2.1 Forces and Free-Body Diagrams

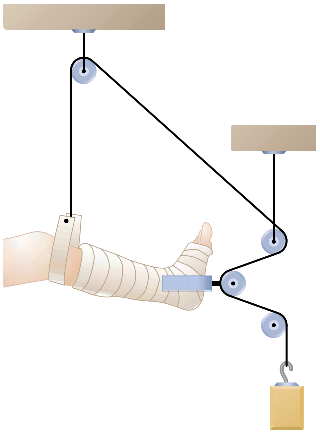

Figure 1
Designing a traction system that prevents leg motion requires an understanding of forces and the components of forces. In this case, the tibia (small lower leg bone) is stabilized by the traction cord attached to a weight and strung through a pulley system.

A **force** is a push or a pull. Forces act on objects, and can result in the acceleration, compression, stretching, or twisting of objects. Forces can also act to stabilize an object. For example, when a person fractures a leg, any motion of the leg can hinder the healing of the bone. To reduce this problem, the leg can be placed in a traction system, like that shown in **Figure 1**. A traction system stabilizes the broken limb and prevents unnecessary motion of its broken parts.

To analyze the forces that are acting on the leg shown in **Figure 1**, several background concepts must be understood. In this section, we will use the following questions to introduce the concepts:

- What are the main types of forces that you experience in everyday situations involving objects at rest or in motion, and how are these forces measured?
- How can you use diagrams to mathematically analyze the forces acting on a body or an object?
- What is the net force, or the resultant force, and how can it be calculated?

Common Forces

When you hold a textbook in your hand, you feel Earth's force of gravity pulling downward on the book. The **force of gravity** is the force of attraction between all objects. It is an action-at-a-distance force, which means that contact between the objects is not required. Gravity exists because matter exists. However, the force of gravity is extremely small unless at least one of the objects is very large. For example, the force of gravity between a 1.0-kg ball and Earth at Earth's surface is 9.8 N, but the force of gravity between two 1.0-kg balls separated by a distance of 1.0 m is only 6.7×10^{-11} N, a negligible amount.

The force of gravity on an object, like a book in your hand, acts downward, toward Earth's centre. However, to keep the book stationary in your hand, there must be an upward force acting on it. That force, called the **normal force**, is the force perpendicular to the two surfaces in contact. As you can see in **Figure 2**, the normal force acts vertically upward when the contact surfaces are horizontal.

Another common force, **tension**, is the force exerted by materials that can be stretched (e.g., ropes, strings, springs, fibres, cables, and rubber bands). The more the material is stretched, like the spring scale shown in **Figure 3**, the greater the tension in the material.

force ($\vec{F}$) a push or a pull

force of gravity ($\vec{F}_g$) force of attraction between all objects

normal force ($\vec{F}_N$) force perpendicular to the surfaces of objects in contact

tension ($\vec{F}_T$) force exerted by materials, such as ropes, fibres, springs, and cables, that can be stretched

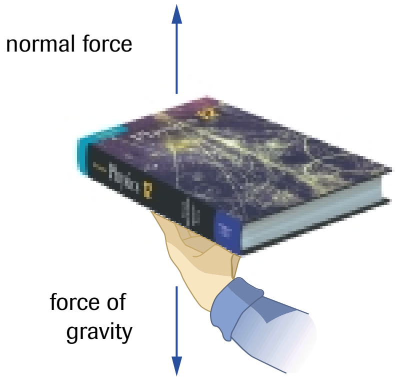

Figure 2
For a book held stationary in your hand, Earth's force of gravity acts downward. Your hand exerts a normal force upward on the book.

