximately how much gravitational potential energy, in terms of *m*, *g*, *N*, and *y*, is stored in the entire pile?

Applying Inquiry Skills

7. Using unit analysis, express the units of gravitational potential energy in terms of base SI units. Compare the result to the SI base units of work and kinetic energy.

Making Connections

8. A barrel of oil contains about 6.1×10^9 J of chemical potential energy.
   (a) Determine to what height above the ground this energy could raise all the students in your school. State all your assumptions and show all your calculations.
   (b) How many joules of chemical potential energy are stored in each litre of oil? (*Hint:* You will have to determine how many litres there are in a barrel.)

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Answers

2. 2.12×10^5 J
3. (a) 0.884 J; 0.0 J
   (b) 0.884 J
4. 4.13×10^5 J
5. 71.3 kg

Case Study: An Environmentally Friendly Way of Generating Electricity

There are many practical applications of gravitational potential energy. For example, hydroelectric generating stations take advantage of the gravitational potential energy of water as it flows or falls from one level to a lower level. At many stations, huge dams store the water, allowing engineers to control the flow of water through pipes into turbines connected to the generators.

One of the major problems with building large dams is that the local ecology is drastically and permanently affected. Large artificial lakes created by dams flood previously dry areas, destroying plant life and animal habitat.

Bhutan, located east of Nepal and north of India (see **Figure 4**), has found a method of generating hydroelectricity without large dams. This small country, with a land area only about 85% that of Nova Scotia, is located in the Himalayan Mountain region. It has very strict environmental laws to protect its great forests, which cover more than 70% of its land. Careful environmental policy is evident in the design of the Chukha electrical generating station, which generates power at a rate of 360 MW. (By comparison, the two huge Robert Beck generating stations and the adjacent pumping-generating station at Niagara Falls generate 1800 MW.) The design of this generating station along the Wong Chu River is called a "run-of-the-river scheme," as shown in **Figure 5**. In this design, a small dam diverts some of the river's water flow into a large entrance tunnel, or *head race,* that is 6.0 km long. This tunnel, drilled through solid granite, is angled downward to the top of the generating station and takes advantage of the drop in elevation to convert the gravitational potential energy of the water into kinetic energy. After the water falls through the turbines at the generating station, it flows through an exit tunnel, or *tail race*, rejoining the river 1.0 km downstream.

Figure 4
Bhutan is a land-locked nation with many mountain glaciers and rivers.

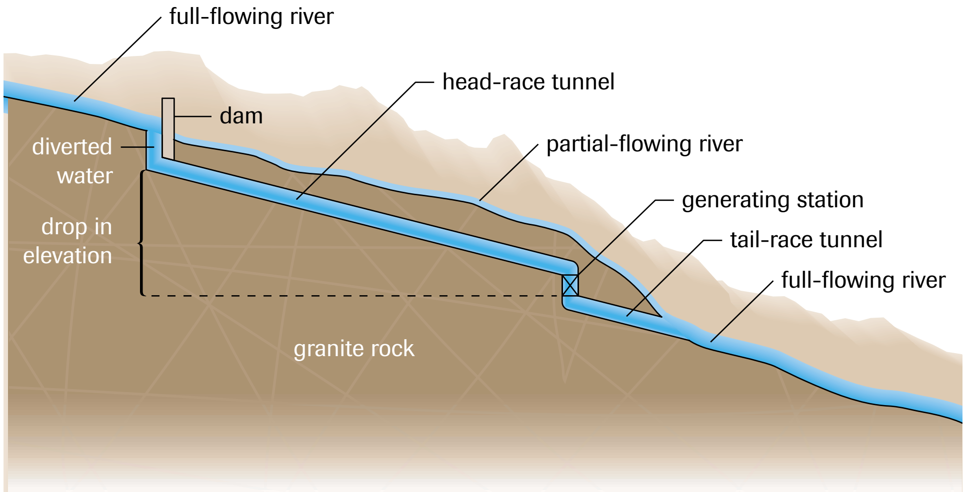

Figure 5
The basic design of the Chukha generating plant

Only about 5% of the electrical energy generated at the Chukha plant is used in Bhutan. The remainder is exported to India along 220-V transmission lines.

