4.1 Work Done by a Constant Force

work (W) the energy transferred to an object when a force acting on the object moves it through a distance

In everyday language, the term "work" has a variety of meanings. In physics, however, **work** is the energy transferred to an object when a force acting on the object moves it through a distance. For example, to raise your backpack from the floor to your desk, you must do work. The work you do in raising the backpack is directly proportional to the magnitude of the displacement and directly proportional to the magnitude of the applied force.

The force needed to raise the backpack and the displacement of the backpack are in the same direction. However, this is often not the case, as shown in **Figure 1** where the force is at some angle θ to the displacement. The component of the force that is parallel to the displacement, $F\cos\theta$, causes the object to undergo the displacement. The resulting mathematical relationship is the defining equation for work:

$$W = (F\cos\theta)\Delta d$$

where W is the work done on an object by a constant force $\vec{F}$, F is the magnitude of that force, θ is the angle between the force and the displacement, and Δd is the magnitude of the displacement.

LEARNING TIP

Scalar Product
The defining equation for the work done by a constant force can be written as the *scalar product*, also called the *dot product*, of the force vector and the displacement vector. In this notation, the equation for work is $W = \vec{F} \cdot \Delta\vec{d}$. To review the properties of scalar products, refer to Appendix A.

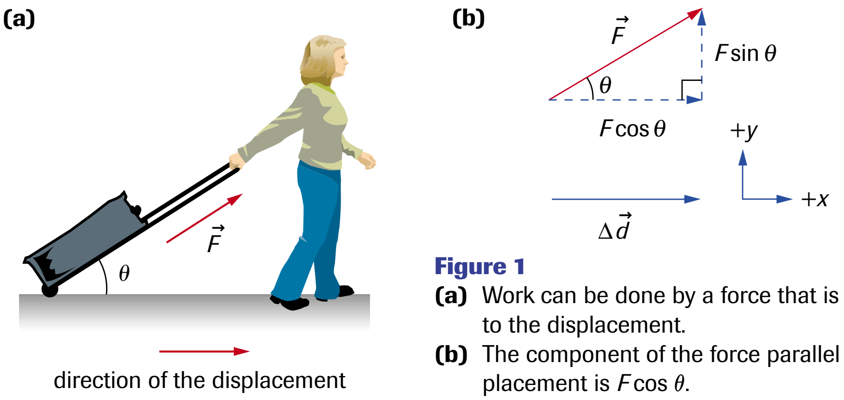


Figure 1
(a) Work can be done by a force that is at an angle to the displacement.
(b) The component of the force parallel to the displacement is $F\cos\theta$.

Work is a scalar quantity—it has no direction. Since force is measured in newtons and displacement in metres, the SI unit for work is the newton metre (N·m). The newton metre is called a **joule** (J) after James Prescott Joule (1818–1889), a British physicist who did pioneering research into the relationship between work and heat. Since the joule is a derived SI unit, it can be expressed in terms of metres, kilograms, and seconds: $1\ J = 1\ kg \cdot m^2/s^2$.

joule (J) SI derived unit for measuring forms of energy and work; equal to the work done when a force of 1 N displaces an object 1 m in the direction of the force

SAMPLE problem 1

An emergency worker applies a force to push a patient horizontally for 2.44 m on a gurney with nearly frictionless wheels.

(a) Determine the work done in pushing the gurney if the force applied is horizontal and of magnitude of 15.5 N.

(b) Determine the work done if the force, of magnitude 15.5 N, is applied at an angle of 25.3° below the horizontal.

(c) Describe the difference in the observed motion between (a) and (b).

Solution

(a) In this case, the force and the displacement are in the same direction.

$F = 15.5\ \text{N}$

$\theta = 0°$

$\Delta d = 2.44\ \text{m}$

$W = ?$

$$\begin{aligned} W &= (F\cos\theta)\Delta d \\ &= (15.5\ \text{N})(\cos 0°)(2.44\ \text{m}) \\ &= 37.8\ \text{N}\cdot\text{m} \\ W &= 37.8\ \text{J} \end{aligned}$$

The work done is 37.8 J.

