What do the following situations have in common?

- A monkey jumps from the branch of one tree to the branch of an adjacent tree.
- A snowboarder glides at top speed off the end of a ramp (**Figure 1**).
- A relief package drops from a low-flying airplane.

In each situation, the body or object moves through the air without a propulsion system along a two-dimensional curved trajectory (**Figure 2(a)**). Such an object is called a **projectile**; the motion of a projectile is called *projectile motion*.

(a)

(b)
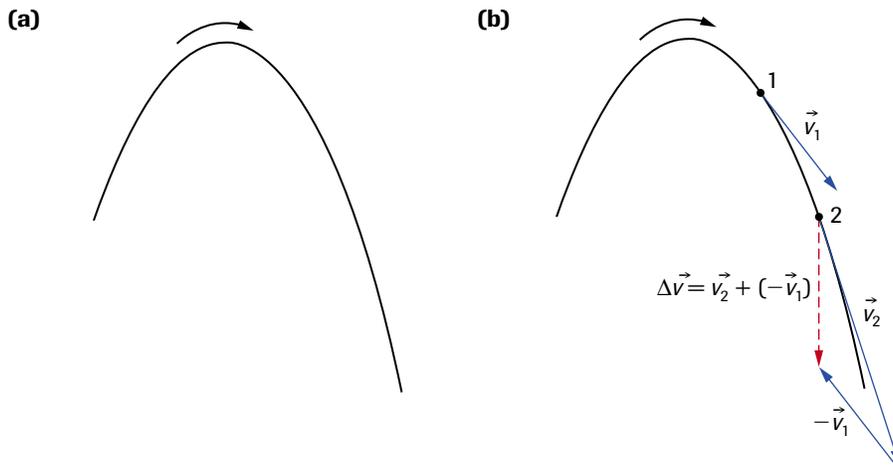
$$\Delta\vec{v} = \vec{v}_2 + (-\vec{v}_1)$$

Figure 2
(a) A typical trajectory followed by a projectile.
(b) The change in velocity between position 1 and position 2 is $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, which is shown as $\Delta\vec{v} = \vec{v}_2 + (-\vec{v}_1)$.
If $\Delta\vec{v}$ is divided by the time Δt required for the motion from position 1 to position 2, the result is the average acceleration for that time interval.

It is evident that a projectile is accelerating because the direction of its instantaneous velocity is continually changing. However, in what direction is that acceleration occurring? Since $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$, $\vec{a}_{av}$ is in the direction of $\Delta\vec{v}$. **Figure 2(b)** shows that the vector subtraction $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ yields a vector directed downward, which indicates that the direction of acceleration is also downward.

In the Try This Activity at the beginning of Chapter 1, you considered two projectiles, balls A and B, which began moving simultaneously. Ball A fell from rest, while ball B was launched horizontally with an initial velocity. Although, as we show in **Figure 3**, B had a longer path than A, the two balls landed simultaneously. The initial horizontal motion of a projectile like ball B does not affect its vertical acceleration.

Other experiments show the same thing. **Figure 4** is a stroboscopic photograph of two balls released simultaneously. The ball on the right was projected horizontally. The interval between strobe flashes was constant. A grid has been superimposed on the photo to facilitate measurement and analysis. In successive equal time intervals, the vertical components of the displacement increase by the same amount for each ball. Note that the projected ball travels a constant horizontal displacement in each time interval. The independent horizontal and vertical motions combine to produce the trajectory.
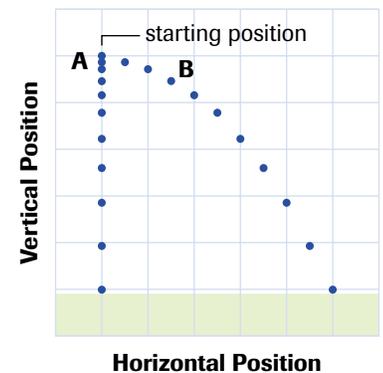

Figure 1
How would you describe the motion of the snowboarder after leaving the ramp?

projectile an object that moves through the air, along a trajectory, without a propulsion system

DID YOU KNOW ?

Dangerous Projectiles
You may have seen soldiers in the television news firing their rifles into the air to celebrate some victory. The bullets travel as high-speed projectiles and, despite air resistance, return to Earth at high enough speeds to be dangerous. Reports indicate that from time to time people are injured by the returning bullets.
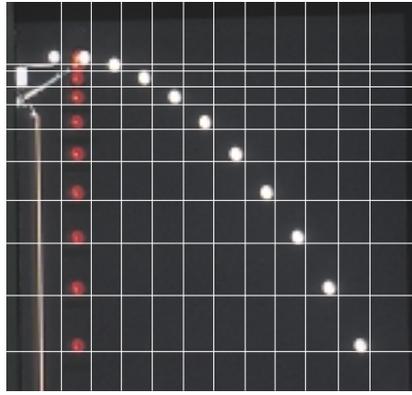

Figure 3
Ball B is projected horizontally at the same instant that ball A is dropped. Although the path of ball B is longer than the path of ball A, the balls land at the same instant.

projectile motion motion with a constant horizontal velocity and a constant vertical acceleration due to gravity

horizontal range (Δx) the horizontal displacement of a projectile

Figure 4
These two balls reached the lowest position at the same instant even though one was projected horizontally. Both balls had an initial vertical velocity of zero, and both experienced free fall.