Practice

Understanding Concepts

9. (a) Starting with energy from the Sun, list all the energy transformations that occur in the production of electrical energy at the Chukha plant.
(b) What is a run-of-the-river generating station?

Making Connections

10. Canada also has run-of-the-river generating stations. Research this way of generating electricity in Canada. Describe any differences between the Chukha plant and Canadian designs.

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11. Research more about Bhutan's generation of electrical energy, from the Internet or other appropriate publications, and report your findings.
(a) What are the sources of water in Bhutan?
(b) Describe how Bhutan is trying to preserve its environment while adapting to growing energy needs.

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12. Set up a debate in which it is resolved "That Canada should place more emphasis on developing environmentally friendly ways of generating electrical energy."

SUMMARY Gravitational Potential Energy at Earth's Surface

- Gravitational potential energy is the energy possessed by an object due to its elevation above Earth's surface. It is a scalar quantity measured in joules (J).
- Gravitational potential energy is always stated relative to a reference level.
- The gravitational potential energy of an object depends on its mass, the gravitational field in which the object is located, and the object's height above a reference level.

Section 4.3 Questions

Understanding Concepts

1. When a construction worker lifts a piece of wood from the ground, does the wood's gravitational potential energy increase or decrease?
2. An astronaut of mass 63 kg climbs a set of stairs with a total vertical rise of 3.4 m.
   (a) What is the astronaut's gravitational potential energy, relative to the bottom of the stairs, if the stairs are located on Earth?
   (b) Repeat (a) if the stairs are located on the Moon where $g = 1.6$ N/kg.
3. A pear of mass 125 g falls from a branch 3.50 m above the ground. What are the respective gravitational potential energies of the pear on the branch and on the ground
   (a) relative to the ground
   (b) relative to the branch
4. After being hit by a bat, a 0.15-kg baseball reaches a maximum height where its gravitational potential energy has increased by 22 J from the point where it was hit. What is the ball's maximum height (above the point where it was hit)?
5. A weightlifter, doing a biceps curl, lifts a 15-kg mass a vertical distance of 66 cm. Acceleration is negligible.
   (a) How much work is done by gravity on the mass?
   (b) How much work is done by the weightlifter on the mass?
   (c) By how much does the gravitational potential energy of the mass increase?

Applying Inquiry Skills

6. Plot, on a single graph, the gravitational potential energy of a 60.0-kg astronaut on Earth, on the Moon, and on Mars, as a function of the vertical elevation above the surface to a maximum height of 10.0 m. Your graph will have three distinct lines. (If necessary, refer to Appendix C for planetary data.)

Making Connections

7. Some Canadian hydroelectric generating facilities, similar to the one in **Figure 6**, convert the gravitational potential energy of water behind a dam into electrical energy.
   (a) Determine the gravitational potential energy relative to the turbines of a lake of volume 32.8 km^3 with an average height of 23.1 m above the turbines. (The density of the water is 1.00×10^3 kg/m^3.)
   (b) Compare your answer in (a) with the annual energy output of the Chukha plant in Bhutan, which is 1.14×10^{15} J.

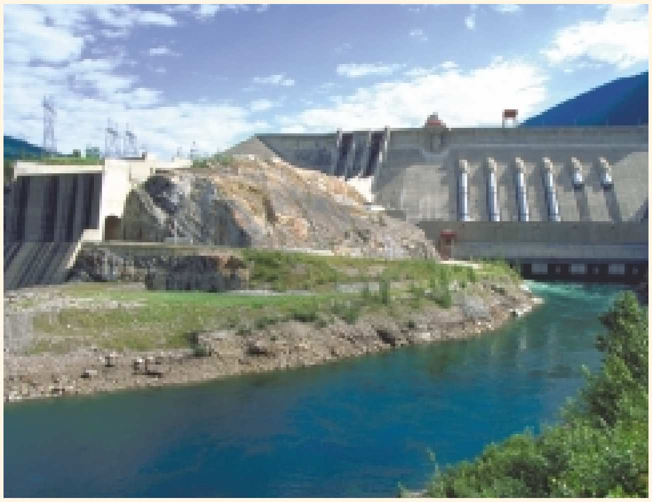

Figure 6
The Revelstoke Dam and Generating Station sits on the Columbia River in British Columbia, 5 km north of the city of Revelstoke, B.C.