(b) $\theta = 25.3°$

$W = ?$

$$\begin{aligned} W &= (F\cos\theta)\Delta d \\ &= (15.5\ \text{N})(\cos 25.3°)(2.44\ \text{m}) \\ W &= 34.2\ \text{J} \end{aligned}$$

The work done is 34.2 J.

(c) Since friction is negligible, the applied force causes an acceleration in the direction of the horizontal component. The greater amount of work accomplished in (a) must result in a greater speed after the gurney has moved 2.44 m. (This example relates to the concept of work changing into kinetic energy, which is presented in Section 4.2.)

In Sample Problem 1, the positive work done caused the speed of the gurney to increase. However, if the force acting on an object in motion is opposite in direction to the displacement, we say that negative work is done. If the only force acting on an object does negative work, the result is a decrease in the speed of the object.

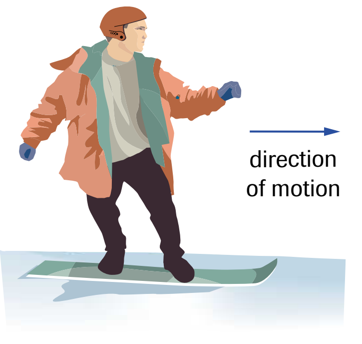


Figure 2
The system diagram of the snowboarder in Sample Problem 2

SAMPLE problem 2

A snowboarder reaches the bottom of a hill, then glides to a stop in 16.4 m along a horizontal surface (**Figure 2**). The total mass of the board and its rider is 64.2 kg. The coefficient of kinetic friction between the snowboard and the snow is 0.106.

(a) Draw an FBD of the snowboarder as the board is gliding to a stop. Determine the magnitude of the kinetic friction.

(b) Calculate the work done by friction in bringing the board to a stop.

Solution

(a) The FBD of the snowboarder is shown in **Figure 3**.

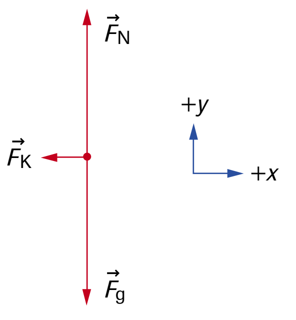


Figure 3
The FBD of the snowboarder in Sample Problem 2

$m = 64.2\ \text{kg}$

$g = |\vec{g}| = 9.80\ \text{N/kg}$

$\mu_K = 0.106$

$|\vec{F}_N| = |m\vec{g}|$ because $\sum \vec{F}_y = ma_y = 0$

$F_K = ?$

Using the magnitudes of the forces (as described in Section 2.4):

$$F_K = \mu_K F_N$$
$$= (\mu_K)(mg)$$
$$= (0.106)(64.2 \text{ kg})(9.80 \text{ N/kg})$$
$$F_K = 66.7 \text{ N}$$

The magnitude of kinetic friction is 66.7 N.

(b) $F_K = 66.7 \text{ N}$

$\Delta d = 16.4 \text{ m}$

$\theta = 180°$

$$W = (F_K \cos\theta)\Delta d$$
$$= (66.7 \text{ N})(\cos 180°)(16.4 \text{ m})$$
$$W = -1.09 \times 10^3 \text{ J}$$

The work done by the kinetic friction is -1.09×10^3 J.

In Sample Problem 2, the work done by the kinetic friction is negative. In this case, the negative work caused a decrease in speed. Can negative work be done in other situations? Consider the work done in lowering an object at a constant speed.