Figure 5
(a) At time $t = 0$, the initial velocity of the projectile, $\vec{v}_i$, has a horizontal component, v_{ix}, and a vertical component, v_{iy}.
(b) After Δt has elapsed, the projectile's velocity, $\vec{v}_f$, has the same horizontal component (neglecting air resistance) and a different vertical component, v_{fy}.

If you look carefully at the grid superimposed on the photograph in **Figure 4**, you can make the following important conclusions about projectile motion:

- The horizontal component of a projectile's velocity is constant. (The horizontal component of acceleration, in other words, is zero.)
- The projectile experiences constant downward acceleration due to gravity.
- The horizontal and vertical motions of a projectile are independent of each other, except they have a common time.

These conclusions are based on the assumption that air resistance can be neglected, an assumption we made when we analyzed the acceleration due to gravity in Section 1.3.

If you were performing an experiment to determine whether the concepts about projectile motion apply to an object on an inclined plane (for example, a puck moving on an air table set up at an angle to the horizontal), what observations would you expect to make? How would you analyze the motion of a projectile on an inclined plane to verify that the horizontal velocity is constant and the vertical acceleration is constant? You will explore these questions in Investigation 1.4.1 in the Lab Activities section at the end of this chapter. ⚛ ▌

Analyzing the Motion of Objects Projected Horizontally

Projectile motion is motion with a constant horizontal velocity combined with a constant vertical acceleration caused by gravity. Since the horizontal and vertical motions are independent of each other, we can apply independent sets of equations to analyze projectile motion. The constant velocity equations from Section 1.1 apply to the horizontal motion, while the constant acceleration equations from Sections 1.2 and 1.3 (with $|\vec{g}| = 9.8 \text{ m/s}^2$) apply to the vertical motion.

Figure 5 shows the initial and final velocity vectors for a projectile, with their horizontal and vertical components. **Table 1** summarizes the kinematics equations for both components. None of the variables has an arrow over it since these variables represent components of vectors, not vectors themselves. For example, v_{ix} represents the x-component (which is not a vector) of the initial velocity and v_y represents the y-component (also not a vector) of the velocity after some time interval Δt. The horizontal displacement, Δx, is called the **horizontal range** of the projectile.

Table 1 Kinematics Equations for Projectile Motion

Horizontal (x) Motion	The constant velocity (zero acceleration) equation is written for the x-component only.	$v_{ix} = \dfrac{\Delta x}{\Delta t}$
Vertical (y) Motion	The five constant acceleration equations involving the acceleration due to gravity are written for the y-component only. The constant acceleration has a magnitude of $\lvert \vec{g} \rvert = g = 9.8 \text{ m/s}^2$.	$a_y = \dfrac{v_{fy} - v_{iy}}{\Delta t}$ or $v_{fy} = v_{iy} + a_y\Delta t$ $\Delta y = v_{iy}\Delta t + \dfrac{1}{2} a_y(\Delta t)^2$ $\Delta y = v_{av,y}\Delta t$ or $\Delta y = \dfrac{1}{2}(v_{fy} + v_{iy})\Delta t$ $v_{fy}{}^2 = v_{iy}{}^2 + 2a_y\Delta y$ $\Delta y = v_{fy}\Delta t - \dfrac{1}{2} a_y(\Delta t)^2$

▶ SAMPLE problem 1

A ball is thrown off a balcony and has an initial velocity of 18 m/s horizontally.

(a) Determine the position of the ball at $t =$ 1.0 s, 2.0 s, 3.0 s, and 4.0 s.

(b) Show these positions on a scale diagram.

(c) What is the mathematical name of the resulting curve?

(a)

Solution

(a) Let the $+x$ direction be to the right and the $+y$ direction be downward (which is convenient since there is no upward motion) (see **Figure 6(a)**).

Horizontally (constant v_{ix}):

$v_{ix} =$ 18 m/s

$\Delta t =$ 1.0 s

$\Delta x =$?

$\Delta x = v_{ix}\Delta t$

$\quad\ = (18 \text{ m/s})(1.0 \text{ s})$

$\Delta x =$ 18 m

Table 2 gives the Δx values for $\Delta t =$ 1.0 s, 2.0 s, 3.0 s, and 4.0 s.

Vertically (constant a_y):

$v_{iy} = 0$ $\qquad\qquad\qquad$ $\Delta t =$ 1.0 s

$a_y = +g =$ 9.8 m/s^2 $\qquad$ $\Delta y =$?