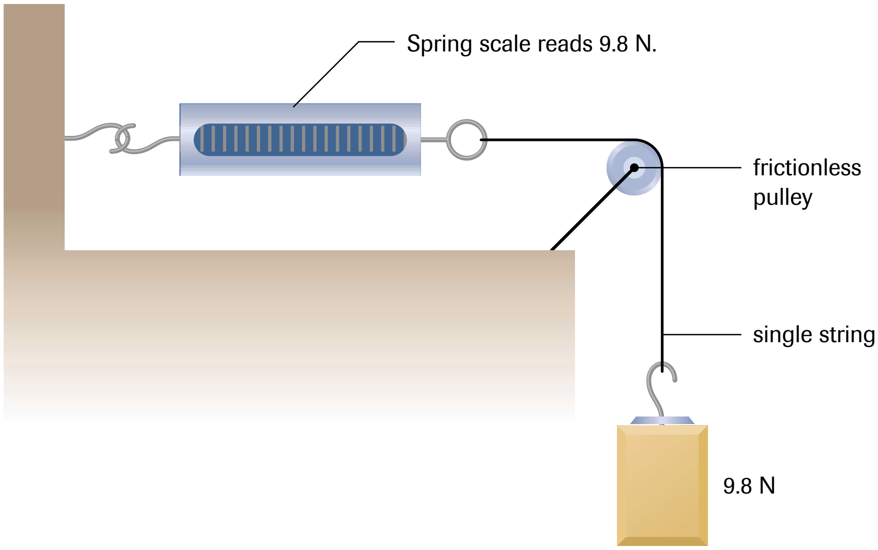

Figure 4
Since tension is constant along a single string, the tension in the horizontal part of this string has the same magnitude as the tension in the vertical part of the string.

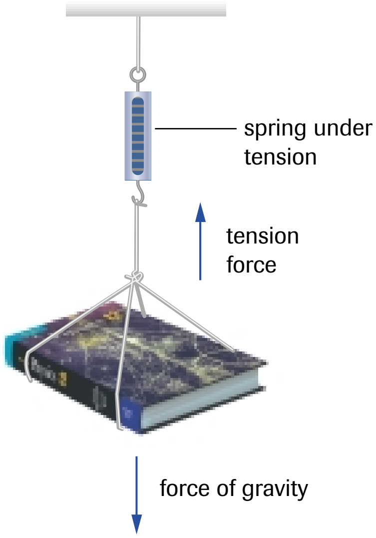

Figure 3
When an extension spring stretches, the tension increases to bring the spring back to its original state. The greater the downward force of gravity on the book, the greater the upward tension in the spring.

An important characteristic of the tension force in a material is that it has the same magnitude everywhere along the length of the material. This is true even if the direction of the force changes, as when a rope or string hangs over a pulley (**Figure 4**).

Another common force is **friction**—the force that resists motion or attempted motion between objects in contact. Friction always acts in the direction opposite to the direction of motion. For example, if you exert a force on your textbook causing it to move eastward across your desk, the force of friction acting on the book is westward. **Static friction** is the force that tends to prevent a stationary object from starting to move. **Kinetic friction** is the force that acts against an object's motion. **Air resistance** is friction that involves an object moving through air; it becomes noticeable at high speeds.

Finally, because there are several possible names for various pushes, pulls, thrusts, and so on, we will use the general term *applied force* for any contact force that does not fit any of the previously described categories.

We will use consistent symbols for the common forces as summarized in **Table 1**. Notice that because force is a vector quantity, we use an arrow above each symbol.

friction ($\vec{F}_f$) force that resists motion or attempted motion between objects in contact; acts in direction opposite to motion or attempted motion

static friction ($\vec{F}_S$) force that tends to prevent a stationary object from starting to move

kinetic friction ($\vec{F}_K$) force that acts against an object's motion

air resistance frictional force that opposes an object's motion through air

Practice

Understanding Concepts

1. Summarize the common forces by completing a table with the following headings: Name of Force, Type of Force, and Example in Daily Life. (Indicate either action-at-a-distance or contact force under Type of Force.)
2. Refer to the traction system shown in **Figure 1**. Assuming that the tension in the cord just above the mass has a magnitude of 18 N, do you expect the tension in the vertical cord above the leg to be less than 18 N, equal to 18 N, or greater than 18 N? Give your reasons.
3. Mechanical and structural engineers say, "You can't push a rope." Rephrase this statement using more formal physics terminology.

Table 1 Common Forces

Force	Symbol
gravity	$\vec{F}_g$
normal	$\vec{F}_N$
tension	$\vec{F}_T$
friction	$\vec{F}_f$
kinetic friction	$\vec{F}_K$
static friction	$\vec{F}_S$
air resistance	$\vec{F}_{air}$
applied force	$\vec{F}_{app}$

free-body diagram (FBD) diagram of a single object showing all the forces acting on that object

Drawing Free-Body Diagrams

A convenient way to analyze situations involving forces is to use diagrams. A **free-body diagram** (FBD) is a diagram of a single object that shows only the forces acting on that object; no other objects are shown in the FBD. The object itself can be represented by a dot or drawn as a small sketch. The directions and the approximate magnitudes of the forces are drawn on the diagram as arrows facing away from the object. A coordinate system is shown in the FBD, with the $+x$ and $+y$ directions indicated.

