4.2 Kinetic Energy and the Work-Energy Theorem

Figure 1
As the propeller of a seaplane exerts a backward force on the air, the air exerts a forward force on the propeller. This force, applied as the plane moves forward by a displacement of magnitude Δd, does work on the plane.

kinetic energy (E_K) energy of motion

In Section 4.1, we learned how to calculate the amount of work done on an object when a force acts on the object as it moves through a displacement. But how is the object different as a result of having work done on it? Consider what happens when a net force causes a seaplane to accelerate as it covers a certain displacement (**Figure 1**). According to the defining equation of work, the total work done on the plane by the net force is $W_{total} = (\Sigma F)(\cos\theta)\Delta d$. In this case, the work done causes the speed of the plane to increase, thereby causing the energy of the plane to increase.

Specifically, the plane gains **kinetic energy** as its speed increases; in simple terms, kinetic energy is energy of motion. Kinetic energy depends on both the mass and the speed of the moving object. Since work is energy transferred to an object, if the work results in an increase in speed, the kinetic energy also increases.

Figure 2 illustrates a situation we will analyze mathematically.

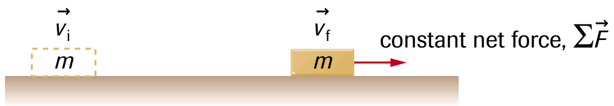

Figure 2
This situation can be used to derive an equation relating work and kinetic energy.

A constant net horizontal force is applied to an object, causing its speed to increase uniformly from v_i to v_f as it moves a distance Δd. The total work done by the net force is

$$W_{total} = \left(\sum F\right)(\cos\theta)\Delta d$$

Since $\theta = 0°$, then $\cos\theta = 1$, and we can simplify the equation:

$$W_{total} = \left(\sum F\right)\Delta d$$

From Newton's second law of motion,

$$\sum F = ma$$

where a is the acceleration. With a constant force, the acceleration is constant. We can rearrange the equation involving constant acceleration, $v_f^2 = v_i^2 + 2a\Delta d$ as $a = \dfrac{v_f^2 - v_i^2}{2\Delta d}$ and substitute for a:

$$\sum F = m\left(\frac{v_f^2 - v_i^2}{2\Delta d}\right)$$

We now substitute this equation into the equation for total work:

$$W_{total} = m\left(\frac{v_f^2 - v_i^2}{2\Delta d}\right)\Delta d$$

$$W_{total} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Our result shows that the total work done on an object equals the change in the quantity $\frac{1}{2}mv^2$. We define $\frac{1}{2}mv^2$ to be the kinetic energy E_K of the object.

Kinetic Energy Equation

$$E_K = \frac{1}{2}mv^2$$

where E_K is the kinetic energy of the object in joules, m is the mass of the object in kilograms, and v is the speed of the object in metres per second. Note that kinetic energy is a scalar quantity.

We can summarize the relationship involving the total work and the kinetic energy as follows:

$$\begin{aligned} W_{total} &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= E_{Kf} - E_{Ki} \\ W_{total} &= \Delta E_K \end{aligned}$$

These relationships are the basis of the *work-energy theorem.*

Work-Energy Theorem

The total work done on an object equals the change in the object's kinetic energy, provided there is no change in any other form of energy (for example, gravitational potential energy).

Although this theorem was derived for one-dimensional motion involving a constant net force, it is also true for motion in two or three dimensions involving varying forces.

It is important to note that a change in the object's kinetic energy equals the work done by the net force $\Sigma\vec{F}$, which is the vector sum of all the forces. If the total work is positive, then the object's kinetic energy increases. If the total work is negative, then the object's kinetic energy decreases.

LEARNING TIP

Comparing Force and Energy

There is no completely satisfactory definition of energy. However, the concept of energy can be more easily understood by comparing it with force: *force is the agent that causes change; energy is a measure of that change.* For example, when a net force causes the speed of an object to change, that change is manifested as a change in the kinetic energy of the object. Remember that the energy of an object is a measure of how much work that object can do.

SAMPLE problem 1

What total work, in megajoules, is required to cause a cargo plane of mass 4.55×10^5 kg to increase its speed in level flight from 105 m/s to 185 m/s?