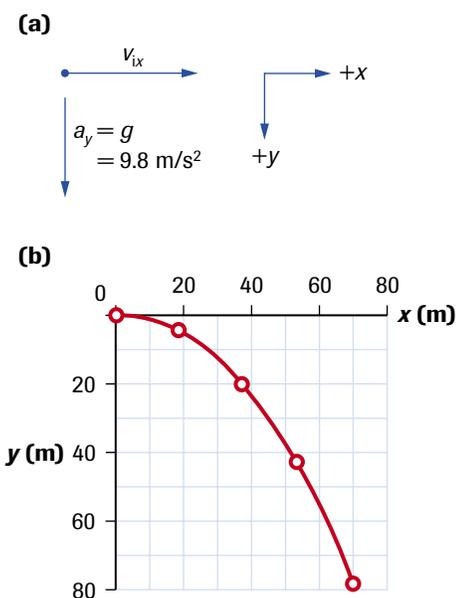$\Delta y = v_{iy}\Delta t + \dfrac{1}{2}a_y(\Delta t)^2$

$\quad\ = \dfrac{1}{2}a_y(\Delta t)^2$

$\quad\ = \dfrac{(9.8 \text{ m/s}^2)(1.0 \text{ s})^2}{2}$

$\Delta y = +4.9 \text{ m}$

Table 2 gives the Δy values for $\Delta t =$ 1.0 s, 2.0 s, 3.0 s, and 4.0 s.

(b) **Figure 6(b)** shows a scale diagram of the ball's position at the required times. The positions are joined with a smooth curve.

(c) The curved path shown in **Figure 6(b)** is a parabola.

(b)
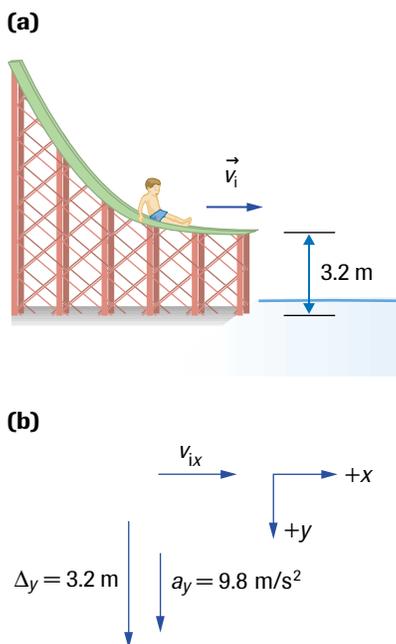

Figure 6
For Sample Problem 1
(a) Initial conditions
(b) Scale diagram of the motion

Table 2 Calculated Positions at Select Times

t (s)	Δx (m)	Δy (m)
0.0	0.0	0.0
1.0	18	4.9
2.0	36	20
3.0	54	44
4.0	72	78

(a)

$\vec{v_i}$

3.2 m

(b)

v_{ix}

$+x$

$+y$

$\Delta y = 3.2$ m $a_y = 9.8$ m/s^2

Figure 7
For Sample Problem 2
(a) The situation
(b) The initial conditions

▶ **SAMPLE** problem **2**

A child travels down a water slide, leaving it with a velocity of 4.2 m/s horizontally, as in **Figure 7(a)**. The child then experiences projectile motion, landing in a swimming pool 3.2 m below the slide.

(a) For how long is the child airborne?

(b) Determine the child's horizontal displacement while in the air.

(c) Determine the child's velocity upon entering the water.

Solution

As shown in **Figure 7(b)**, $+x$ is to the right and $+y$ is downward. The initial position is the position where the child leaves the slide.

(a) *Horizontally (constant v_{ix}):*

$v_{ix} = 4.2$ m/s
$\Delta x = ?$
$\Delta t = ?$

Vertically (constant a_y):

$v_{iy} = 0$ $\qquad\qquad$ $\Delta y = 3.2$ m
$a_y = +g = 9.8$ m/s^2 $\qquad$ $v_{fy} = ?$
$\Delta t = ?$

The horizontal motion has two unknowns and only one equation $\Delta x = v_{ix}\Delta t$. We can analyze the vertical motion to determine Δt:

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$\Delta y = \frac{1}{2}a_y(\Delta t)^2$$

$$(\Delta t)^2 = \frac{2\Delta y}{a_y}$$

$$\Delta t = \pm\sqrt{\frac{2\Delta y}{a_y}}$$

$$= \pm\sqrt{\frac{2(3.2 \text{ m})}{9.8 \text{ m/s}^2}}$$

$$\Delta t = \pm 0.81 \text{ s}$$

Since we are analyzing a trajectory that starts at $t = 0$, only the positive root applies. The child is in the air for 0.81 s.