In solving word problems, especially complex ones, it is sometimes helpful to sketch a diagram of the system, called a *system diagram*, before drawing an FBD.

SAMPLE problem 1

You toss a ball vertically upward. Draw an FBD of the ball just before it leaves your hand.

Solution

Only two forces act on the ball (**Figure 5**). Gravity acts downward. The normal force applied by your hand (we may call this the applied force, since it comes from you) acts upward. Since there are no horizontal components of forces in this situation, our FBD shows a $+y$ direction, but no $+x$ direction.

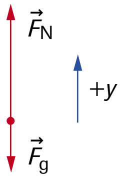

Figure 5
The FBD of the ball in Sample Problem 1

LEARNING TIP

Choosing Positive Directions
There is no right or wrong choice for $+x$ and $+y$ directions, although one choice may be more convenient than another. If there is acceleration in an obvious direction, it is convenient to choose that direction as positive. The direction of the other component is then perpendicular to that direction.

SAMPLE problem 2

A child is pushing with a horizontal force against a chair that remains stationary. Draw a system diagram of the overall situation and an FBD of the chair.

Solution

The system diagram in **Figure 6(a)** shows the four forces acting on the chair: gravity, the normal force, the applied force (the push delivered by the child), and the force of static friction. The $+x$ direction is chosen in the direction of the attempted motion. **Figure 6(b)** is the corresponding FBD, showing these same four forces.

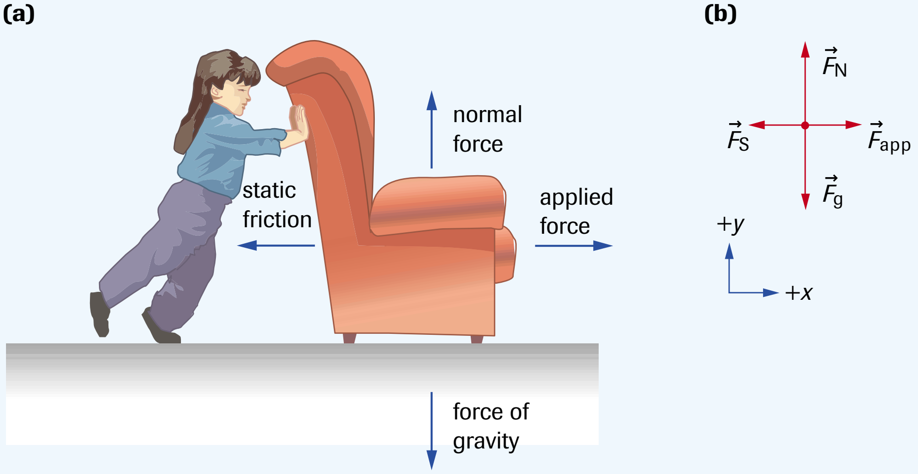

Figure 6
(a) The forces acting on the chair
(b) The FBD of the stationary chair

SAMPLE problem 3

A child pulls a sleigh up a snow-covered hill at a constant velocity with a force parallel to the hillside. Draw a system diagram of the overall situation and an FBD of the sleigh.

Solution

The system diagram of **Figure 7(a)** shows the four forces acting on the sleigh: gravity, tension in the rope, kinetic friction, and the normal force. The $+x$ direction is the direction of motion and the $+y$ direction is perpendicular to that motion. **Figure 7(b)** is the corresponding FBD, including the components of the force of gravity.

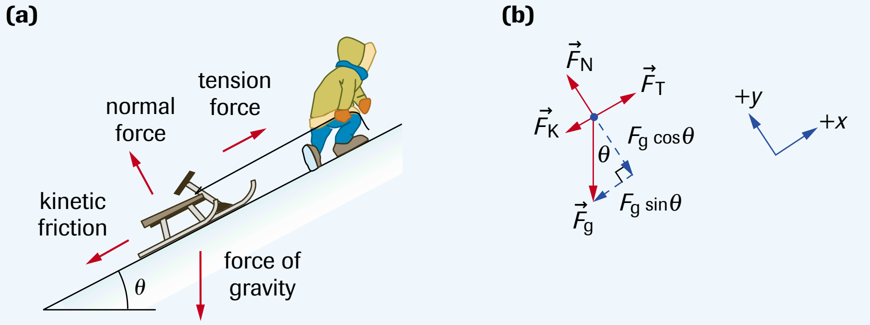

Figure 7
(a) The forces acting on the sleigh
(b) The FBD of the sleigh

LEARNING TIP

Components of Forces
Whenever there is a force in an FBD that is not parallel to either the $+x$ direction or the $+y$ direction, draw and label the components of that force, as in **Figure 7(b)**. Appendix A discusses vector components.

