2.5 Inertial and Noninertial Frames of Reference

Imagine that you are travelling on a bus at a constant speed along a straight, smooth road. If you place a ball on the floor of the bus, it stays at rest relative to you and the bus, just as it would if you placed it on the classroom floor (**Figure 1**). Initially, the ball is stationary and it remains that way because there is no net force acting on it. However, if the bus driver suddenly applies the brakes, the ball appears to accelerate forward relative to the bus, even though there is still no net force acting on it. (There is no force actually pushing the ball forward.)

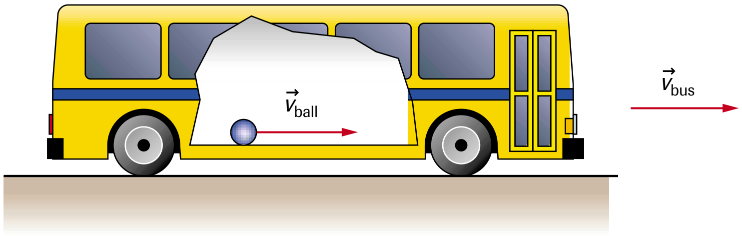

Figure 1
The bus and the ball move at a constant velocity. Relative to the bus, the ball is at rest.

Relative to your classroom, or relative to a bus travelling at a constant velocity, the ball stays at rest if there is no net force acting on it; in other words, the ball obeys Newton's first law of motion, the law of inertia. Therefore, we call your classroom or the bus moving at a constant velocity an inertial frame of reference. A frame of reference (defined in Chapter 1) is an object, such as a room, a bus, or even an atom, relative to which the positions, velocities, accelerations, etc., of other objects can be measured. An **inertial frame of reference** is a frame in which the law of inertia and other physics laws are valid. Any frame moving at a constant velocity relative to the first frame is also an inertial frame.

inertial frame of reference a frame in which the law of inertia is valid

noninertial frame of reference a frame in which the law of inertia is not valid

When the brakes are applied to the bus, the bus undergoes acceleration. Thus, it is a **noninertial frame of reference**, one in which the law of inertia does not hold. Although the ball accelerates toward the front of the bus when the brakes are applied, there is no net force causing that acceleration. The reason there appears to be a net force on the ball is that we are observing the motion from the accelerated frame of reference inside the bus (a noninertial frame). The situation is much easier to explain if we consider it from an inertial frame, such as the road. Relative to the road, when the brakes are applied to the bus, the ball tends to continue to move forward at a constant velocity, as explained by the law of inertia. Since the bus is slowing down and the ball is not, the ball accelerates toward the front, relative to the bus (**Figure 2**).

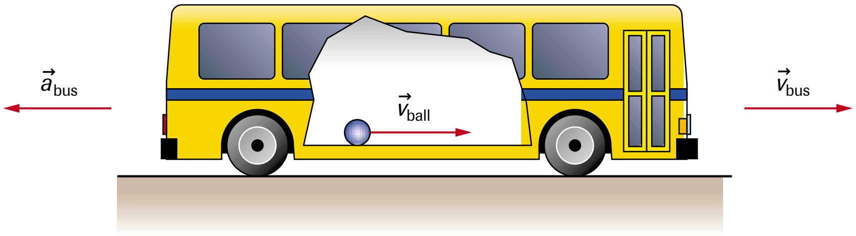

Figure 2
When the brakes are applied, the bus slows down, but the ball tends to continue moving forward at a constant velocity relative to the ground. Thus, relative to the bus, the ball accelerates forward.

To explain the ball's observed motion in the bus, we have to invent a force toward the front of the bus. This **fictitious force** is an invented force that we can use to explain observed motion in an accelerating frame of reference. In the case of the ball, the fictitious force is in the opposite direction to the acceleration of the noninertial frame itself.

fictitious force an invented force used to explain motion in an accelerating frame of reference

SAMPLE problem 1

Draw an FBD for the ball shown in (a) **Figure 1** and (b) **Figure 2**. Indicate the fictitious force in (b) relative to the frame of reference of the bus.

Solution

Figure 3 shows the required diagrams. We use the symbol $\vec{F}_{\text{fict}}$ to represent the fictitious force.