(b) We can substitute $\Delta t = 0.81$ s into the equation for horizontal motion.

$$\Delta x = v_{ix}\Delta t$$
$$= (4.2 \text{ m/s})(0.81 \text{ s})$$
$$\Delta x = 3.4 \text{ m}$$

The child reaches the water 3.4 m horizontally from the end of the slide. In other words, the child's horizontal displacement is 3.4 m.

(c) To find the child's final velocity, a vector quantity, we must first determine its horizontal and vertical components. The x-component is constant at 4.2 m/s. We find the y-component as follows:

$$v_{fy} = v_{iy} + a_y\Delta t$$
$$= 0 \text{ m/s} + (9.8 \text{ m/s}^2)(0.81 \text{ s})$$
$$v_{fy} = 7.9 \text{ m/s}$$

We now apply the law of Pythagoras and trigonometry to determine the final velocity as shown in **Figure 8**.

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$
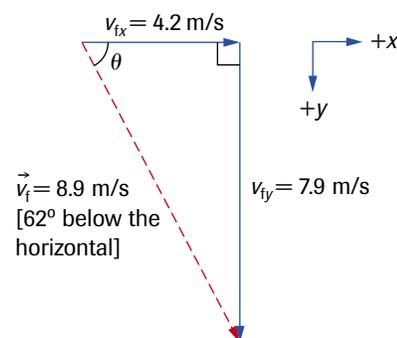$$= \sqrt{(4.2 \text{ m/s})^2 + (7.9 \text{ m/s})^2}$$
$$v_f = 8.9 \text{ m/s}$$

$$\theta = \tan^{-1} \frac{v_{fy}}{v_{fx}}$$
$$= \tan^{-1} \frac{7.9 \text{ m/s}}{4.2 \text{ m/s}}$$
$$\theta = 62°$$

The final velocity is 8.9 m/s at an angle of 62° below the horizontal.

Figure 8
The solution for part (c) of Sample Problem 2

▶ SAMPLE problem 3

A helicopter, travelling horizontally, is 82 m above the ground. The pilot prepares to release a relief package intended to land on the ground 96 m horizontally ahead. Air resistance is negligible. The pilot does not throw the package, but lets it drop. What is the initial velocity of the package relative to the ground?

Solution

Figure 9 shows the situation, with the initial position chosen as the point of release, +x chosen to the right, and +y chosen downward. Since the pilot does not throw the package, the initial horizontal velocity of the package is the same as the horizontal velocity of the helicopter.

Horizontally (constant v_{ix}):

$\Delta x = 96$ m

$\Delta t = ?$

$v_{ix} = ?$

Vertically (constant a_y):

$v_{iy} = 0$ m/s $\Delta y = 82$ m

$a_y = +g = 9.8$ m/s² $\Delta t = ?$

As in Sample Problem 2, we can determine Δt from the equations for vertical motion. The appropriate equation is

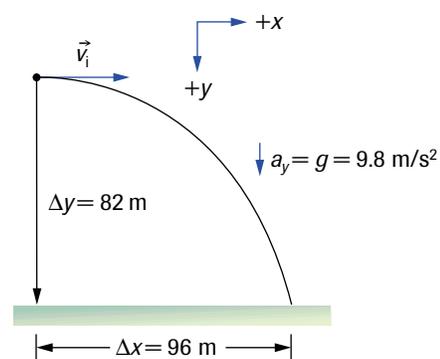$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$\Delta y = \frac{1}{2}a_y(\Delta t)^2$$
$$(\Delta t)^2 = \frac{2\Delta y}{a_y}$$
$$\Delta t = \pm \sqrt{\frac{2\Delta y}{a_y}}$$
$$= \pm \sqrt{\frac{2(82 \text{ m})}{9.8 \text{ m/s}^2}}$$
$$\Delta t = 4.1 \text{ s}$$

Figure 9
The situation for Sample Problem 3

Since we only consider events after the release of the package at $t = 0$, only the positive root applies.

$$v_{ix} = \frac{\Delta x}{\Delta t}$$

$$= \frac{96 \text{ m}}{4.1 \text{ s}}$$

$$v_{ix} = 23 \text{ m/s}$$

The initial velocity of the package is 23 m/s [horizontally].

▶ *Practice*

Understanding Concepts

1. Explain why an airplane moving through the air is not an example of projectile motion.

2. A stone is thrown horizontally under negligible air resistance. What are its vertical acceleration and its horizontal acceleration?

3. A marble rolls off a table with a velocity of 1.93 m/s [horizontally]. The tabletop is 76.5 cm above the floor. If air resistance is negligible, determine
   (a) how long the marble is airborne
   (b) the horizontal range
   (c) the velocity at impact

4. A stone is thrown horizontally with an initial speed of 8.0 m/s from a cliff. Air resistance is negligible.
   (a) Determine the horizontal and vertical components of displacement and instantaneous velocity at $t = 0.0$ s, 1.0 s, 2.0 s, and 3.0 s.
   (b) Draw a scale diagram showing the path of the stone.
   (c) Draw the instantaneous velocity vector at each point on your diagram.
   (d) Determine the average acceleration between 1.0 s and 2.0 s, and between 2.0 s and 3.0 s. What do you conclude?