Practice

Understanding Concepts

4. Draw an FBD for objects A, B, C, and D.
  (a) A hot dog (object A) sits on a table.
  (b) A length of railway track (B) is being raised by a cable connected to a crane.
  (c) A pencil (C) has just begun falling from a desk to the floor. Air resistance is negligible.
  (d) A stove (D) is being pulled up a ramp into a delivery truck by a cable parallel to the ramp. The ramp is at an angle of 18° above the horizontal.

5. You throw a ball vertically upward from your hand. Air resistance is negligible. Draw an FBD of the ball (a) shortly after it leaves your hand, (b) when it is at the top of its motion, and (c) as it is falling back down.

6. Draw an FBD of the ball from question 5 as it falls back down, assuming there is a force of friction caused by air resistance.

7. The tourist in **Figure 8** is pulling a loaded suitcase at a constant velocity to the right with a force applied to the handle at an angle θ above the horizontal. A small force of friction resists the motion.
  (a) Draw an FBD of the suitcase, labelling the components of the appropriate forces. Choose the direction of motion to be the $+x$ direction.
  (b) Draw an FBD of the suitcase, labelling the components of the appropriate forces. Choose the $+x$ direction as the direction in which the handle is pointing.
  (c) Which choice of $+x$ is more convenient? Why? (*Hint:* Did you show the components of all the forces in your FBDs?)

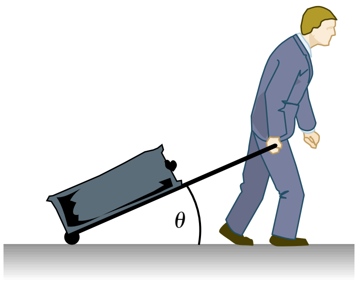

Figure 8
For question 7

Applying Inquiry Skills

8. You are watching a skydiver whose parachute is open and whose instantaneous height above ground level is indicated by digits on an electronic screen. The skydiver has reached terminal speed. (For this question, assume the skydiver and the parachute together act as one body.)
  (a) Draw a system diagram and an FBD for the situation.
  (b) Describe how you would estimate the force of air resistance acting on the skydiver and parachute. Include any assumptions and calculations.

Analyzing Forces on Stationary Objects

net force $(\Sigma\vec{F})$ the sum of all forces acting on an object

LEARNING TIP

Net Force Symbols

The net force can have a variety of symbols, such as $\vec{F}_{\text{net}}$, $\vec{F}_{\text{R}}$ (for resultant force), $\vec{F}_{\text{total}}$, $\vec{F}_{\text{sum}}$, and the symbol used in this text, $\Sigma\vec{F}$.

When analyzing a problem involving forces acting on an object, you must find the sum of all the forces acting on that object. The sum of all forces acting on an object has a variety of names, including net force, resultant force, total force, or sum of the forces; in this text, we will use the term **net force**. The symbol for net force is $\Sigma\vec{F}$, where the Greek letter sigma (Σ) serves as a reminder to add, or "sum," all the forces.

Determining the sum of all the forces is straightforward if all the forces are linear or perpendicular to each other, but it is somewhat more complex if some forces are at angles other that 90°. In two-dimensional situations, it is often convenient to analyze the components of the forces, in which case the symbols ΣF_x and ΣF_y are used instead of $\Sigma\vec{F}$.

SAMPLE problem 4

In hitting a volleyball, a player applies an average force of 9.9 N [33° above the horizontal] for 5.0 ms. The force of gravity on the ball is 2.6 N [down]. Determine the net force on the ball as it is being struck.

Solution

The relevant given information is shown in the FBD of the ball in **Figure 9(a)**. (Notice that the time interval of 5.0 ms is not shown because it is not needed for this solution.) The net force on the ball is the vector sum $\vec{F}_{\text{g}} + \vec{F}_{\text{app}}$. We calculate the net force by taking components with the $+x$ and $+y$ directions as in **Figure 9(b)**.