SAMPLE problem 3

A store employee raises an 8.72-kg case of cola at a constant velocity from the floor to a shelf 1.72 m above the floor. Later, a customer lowers the case 1.05 m from the shelf to a cart at a constant velocity. (We can neglect the short periods of acceleration at the beginning and end of the raising and lowering of the case.) Determine the work done on the case

(a) by the employee as the case rises

(b) by gravity as the case rises

(c) by the customer as the case descends

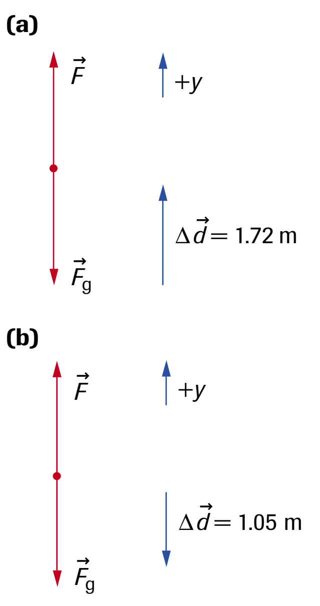


Figure 4
System diagrams and FBDs for Sample Problem 3
(a) Employee raising a case of cola
(b) Customer lowering a case of cola into a cart

Solution

(a) **Figure 4(a)** shows that the force needed to raise the case at constant velocity is equal in magnitude to the weight of the case, $|m\vec{g}|$.

$\theta = 0°$ $\quad g = 9.80$ N/kg $\quad W = ?$

$m = 8.72$ kg $\quad \Delta d = 1.72$ m

$$W = (F\cos\theta)\Delta d$$
$$= (mg\cos\theta)\Delta d$$
$$= (8.72 \text{ kg})(9.80 \text{ N/kg})(\cos 0°)(1.72 \text{ m})$$
$$W = 1.47 \times 10^2 \text{ J}$$

The work done on the case by the employee is 1.47×10^2 J.

(b) Since the force of gravity is opposite in direction to the displacement, $\theta = 180°$.

$$W = (F\cos\theta)\Delta d$$
$$= (mg\cos\theta)\Delta d$$
$$= (8.72 \text{ kg})(9.80 \text{ N/kg})(\cos 180°)(1.72 \text{ m})$$
$$W = -1.47 \times 10^2 \text{ J}$$

The work done by gravity is -1.47×10^2 J.

(c) **Figure 4(b)** shows that the force needed to lower the case at constant velocity is upward while the displacement is downward.

$\theta = 180°$

$\Delta d = 1.05$ m

$$W = (F \cos \theta)\Delta d$$
$$= (mg \cos \theta)\Delta d$$
$$= (8.72 \text{ kg})(9.80 \text{ N/kg})(\cos 180°)(1.05 \text{ m})$$
$$W = -8.97 \times 10^1 \text{ J}$$

The work done on the case by the customer is -8.97×10^1 J.

Notice that in Sample Problem 3, the work done on the case by an upward force as the case rises (at constant velocity) is positive, but the work done by a downward force (gravity) as the case rises is negative. Likewise, the work done on the case by an upward force as the case moves downward (at constant velocity) is negative. We can conclude that the equation $W = (F \cos \theta)\Delta d$ yields positive work when the force and displacement are in the same direction, and negative work when the force and displacement are in opposite directions. This conclusion also applies when considering the components of the forces involved.