5. A baseball pitcher throws a ball horizontally under negligible air resistance. The ball falls 83 cm in travelling 18.4 m to the home plate. Determine the ball's initial horizontal speed.

Applying Inquiry Skills

6. **Figure 10** shows a trajectory apparatus. A vertical target plate allows the horizontal position to be adjusted from one side of the graph paper to the other.
   (a) Describe how this apparatus is used to analyze projectile motion.
   (b) What would you expect to see plotted on graph paper? Draw a diagram. If you have access to a trajectory apparatus, use it to check your prediction.

Making Connections

7. When characters in cartoons run off the edge of a cliff, they hang suspended in the air for a short time before plummeting. If cartoons obeyed the laws of physics, what would they show instead?

Answers

3. (a) 0.395 s
   (b) 76.3 cm
   (c) 4.33 m/s [63.5° below the horizontal]

4. (a) At 3.0 s, $\Delta x = 24$ m, $\Delta y = 44$ m, and $\vec{v} = 3.0 \times 10^1$ m/s [75° below the horizontal].
   (d) 9.8 m/s² [down]

5. 45 m/s

Figure 10
When the steel ball is launched from the ramp and collides with the target plate, the point of contact is recorded on the target paper.

Analyzing More Complex Projectile Motion

In the projectile problems we have solved so far, the initial velocity was horizontal. The same kinematics equations can be used to analyze problems where the initial velocity is at some angle to the horizontal. Since $v_{iy} \neq 0$, you must take care with your choice of positive and negative directions for the vertical motion. For example, a fly ball in baseball

(**Figure 11**) has an initial velocity with an upward vertical component. If the $+y$ direction is chosen to be upward, then v_{iy} is positive, and the vertical acceleration a_y is negative because the gravitational acceleration is downward. Conversely, if the $+y$ direction is chosen to be downward, then v_{iy} is negative and a_y is positive.

(a)

$\Delta y > 0$

$a_y < 0$

(b)

$\Delta y < 0$

$a_y > 0$
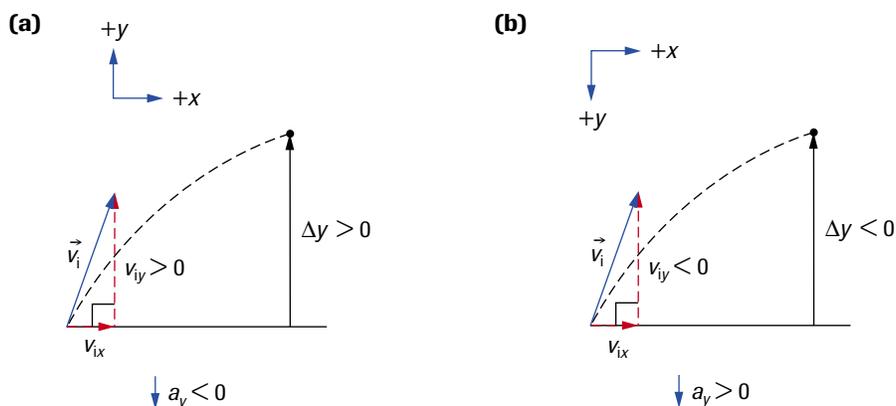
Figure 11
(a) The $+y$ direction is upward.
(b) The $+y$ direction is downward.

▶ **SAMPLE problem 4**

A golfer strikes a golf ball on level ground. The ball leaves the ground with an initial velocity of 42 m/s [32° above the horizontal]. The initial conditions are shown in **Figure 12**. If air resistance is negligible, determine the ball's

(a) horizontal range (assuming that it lands at the same level from which it started)
(b) maximum height
(c) horizontal displacement when it is 15 m above the ground

Solution

(a) We begin by finding the horizontal and vertical components of the initial velocity.

$$v_{ix} = |\vec{v}_i| \cos \theta \qquad\qquad v_{iy} = |\vec{v}_i| \sin \theta$$
$$= (42 \text{ m/s})(\cos 32°) \qquad = (42 \text{ m/s})(\sin 32°)$$
$$v_{ix} = 36 \text{ m/s} \qquad\qquad v_{iy} = 22 \text{ m/s}$$

Horizontally (constant v_{ix}):
$$v_{ix} = 36 \text{ m/s}$$
$$\Delta x = ?$$
$$\Delta t = ?$$

Vertically (constant a_y):
$$a_y = -g = -9.8 \text{ m/s}^2 \qquad \Delta y = 0$$
$$v_{iy} = 22 \text{ m/s} \qquad\qquad \Delta t = ?$$
$$v_{fy} = -22 \text{ m/s}$$