First, we take components of $\vec{F}_{\text{app}}$:

$$F_{\text{app},x} = (9.9\ \text{N})(\cos 33^\circ) \qquad F_{\text{app},y} = (9.9\ \text{N})(\sin 33^\circ)$$
$$F_{\text{app},x} = 8.3\ \text{N} \qquad F_{\text{app},y} = 5.4\ \text{N}$$

Next, we take components of $\vec{F}_{\text{g}}$:

$$F_{\text{g}x} = 0.0\ \text{N} \qquad F_{\text{g}y} = -2.6\ \text{N}$$

We add the components to determine the net force:

$$\begin{aligned} \Sigma F_x &= F_{\text{app},x} + F_{\text{g}x} & \Sigma F_y &= F_{\text{app},y} + F_{\text{g}y} \\ &= 8.3\ \text{N} + 0.0\ \text{N} & &= 5.4\ \text{N} + (-2.6\ \text{N}) \\ \Sigma F_x &= 8.3\ \text{N} & \Sigma F_y &= 2.8\ \text{N} \end{aligned}$$

Figure 9(c) shows how we determine the magnitude of the net force:

$$\begin{aligned} |\Sigma\vec{F}| &= \sqrt{(8.3\ \text{N})^2 + (2.8\ \text{N})^2} \\ |\Sigma\vec{F}| &= 8.8\ \text{N} \end{aligned}$$

The direction of $\Sigma\vec{F}$ is given by the angle ϕ in the diagram:

$$\begin{aligned} \phi &= \tan^{-1}\frac{2.8\ \text{N}}{8.3\ \text{N}} \\ \phi &= 19^\circ \end{aligned}$$

The net force on the ball is 8.8 N [19° above the horizontal].

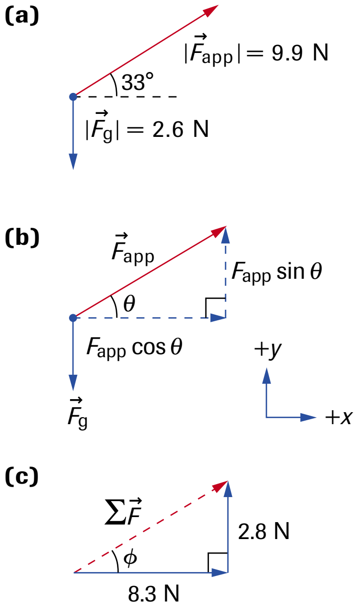

Figure 9
(a) FBD of the ball
(b) The components of the forces
(c) The net force

SAMPLE problem 5

The boat in **Figure 10** is secured to a lakeside pier with two horizontal ropes. A wind is blowing offshore. The tensions in the ropes are $\vec{F}_1 = 48$ N [16° N of E] and $\vec{F}_2 = 48$ N [16° S of E].

(a) Use a vector scale diagram to determine the sum of the tension forces in the two ropes.

(b) Assuming that the net horizontal force on the boat is zero, determine the force of the wind on the boat.

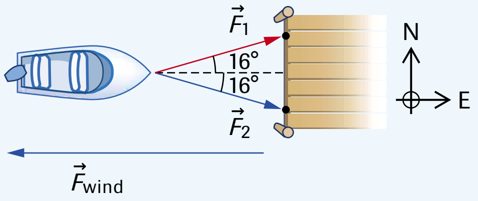

Figure 10
The horizontal forces acting on the boat

LEARNING TIP

Adding Force Vectors
As you know from Chapter 1 and Appendix A, this text uses three methods of adding vectors. For understanding concepts, using vector scale diagrams is highly recommended. For high accuracy and relatively quick solutions, trigonometry is an excellent method, but the laws of cosines and sines can be applied easily only to the addition (or subtraction) of two vectors. The component technique is highly accurate and appropriate for the addition (or subtraction) of any number of vectors.

Solution

(a) $\vec{F}_1 = 48$ N [16° N of E]

$\vec{F}_2 = 48$ N [16° S of E]

$\vec{F}_1 + \vec{F}_2 = ?$

This vector addition is shown in **Figure 11**. Measurement with a ruler indicates that the sum of the tensions in the ropes is 92 N [E].

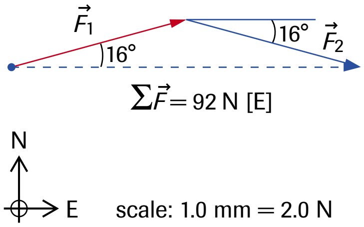

Figure 11
Vector scale diagram showing the sum of the tensions for Sample Problem 5

(b) Using the symbol $\vec{F}_{\text{wind}}$ for the force of the wind on the boat, we know that:

$$\Sigma\vec{F} = 0$$

$$\vec{F}_1 + \vec{F}_2 = 92 \text{ N [E]}$$

$$\begin{aligned}\Sigma\vec{F} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_{\text{wind}}\\ \vec{F}_{\text{wind}} &= \Sigma\vec{F} - (\vec{F}_1 + \vec{F}_2)\\ &= 0.0 \text{ N} - 92 \text{ N [E]}\\ \vec{F}_{\text{wind}} &= 92 \text{ N [W]}\end{aligned}$$

The force of the wind on the boat is 92 N [W].