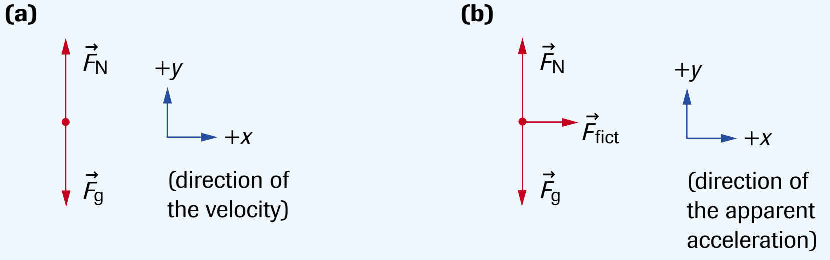

Figure 3
(a) The FBD in a fixed frame of reference. The $+x$ direction is chosen to be the direction of the velocity.
(b) The FBD in the accelerating frame of reference. The $+x$ direction is chosen to be the direction of the apparent acceleration due to the fictitious force $\vec{F}_{\text{fict}}$ relative to the frame of reference of the bus.

SAMPLE problem 2

A teacher suspends a small rubber stopper from the roof of a bus, as in **Figure 3(a)** from the chapter opener. The suspending cord makes an angle of 8.5° from the vertical as the bus is accelerating forward. Determine the magnitude of the acceleration of the bus.

Solution

To solve this problem, we will look at the situation from Earth's frame of reference because it is an inertial frame. We begin by drawing the system diagram and the FBD in that frame, as shown in **Figure 4**.

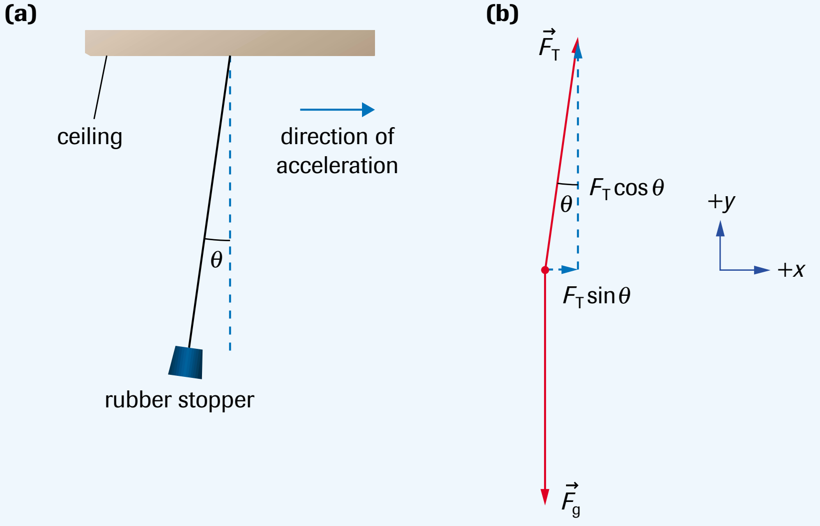

Figure 4
(a) System diagram of improvised rubber-stopper accelerometer
(b) FBD of accelerometer bob

LEARNING TIP

Fictitious Forces
Fictitious forces are sometimes called pseudoforces or inertial forces. Fictitious forces are not needed in an inertial frame of reference.

It is the horizontal component of the tension that causes the acceleration. Since both it and the horizontal acceleration are unknowns, we must use two equations. We start with the vertical components:

$$\sum F_y = ma_y = 0$$
$$F_T \cos\theta - F_g = 0$$
$$F_T \cos\theta = F_g \text{ where } F_g = mg$$
$$F_T = \frac{mg}{\cos\theta}$$

This expression for F_T can now be substituted into the equation for the horizontal components:

$$\sum F_x = ma_x$$
$$F_T \sin\theta = ma_x$$
$$a_x = (F_T)\left(\frac{\sin\theta}{m}\right)$$
$$= \left(\frac{mg}{\cos\theta}\right)\left(\frac{\sin\theta}{m}\right)$$
$$= g\left(\frac{\sin\theta}{\cos\theta}\right)$$
$$= g\tan\theta$$
$$= (9.8 \text{ m/s}^2)(\tan 8.5°)$$
$$a_x = 1.5 \text{ m/s}^2$$

The magnitude of the acceleration is 1.5 m/s^2.