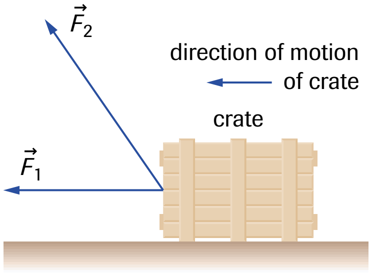


Figure 5
For question 1

Practice

Understanding Concepts

1. **Figure 5** shows a scale diagram of two applied forces, $\vec{F}_1$ and $\vec{F}_2$, acting on a crate and causing it to move horizontally. Which force does more work on the crate? Explain your reasoning.
2. Can the work done by the force of kinetic friction on an object ever be positive? If "yes," give an example. If "no," explain why not.
3. Can the work done by the force of Earth's gravity on an object ever be positive? If "yes," give an example. If "no," explain why not.
4. A 2.75-kg potted plant rests on the floor. Determine the work required to move the plant at a constant speed
   (a) to a shelf 1.37 m above the floor
   (b) along the shelf for 1.07 m where the coefficient of kinetic friction is 0.549
5. A loaded grocery cart of mass 24.5 kg is pushed along an aisle by an applied force of 14.2 N [22.5° below the horizontal]. How much work is done by the applied force if the aisle is 14.8 m long?
6. A tension force of 12.5 N [19.5° above the horizontal] does 225 J of work in pulling a toboggan along a smooth, horizontal surface. How far does the toboggan move?
7. **Figure 6** is a graph of the net horizontal forces acting on an object as a function of displacement along a horizontal surface.
   (a) Determine the area under the line up to a displacement of 2.0 m [E]. What does that area represent?
   (b) Determine the total area up to 6.0 m [E].
   (c) Describe a physical situation that could result in this graph.

Applying Inquiry Skills

8. How would you demonstrate, using a pen and a sheet of paper, that static friction can do positive work on a pen?

Answers

4. (a) 36.9 J
   (b) 15.8 J
5. 194 J
6. 19.1 m
7. (a) 8.0 J
   (b) 0.0 J

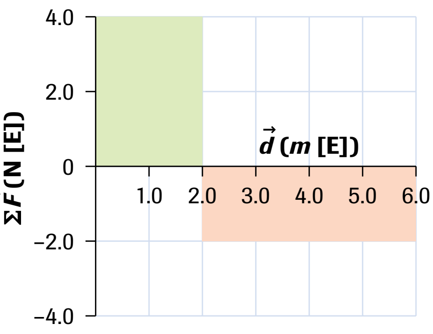


Figure 6
For question 7

Zero Work

If you push against a tree, are you doing work? Since the force you apply does not move the tree, you do zero work on the tree. Whenever the magnitude of displacement Δd is zero, the work, $W = (F \cos \theta)\Delta d$, is also zero. No energy is transferred to the object.

Zero work due to zero displacement is just one way in which no work is done on an object. Consider a space probe that has travelled beyond the farthest planet in the solar system and is so far from any noticeable forces that the net force on it is negligible. Although the probe undergoes a displacement, since the force acting on it is zero the work done on the probe is zero. In this case, too, no energy is transferred to the object.

Finally, consider the third variable in the work equation, the angle θ between the force and the displacement. When the force and the displacement are perpendicular, cos 90° is zero. Thus, no work is done by the force on the object, even if the object moves, and no energy is transferred to the object. This can happen, for example, when you carry a heavy bag of sand (**Figure 7**): an upward force is exerted on the bag of sand to keep it from falling as you move horizontally. The normal force is perpendicular to the displacement, so the work done by that normal force on the bag of sand is zero. Can you think of other situations in which the force is perpendicular to the motion?

Figure 7
As a volunteer carries a heavy sandbag toward the bank of a flooding river, the normal force on the sandbag is upward and the displacement is horizontal.

SAMPLE problem 4

In performing a centripetal-acceleration investigation, you twirl a rubber stopper in a horizontal circle around your head. How much work is done on the stopper by the tension in the string in half a revolution?

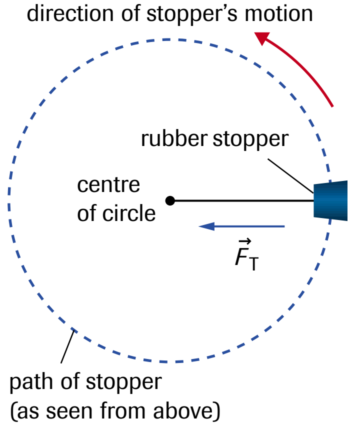
Solution

Figure 8(a) shows the situation described. The force causing the centripetal acceleration is the tension $\vec{F}_T$. This force changes direction continually as the stopper travels in a circle. However, at any particular instant as shown in **Figure 8(b)**, the instantaneous velocity is perpendicular to the tension. Consequently, the displacement over a very short time interval is also at an angle of 90° to the tension force applied by the string to the stopper. Thus, for any short time interval during the rotation,

$$W = (F \cos \theta)\Delta d$$
$$= F(\cos 90°)\Delta d$$
$$W = 0.0 \text{ J}$$

Adding the work done over all the short time intervals in half a revolution yields 0.0 J. Therefore, the work done by the tension on the stopper is zero.