Practice

Understanding Concepts

9. Determine the net force on objects E, F, and G.

(a) At a particular instant, a soaring bird (E) is subject to an upward lift of 3.74 N, a downward gravitational force of 3.27 N, and a horizontal air resistance force of 0.354 N.

(b) A long-jump contestant (F) experiences at the instant of landing a gravitational force of 538 N [down] and a force, applied by the ground to the feet, of 6382 N [28.3° above the horizontal].

(c) In a football game, a quarterback (G), hit simultaneously by two linebackers, experiences horizontal forces of 412 N [27.0° W of N] and 478 N [36.0° N of E]. (Consider only the horizontal forces and neglect friction. Note that we can ignore the vertical forces because they are equal in magnitude, but opposite in direction.)

Answers

9. (a) 0.59 N [53° above the horizontal]

(b) 6.15×10^3 N [23.9° above the horizontal]

(c) 678 N [17.1° E of N]

Answer

11. 31 N [30° S of E], to two significant digits

10. Solve Sample Problem 5(a) using (a) components and (b) trigonometry.

11. A crate is being dragged across a horizontal icy sidewalk by two people pulling horizontally on cords (**Figure 12**). The net horizontal force on the crate is 56 N [16° S of E]. The tension in cord 1 is 27 N [E]. If friction is negligible, determine the tension in cord 2.

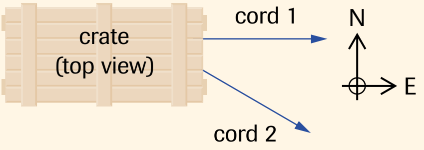

Figure 12

SUMMARY Forces and Free-Body Diagrams

- We commonly deal with Earth's force of gravity, the normal force, tension forces, and friction forces.
- Static friction tends to prevent a stationary object from starting to move; kinetic friction acts against an object's motion. Air resistance acts against an object moving through air.
- The free-body diagram (FBD) of an object shows all the forces acting on that object. It is an indispensable tool in helping to solve problems involving forces.
- The net force $\Sigma\vec{F}$ is the vector sum of all the forces acting on an object.

Section 2.1 Questions

Understanding Concepts

1. You push your ruler westward at a constant speed across your desk by applying a force at an angle of 25° above the horizontal.
  (a) Name all the forces acting on the ruler and state which ones are contact forces.
  (b) What fundamental force is responsible for the contact forces?
  (c) Draw an FBD of the ruler in this situation. Where appropriate, include the components of forces.

2. Draw an FBD for objects H, I, J, and K.
  (a) a cup (H) hanging from a hook
  (b) a person (I) standing in an elevator that is moving downward
  (c) a curling rock (J) sliding freely in a straight line on a rink
  (d) a crate (K) being dragged across a floor, with significant friction, by a person pulling on a rope at an angle of 23° above the horizontal

3. The force of gravity on a textbook is 18 N [down].
  (a) What is the net force on the book if it is held stationary in your hand?
  (b) Neglecting air resistance, what is the net force acting on the book if you suddenly remove your hand?

4. At one particular instant in its flight, a ball experiences a gravitational force $\vec{F}_g$ = 1.5 N [down] and an air resistance force $\vec{F}_{air}$ = 0.50 N [32° above the horizontal]. Calculate the net force on the ball.

5. Given the following force vectors, $\vec{F}_A$ = 3.6 N [28° W of S], $\vec{F}_B$ = 4.3 N [15° N of W], and $\vec{F}_C$ = 2.1 N [24° E of S], determine
  (a) $\vec{F}_A + \vec{F}_B + \vec{F}_C$, using a vector scale diagram
  (b) $\vec{F}_A + \vec{F}_B + \vec{F}_C$, using components
  (c) $\vec{F}_A - \vec{F}_B$, using a vector scale diagram
  (d) $\vec{F}_A - \vec{F}_B$, using trigonometry

6. Given $\vec{F}_1$ = 36 N [25° N of E] and $\vec{F}_2$ = 42 N [15° E of S], determine the force $\vec{F}_3$ that must be added to the sum of $\vec{F}_1 + \vec{F}_2$ to produce a net force of zero.