Since the horizontal motion has two unknowns and only one equation, we can use the vertical motion to solve for Δt:

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$0 = 22 \text{ m/s } \Delta t - 4.9 \text{ m/s}^2 \, (\Delta t)^2$$
$$0 = \Delta t \, (22 \text{ m/s} - 4.9 \text{ m/s}^2 \, \Delta t)$$

$|\vec{v}_i| = 42 \text{ m/s}$

$v_{iy} = |\vec{v}_i| \sin \theta$

$v_{ix} = |\vec{v}_i| \cos \theta$

$a_y = -g = -9.8 \text{ m/s}^2$
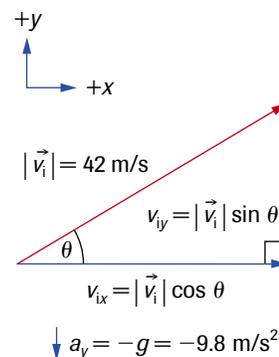
Figure 12
Initial conditions for Sample Problem 4. The golf tee is chosen as the initial position, and the $+y$ direction is chosen as upward.

Therefore, the ball was hit at $\Delta t = 0$ and the ball lands at 22 m/s – 4.9 m/s² $\Delta t = 0$. Solving for Δt, we find that $\Delta t = 4.5$ s, which we can use to find the horizontal range.

$$\Delta x = v_{ix}\Delta t$$
$$= (36 \text{ m/s})(4.5 \text{ s})$$
$$\Delta x = 1.6 \times 10^2 \text{ m}$$

The horizontal range is 1.6×10^2 m.

(b) To determine the maximum height, we start by noting that at the highest position, $v_{fy} = 0$ m/s. (This also happens when an object thrown directly upward reaches the top of its flight.)

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$$
$$0 = v_{iy}^2 + 2a_y\Delta y$$

$$\Delta y = \frac{v_{iy}^2}{-2a_y}$$

$$= \frac{(22 \text{ m/s})^2}{-2(-9.8 \text{ m/s}^2)}$$

$$\Delta y = 25 \text{ m}$$

The maximum height is 25 m.

(c) To find the horizontal displacement when $\Delta y = 15$ m, we must find the time interval Δt between the start of the motion and when $\Delta y = 15$ m. We can apply the quadratic formula:

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$15 \text{ m} = 22 \text{ m/s } \Delta t - 4.9 \text{ m/s}^2 (\Delta t)^2$$

$$4.9 \text{ m/s}^2 (\Delta t)^2 - 22 \text{ m/s } \Delta t + 15 \text{ m} = 0$$

Using the quadratic formula,

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{where } a = 4.9 \text{ m/s}^2, b = -22 \text{ m/s, and } c = 15 \text{ m}$$

$$= \frac{-(-22 \text{ m/s}) \pm \sqrt{(-22 \text{ m/s})^2 - 4(4.9 \text{ m/s}^2)(15 \text{ m})}}{2(4.9 \text{ m/s}^2)}$$

$$\Delta t = 3.7 \text{ s or } 0.84 \text{ s}$$

Thus, the ball is 15 m above the ground twice: when rising and when descending. We can determine the corresponding horizontal positions:

$$\Delta x_{up} = v_{ix}\Delta t \qquad\qquad \Delta x_{down} = v_{ix}\Delta t$$
$$= (36 \text{ m/s})(0.84 \text{ s}) \qquad\qquad = (36 \text{ m/s})(3.7 \text{ s})$$
$$\Delta x_{up} = 3.0 \times 10^1 \text{ m} \qquad\qquad \Delta x_{down} = 1.3 \times 10^2 \text{ m}$$

The horizontal position of the ball is either 3.0×10^1 m or 1.3×10^2 m when it is 15 m above ground.

As you learned in the solution to Sample Problem 4, the range of a projectile can be found by applying the kinematics equations step by step. We can also derive a general equation for the horizontal range Δx of a projectile, given the initial velocity and the angle of launch. What, for example, happens when a projectile lands at the same level from which it began ($\Delta y = 0$), as shown in **Figure 13**? For the horizontal range, the motion is found using the equation $\Delta x = v_{ix}\Delta t$, where the only known variable is v_{ix}. To find the other variable, Δt, we use the vertical motion:

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

where $\Delta y = 0$ because we are considering the situation where the final level is the same as the initial level.

$$v_{iy} = v_i \sin \theta$$
$$a_y = -g$$
$$0 = v_i \sin \theta \Delta t - \frac{1}{2}g(\Delta t)^2$$
$$0 = \Delta t\left(v_i \sin \theta - \frac{1}{2}g(\Delta t)\right)$$

Therefore, either $\Delta t = 0$ (on takeoff) or

$v_i \sin \theta - \frac{1}{2}g\Delta t = 0$ (on landing).