(a)



(b)

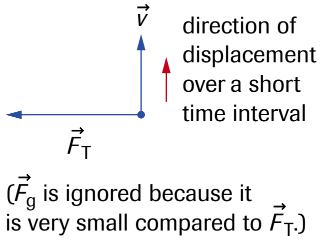


Figure 8
(a) The situation for Sample Problem 4
(b) An instantaneous view of the force on the stopper, the velocity, and the direction of a small displacement

Using Sample Problem 4 as an example, we conclude that *the work done by any centripetal force acting on an object in circular motion is zero.* This applies because the orbit can be considered to be a series of individual small displacements perpendicular to the force, even though the force on the object in circular motion continually changes its direction.

Practice

Understanding Concepts

9. Describe four different situations in a physics classroom or laboratory in which the equation $W = (F \cos \theta)\Delta d$ yields a zero result.
10. Describe, with the use of diagrams, a situation in which one component of a force does zero work while the other component does (a) positive work and (b) negative work.

SUMMARY Work Done by a Constant Force

- Work is the energy transferred to an object when a force $\vec{F}$, acting on the object, moves it through a distance Δd.
- The SI unit of work is the joule (J).
- If the force causing an object to undergo a displacement is at an angle to the displacement, only the component of the force in the direction of the displacement does work on the object.
- Under certain conditions, zero work is done on an object even if the object experiences an applied force or is in motion.

Section 4.1 Questions

Understanding Concepts

1. In what ways is the everyday usage of the word "work" different from the physics usage? In what ways is it the same?

2. Is it possible for a centripetal force to do work on an object? Explain your answer.

3. Describe why you can get tired pushing on a wall even though you are not doing work on the wall.

4. Estimate the work you would do in climbing a vertical ladder equal in length to the height of your classroom.

5. **Figure 9** shows a girl and a boy each exerting a force to move a crate horizontally a distance of 13 m at a constant speed. The girl's applied force is 75 N [22° below the horizontal]. The tension in the rope pulled by the boy is 75 N [32° above the horizontal].
   (a) Determine the total work done by the girl and the boy on the crate.
   (b) How much work is done by the floor on the crate?

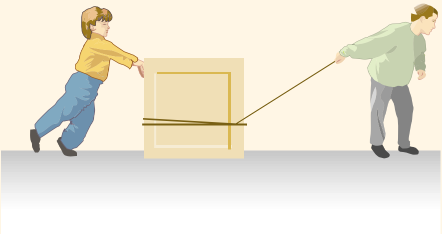


Figure 9

6. A child does 9.65×10^2 J of work in pulling a friend on a sleigh 45.3 m along a snowy horizontal surface (**Figure 10**). The force applied by the child is 24.1 N [parallel to the handle of the sleigh]. What angle relative to the snowy surface is the handle oriented?

Figure 10

7. A parent pulls a toboggan with three children at a constant velocity for 38 m along a horizontal snow-covered trail. The total mass of the children and the toboggan is 66 kg. The force the parent exerts is 58 N [18° above the horizontal].
   (a) Draw an FBD of the toboggan, and determine the magnitude of the normal force and the coefficient of kinetic friction.
   (b) Determine the work done on the toboggan by kinetic friction.
   (c) List three forces, or components of forces, that do zero work on the toboggan.
   (d) How much work does the parent do on the toboggan in pulling it the first 25 m?

Applying Inquiry Skills

8. Plot the cosine of an angle as a function of the angle (from 0° to 180°). Describe how the graph indicates when the work done on an object is positive, negative, or zero.

Making Connections

9. Describe the environmental effects that result from work done by friction.