Solution

$m = 4.55 \times 10^5$ kg

$v_i = 105$ m/s

$v_f = 185$ m/s

$W_{total} = ?$

$$\begin{aligned} W_{total} &= \Delta E_K \\ &= E_{Kf} - E_{Ki} \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}m\left(v_f^2 - v_i^2\right) \\ &= \frac{1}{2}\left(4.55 \times 10^5\ \text{kg}\right)\left((185\ \text{m/s})^2 - (105\ \text{m/s})^2\right) \\ &= 5.28 \times 10^9\ \text{J}\left(\frac{1\ \text{MJ}}{10^6\ \text{J}}\right) \\ W_{total} &= 5.28 \times 10^3\ \text{MJ} \end{aligned}$$

The total work required is 5.28×10^3 MJ.

SAMPLE problem 2

A fire truck of mass 1.6×10^4 kg, travelling at some initial speed, has -2.9 MJ of work done on it, causing its speed to become 11 m/s. Determine the initial speed of the fire truck.

Solution

$m = 1.6 \times 10^4$ kg

$\Delta E_K = -2.9 \text{ MJ} = -2.9 \times 10^6$ J

$v_f = 11$ m/s

$v_i = ?$

$$\Delta E_K = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$2\Delta E_K = mv_f^2 - mv_i^2$$

$$mv_i^2 = mv_f^2 - 2\Delta E_K$$

$$v_i^2 = \frac{mv_f^2 - 2\Delta E_K}{m}$$

$$v_i = \pm\sqrt{\frac{mv_f^2 - 2\Delta E_K}{m}}$$

$$= \pm\sqrt{\frac{(1.6 \times 10^4 \text{ kg})(11 \text{ m/s})^2 - 2(-2.9 \times 10^6 \text{ J})}{1.6 \times 10^4 \text{ kg}}}$$

$$v_i = \pm 22 \text{ m/s}$$

We choose the positive root because speed is always positive. The initial speed is thus 22 m/s.

Practice

Understanding Concepts

1. Could a slow-moving truck have more kinetic energy than a fast-moving car? Explain your answer.

2. By what factor does a cyclist's kinetic energy increase if the cyclist's speed:
(a) doubles
(b) triples
(c) increases by 37%

3. Calculate your kinetic energy when you are running at your maximum speed.

4. A 45-g golf ball leaves the tee with a speed of 43 m/s after a golf club strikes it.
(a) Determine the work done by the club on the ball.
(b) Determine the magnitude of the average force applied by the club to the ball, assuming that the force is parallel to the motion of the ball and acts over a distance of 2.0 cm.

5. A 27-g arrow is shot horizontally. The bowstring exerts an average force of 75 N on the arrow over a distance of 78 cm. Determine, using the work-energy theorem, the maximum speed reached by the arrow as it leaves the bow.

6. A deep-space probe of mass $4.55. \times 10^4$ kg is travelling at an initial speed of 1.22×10^4 m/s. The engines of the probe exert a force of magnitude 3.85×10^5 N over 2.45×10^6 m. Determine the probe's final speed. Assume that the decrease in mass of the probe (because of fuel being burned) is negligible.

Answers

2. (a) 4
   (b) 9
   (c) 1.9
4. (a) 42 J
   (b) 2.1×10^3 N
5. 66 m/s
6. 1.38×10^4 m/s

7. A delivery person pulls a 20.8-kg box across the floor. The force exerted on the box is 95.6 N [35.0° above the horizontal]. The force of kinetic friction on the box has a magnitude of 75.5 N. The box starts from rest. Using the work-energy theorem, determine the speed of the box after being dragged 0.750 m.
8. A toboggan is initially moving at a constant velocity along a snowy horizontal surface where friction is negligible. When a pulling force is applied parallel to the ground over a certain distance, the kinetic energy increases by 47%. By what percentage would the kinetic energy have changed if the pulling force had been at an angle of 38° above the horizontal?