Solving the latter equation for Δt gives

$$\Delta t = \frac{2v_i \sin \theta}{g}$$

Now we return to the horizontal motion:

$$\Delta x = v_{ix}\Delta t$$
$$= (v_i \cos \theta)\Delta t$$
$$= v_i \cos \theta \left(\frac{2v_i \sin \theta}{g}\right)$$
$$\Delta x = \frac{v_i^2}{g} 2\sin \theta \cos \theta$$

Since $2\sin \theta \cos \theta = \sin 2\theta$ (as shown in the trigonometric identities in Appendix A), the horizontal range is

$$\Delta x = \frac{v_i^2}{g} \sin 2\theta$$

where v_i is the magnitude of the initial velocity of a projectile launched at an angle θ to the horizontal. Note that this equation applies only if $\Delta y = 0$.

All of the previous discussion and examples of projectile motion have assumed that air resistance is negligible. This is close to the true situation in cases involving relatively dense objects moving at low speeds, such as a shot used in shot put competition. However, for many situations, air resistance cannot be ignored. When air resistance is considered, the analysis of projectile motion becomes more complex and is beyond the intention of this text. The concept of "hang time" in certain sports, especially football, is important and is explored in Lab Exercise 1.4.1 in the Lab Activities Section at the end of this chapter. 🖍🟩
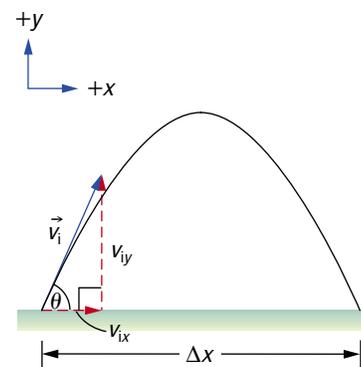
Figure 13
Initial conditions for deriving the horizontal range of a projectile in terms of launch angle and initial velocity

▶ **TRY THIS** *activity*

Comparing Horizontal Range

As a class or in a large group, set up a table using these titles: Launch Angle, Time of Flight, Maximum Height, and Horizontal Range. Complete the table for a projectile that has an initial velocity of magnitude 25.00 m/s and lands at the same level from which it was launched. Perform the calculations using four significant digits, using every third degree from 3° to 87° (i.e., 3°, 6°, 9°, … 81°, 84°, 87°). Write conclusions about maximizing height and horizontal range.

🖍 **LAB EXERCISE 1.4.1**

Hang Time in Football (p. 58)
"Hang time" in sports is the time interval between the launch of a ball and the landing or catching of the ball. In football, when a punt is needed, the punter tries to maximize the hang time of the ball to give his teammates time to race downfield to tackle the punt receiver. Of course, at the same time the punter tries to maximize the horizontal range to give his team better field position.

Write your hypothesis and predictions to the following questions, and then explore these concepts further by conducting the lab exercise.
(a) What factors affect the hang time of a punted football? How do they affect hang time?
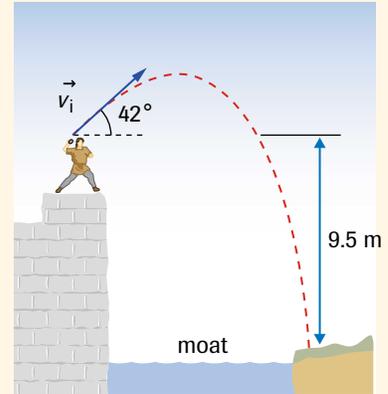(b) What launch angle of a punt maximizes the hang time of a football?

Understanding Concepts

8. A field hockey ball is struck and undergoes projectile motion. Air resistance is negligible.
   (a) What is the vertical component of velocity at the top of the flight?
   (b) What is the acceleration at the top of the flight?
   (c) How does the rise time compare to the fall time if the ball lands at the same level from which it was struck?

9. A cannon is set at an angle of 45° above the horizontal. A cannonball leaves the muzzle with a speed of 2.2×10^2 m/s. Air resistance is negligible. Determine the cannonball's
   (a) maximum height
   (b) time of flight
   (c) horizontal range (to the same vertical level)
   (d) velocity at impact

10. A medieval prince trapped in a castle wraps a message around a rock and throws it from the top of the castle wall with an initial velocity of 12 m/s [42° above the horizontal]. The rock lands just on the far side of the castle's moat, at a level 9.5 m below the initial level (**Figure 14**). Determine the rock's
    (a) time of flight
    (b) width of the moat
    (c) velocity at impact

Answers

9. (a) 1.2×10^3 m
   (b) 32 s
   (c) 4.9×10^3 m
   (d) 2.2×10^2 m/s [45° below the horizontal]

10. (a) 2.4 s
    (b) 22 m
    (c) 18 m/s [60° below the horizontal]

Figure 14
The situation for question 10

SUMMARY **Projectile Motion**

- A projectile is an object moving through the air in a curved trajectory with no propulsion system.
- Projectile motion is motion with a constant horizontal velocity combined with a constant vertical acceleration.
- The horizontal and vertical motions of a projectile are independent of each other except they have a common time.
- Projectile motion problems can be solved by applying the constant velocity equation for the horizontal component of the motion and the constant acceleration equations for the vertical component of the motion.