LEARNING TIP

The Validity of Physics Laws
All the laws of physics are valid in any inertial frame of reference, whether it is your classroom, a bus moving at a constant velocity, or Canada. There is no single frame that is better than others. However, the velocity of an object in one frame may be different than the velocity of the same object in another frame. For example, if you are riding a bicycle and holding a ball in your hand, the velocity of the ball in your frame of reference is zero, but it is not zero relative to the road. It is evident that without a frame of reference, it would be impossible to measure such quantities as velocity and position.

We have so far considered inertial and noninertial frames of reference for linear motion. Comparing these frames of reference when the acceleration involves changing direction, such as when you are in a car following a curve on a highway, is presented in Chapter 3.

Practice

Understanding Concepts

1. You push an air-hockey puck along a surface with negligible friction while riding in a truck as it moves at a constant velocity in Earth's frame of reference. What do you observe? Why?
2. You are in a school bus initially travelling at a constant velocity of 12 m/s [E]. You gently place a tennis ball in the aisle beside your seat.
   (a) What happens to the ball's motion? Why?
   (b) Draw an FBD of the ball in the frame of reference of the road, and an FBD of the ball in the frame of reference of the bus.
   (c) The bus driver presses down on the accelerator pedal, causing the bus to accelerate forward with a constant acceleration. Describe the ball's motion.
   (d) Draw an FBD and explain the ball's motion in (c) from the frame of reference of the road, and from your frame of reference in the bus. Indicate which frame is noninertial, labelling any fictitious forces.
3. A rubber stopper of mass 25 g is suspended by string from the handrail of a subway car travelling directly westward. As the subway train nears a station, it begins to slow down, causing the stopper and string to hang at an angle of 13° from the vertical.
   (a) What is the acceleration of the train? Is it necessary to know the mass of the stopper? Why or why not?
   (b) Determine the magnitude of the tension in the string. Is it necessary to know the mass of the stopper? Why or why not?

Answers

3. (a) 2.3 m/s^2 [E]
   (b) 0.25 N

SUMMARY Inertial and Noninertial Frames of Reference

- An inertial frame of reference is one in which the law of inertia (Newton's first law of motion) holds.
- An accelerating frame of reference is a noninertial frame where the law of inertia does not hold.
- In a noninertial frame of reference, fictitious forces are often invented to account for observations.

Section 2.5 Questions

Understanding Concepts

1. What phrases can you think of that mean the same as "noninertial frame of reference"?
2. You are a passenger in a vehicle heading north. You are holding a horizontal accelerometer like the one shown in **Figure 5**.
   (a) How will you hold the accelerometer so that it can indicate the acceleration?
   (b) Describe what happens to the beads in the accelerometer when the vehicle
      (i) is at rest
      (ii) is accelerating northward
      (iii) is moving with a constant velocity
      (iv) begins to slow down while moving northward
   (c) Draw an FBD of the beads for an instant at which the vehicle is travelling with a constant acceleration northward, from the frame of reference of the road.
   (d) Repeat (c) from your frame of reference in the vehicle.
   (e) If the beads are at an angle of 11° from the vertical, what is the magnitude of the acceleration of the vehicle?
   (f) Determine the magnitude of the normal force acting on the middle bead, which has a mass of 2.2 g.

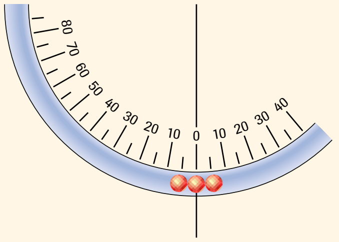

Figure 5
A typical horizontal accelerometer

Applying Inquiry Skills

3. An ornament hangs from the interior rear-view mirror of your car. You plan to use this ornament as a pendulum-style accelerometer to determine the acceleration of the car as it is speeding up in a straight line.
   (a) Draw a system diagram, an FBD of the ornament from the frame of reference of the road, and an FBD of the ornament from the frame of reference of the car.
   (b) Describe how you would determine the acceleration, indicating what measurement(s) you would take and what calculations you would perform. Explain your calculations by referring to one of the two FBDs from part (a).

Making Connections

4. You are a passenger in a car stopped at an intersection. Although the stoplight is red and the driver's foot is still firmly on the brake, you suddenly feel as if the car is moving backward.
   (a) Explain your feeling. (*Hint:* Think about the motion of the car next to you.)
   (b) How could the sensation you feel be applied to the design of an amusement ride in which the riders remain stationary, but have the sensations of motion?