Applying Inquiry Skills

9. Use either unit analysis or dimensional analysis to show that kinetic energy and work are measured in the same units.

Making Connections

10. Many satellites move in elliptical orbits (**Figure 3**). Scientists must understand the energy changes that a satellite experiences in moving from the farthest position A to the nearest position B. A satellite of mass 6.85×10^3 kg has a speed of 2.81×10^3 m/s at position A, and speed of 8.38×10^3 m/s at position B. Determine the work done by Earth's gravity as the satellite moves
    (a) from A to B
    (b) from B to A

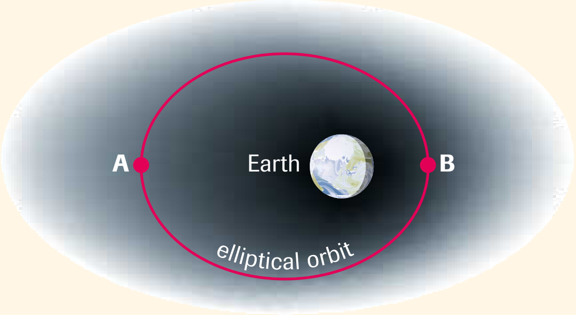

Figure 3

Answers

7. 0.450 m/s
8. 16%
10. (a) 2.13×10^{11} J
    (b) -2.13×10^{11} J

SUMMARY Kinetic Energy and the Work-Energy Theorem

- Kinetic energy E_K is energy of motion. It is a scalar quantity, measured in joules (J).
- The work-energy theorem states that the total work done on an object equals the change in the object's kinetic energy, provided there is no change in any other form of energy.

Section 4.2 Questions

Understanding Concepts

1. As a net external force acts on a certain object, the speed of the object doubles, and then doubles again. How does the work done by the net force in the first doubling compare with the work done in the second doubling? Justify your answer mathematically.
2. What is the kinetic energy of a car of mass 1.50×10^3 kg moving with a velocity of 18.0 m/s [E]?
3. (a) If the velocity of the car in question 2 increases by 15.0%, what is the new kinetic energy of the car?
   (b) By what percentage has the kinetic energy of the car increased?
   (c) How much work was done on the car to increase its kinetic energy?
4. A 55-kg sprinter has a kinetic energy of 3.3×10^3 J. What is the sprinter's speed?
5. A basketball moving with a speed of 12 m/s has a kinetic energy of 43 J. What is the mass of the ball?
6. A plate of mass 0.353 kg falls from rest from a table to the floor 89.3 cm below.
   (a) What is the work done by gravity on the plate during the fall?
   (b) Use the work-energy theorem to determine the speed of the plate just before it hits the floor.
7. A 61-kg skier, coasting down a hill that is at an angle of 23° to the horizontal, experiences a force of kinetic friction of magnitude 72 N. The skier's speed is 3.5 m/s near the top of the slope. Determine the speed after the skier has travelled 62 m downhill. Air resistance is negligible.
8. An ice skater of mass 55.2 kg falls and slides horizontally along the ice travelling 4.18 m before stopping. The coefficient of kinetic friction between the skater and the ice is 0.27. Determine, using the work-energy theorem, the skater's speed at the instant the slide begins.

Applying Inquiry Skills

9. The heaviest trucks in the world travel along relatively flat roads in Australia. A fully loaded "road-train" (**Figure 4**) has a mass of 5.0×10^2 t, while a typical car has a mass of about 1.2 t.

Figure 4
Australian "road-trains" are massive transport trucks comprised of three or more trailers hooked up together. The long, flat desert roads of central Australia make this an ideal form for transporting goods.

   (a) Prepare a table to compare the kinetic energies of these two vehicles travelling at ever-increasing speeds, up to a maximum of 40.0 m/s.
   (b) Prepare a single graph of kinetic energy as a function of speed, plotting the data for both vehicles on the same set of axes.
   (c) Based on your calculations and the graph, write conclusions about the masses, speeds, and kinetic energies of moving vehicles.

Making Connections

10. You may have heard the expression "Speed kills" in discussions of traffic accidents. From a physics perspective, a better expression might be "Kinetic energy kills."
   (a) When a traffic accident involving at least one moving vehicle occurs, the initial kinetic energy of the vehicle(s) must transform into something. Where do you think the energy goes?
   (b) Explain the expression "Kinetic energy kills."