▶ **Section 1.4 Questions**

Understanding Concepts

1. What is the vertical acceleration of a projectile on its way up, at the top of its trajectory, and on its way down?

2. (a) For a projectile with the launch point lower than the landing point, in what part of the flight is the magnitude of the velocity at a maximum? a minimum?
   (b) In what part of the flight is the magnitude of the velocity at a maximum, and in what part is it at a minimum, for a projectile with the launch point higher than the landing point?

3. A projectile launched horizontally moves 16 m in the horizontal plane while falling 1.5 m in the vertical plane. Determine the projectile's initial velocity.

4. A tennis player serves a ball horizontally, giving it a speed of 24 m/s from a height of 2.5 m. The player is 12 m from the net. The top of the net is 0.90 m above the court surface. The ball clears the net and lands on the other side. Air resistance is negligible.

(a) For how long is the ball airborne?
(b) What is the horizontal displacement?
(c) What is the velocity at impact?
(d) By what distance does the ball clear the net?

5. A child throws a ball onto the roof of a house, then catches it with a baseball glove 1.0 m above the ground, as in **Figure 15**. The ball leaves the roof with a speed of 3.2 m/s.
   (a) For how long is the ball airborne after leaving the roof?
   (b) What is the horizontal distance from the glove to the edge of the roof?
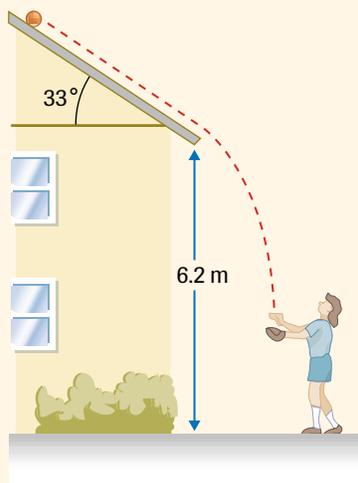   (c) What is the velocity of the ball just before it lands in the glove?

33°

6.2 m

Figure 15

6. For a projectile that lands at the same level from which it starts, state another launch angle above the horizontal that would result in the same range as a projectile launched at an angle of 36°, 16°, and 45.6°. Air resistance is negligible.

7. During World War I, the German army bombarded Paris with a huge gun referred to, by the Allied Forces, as "Big Bertha." Assume that Big Bertha fired shells with an initial velocity of 1.1×10^3 m/s [45° above the horizontal].
   (a) How long was each shell airborne, if the launch point was at the same level as the landing point?
   (b) Determine the maximum horizontal range of each shell.
   (c) Determine the maximum height of each shell.

8. An astronaut on the Moon, where $|\vec{g}| = 1.6$ m/s^2, strikes a golf ball giving the ball a velocity of 32 m/s [35° above the Moon's horizontal]. The ball lands in a crater floor that is 15 m below the level where it was struck. Determine
   (a) the maximum height of the ball
   (b) the time of flight of the ball
   (c) the horizontal range of the ball

Applying Inquiry Skills

9. A garden hose is held with its nozzle horizontally above the ground (**Figure 16**). The flowing water follows projectile motion. Given a metre stick and a calculator, describe how you would determine the speed of the water coming out of the nozzle.

Figure 16
Projectile motion in the garden

10. Describe how you would build and test a device made of simple, inexpensive materials to demonstrate that two coins launched simultaneously from the same level, one launched horizontally and the other dropped vertically, land at the same instant.

Making Connections

11. In real-life situations, projectile motion is often more complex than what has been presented in this section. For example, to determine the horizontal range of a shot in shot put competitions, the following equation is used:

$$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3$$

$$\Delta x = 0.30 \text{ m} + \frac{2v_i^2 \sin \theta \cos \theta}{g} + \frac{v_i \sin \theta \sqrt{v_i^2 \sin^2 \theta + |2 g \Delta y|}}{g}$$

where 0.30 m is the average distance the athlete's hand goes beyond the starting line, v_i is the magnitude of the initial velocity, θ is the angle of launch above the horizontal, Δy is the height above the ground where the shot leaves the hand, and g is the magnitude of the acceleration due to gravity (**Figure 17**).
   (a) Determine the range of a shot released 2.2 m above the ground with an initial velocity of 13 m/s [42° above the horizontal].
   (b) Compare your answer in (a) to the world record for the shot put (currently about 23.1 m).
   (c) Why do you think the equation given here differs from the equation for horizontal range derived in this section?
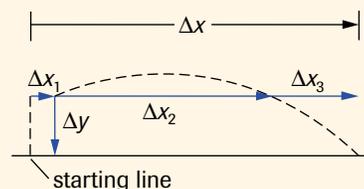

Figure 17