1.2 Acceleration in One and Two Dimensions

Have you noticed that when you are in a car, you have to accelerate on the ramp of an expressway to merge safely (**Figure 1**)? Drivers experience acceleration whenever their vehicles speed up, slow down, or change directions.

There have been concerns that vehicles using alternative energy resources may not be able to accelerate as quickly as vehicles powered by conventional fossil-fuel engines. However, new designs are helping to dispel that worry. For example, the electric limousine shown in **Figure 2** can quickly attain highway speeds.

Figure 1
As they merge onto the main expressway lanes, cars and motorcycles accelerate more easily than trucks.

Figure 2
This experimental electric limousine, with a mass of 3.0×10^3 kg, can travel 300 km on an hour's charge of its lithium battery.

DID YOU KNOW?

The Jerk
Sometimes the instantaneous acceleration varies, such as when rockets are launched into space. The rate of change of acceleration is known as the "jerk," which can be found using the relation jerk $= \frac{\Delta \vec{a}}{\Delta t}$ or by finding the slope of the line on an acceleration-time graph. What is the SI unit of jerk?

Acceleration and Average Acceleration in One Dimension

Acceleration is the rate of change of velocity. Since velocity is a vector quantity, acceleration is also a vector quantity. **Average acceleration**, $\vec{a}_{av}$, is the change in velocity divided by the time interval for that change:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

where $\vec{v}_f$ is the final velocity, $\vec{v}_i$ is the initial velocity, and Δt is the time interval.

The acceleration at any particular instant, or **instantaneous acceleration**—but often just referred to as acceleration—is found by applying the equation:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

In other words, as Δt approaches zero, the average acceleration $\left(\frac{\Delta \vec{v}}{\Delta t}\right)$ approaches the instantaneous acceleration.

As you can see from the following sample problems, any unit of velocity divided by a unit of time is a possible unit of average acceleration and instantaneous acceleration.

acceleration ($\vec{a}$) rate of change of velocity

average acceleration ($\vec{a}_{av}$) change in velocity divided by the time interval for that change

instantaneous acceleration acceleration at a particular instant

SAMPLE problem 1

A racing car accelerates from rest to 96 km/h [W] in 4.1 s. Determine the average acceleration of the car.

Solution

$\vec{v}_i = 0.0$ km/h

$\vec{v}_f = 96$ km/h [W]

$\Delta t = 4.1$ s

$\vec{a}_{av} = ?$

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$= \frac{96 \text{ km/h [W]} - 0.0 \text{ km/h}}{4.1 \text{ s}}$$

$$\vec{a}_{av} = 23 \text{ (km/h)/s [W]}$$

The average acceleration of the car is 23 (km/h)/s [W].

LEARNING TIP

Comparing Symbols

We have already been using the symbols $\vec{v}_{av}$ and $\vec{v}$ for average velocity and instantaneous velocity. In a parallel way, we use $\vec{a}_{av}$ and $\vec{a}$ for average acceleration and instantaneous acceleration. When the acceleration is constant, the average and instantaneous accelerations are equal, and the symbol $\vec{a}$ can be used for both.

SAMPLE problem 2

A motorcyclist travelling at 23 m/s [N] applies the brakes, producing an average acceleration of 7.2 m/s^2 [S].

(a) What is the motorcyclist's velocity after 2.5 s?

(b) Show that the equation you used in (a) is dimensionally correct.

Solution

(a) $\vec{v}_i = 23$ m/s [N]

$\vec{a}_{av} = 7.2 \text{ m/s}^2 \text{ [S]} = -7.2 \text{ m/s}^2 \text{ [N]}$

$\Delta t = 2.5$ s

$\vec{v}_f = ?$

From the equation $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$,

$$\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t$$

$$= 23 \text{ m/s [N]} + (-7.2 \text{ m/s}^2 \text{ [N]})(2.5 \text{ s})$$

$$= 23 \text{ m/s [N]} - 18 \text{ m/s [N]}$$

$$\vec{v}_f = 5 \text{ m/s [N]}$$

The motorcyclist's final velocity is 5 m/s [N].

(b) We can use a question mark above an equal sign $\left(\overset{?}{=}\right)$ to indicate that we are checking to see if the dimensions on the two sides of the equation are the same.

$$\vec{v}_f \overset{?}{=} \vec{v}_i + \vec{a}_{av}\Delta t$$

$$\frac{L}{T} \overset{?}{=} \frac{L}{T} + \left(\frac{L}{T^2}\right)T$$

$$\frac{L}{T} \overset{?}{=} \frac{L}{T} + \frac{L}{T}$$

The dimension on the left side of the equation is equal to the dimension on the right side of the equation.

LEARNING TIP

Positive and Negative Directions

In Sample Problem 2, the average acceleration of 7.2 m/s^2 [S] is equivalent to -7.2 m/s^2 [N]. In this case, the positive direction of the motion is north: $\vec{v}_i = +23$ m/s [N]. Thus, a positive southward acceleration is equivalent to a negative northward acceleration, and both represent slowing down. Slowing down is sometimes called *deceleration*, but in this text we will use the term "negative acceleration." This helps to avoid possible sign errors when using equations.

Answers

4. $2.4\ m/s^2$ [fwd]
5. (a) 2.80 s
   (b) 96.1 km/h
6. 108 km/h [fwd]
7. 42.8 m/s [E]

Practice

Understanding Concepts

1. Which of the following can be used as units of acceleration?
   (a) (km/s)/h (b) $mm \cdot s^{-2}$ (c) Mm/min^2 (d) km/h^2 (e) km/min/min
2. (a) Is it possible to have an eastward velocity with a westward acceleration? If "no," explain why not. If "yes," give an example.
   (b) Is it possible to have acceleration when the velocity is zero? If "no," explain why not. If "yes," give an example.
3. A flock of robins is migrating southward. Describe the flock's motion at instants when its acceleration is (a) positive, (b) negative, and (c) zero. Take the southward direction as positive.
4. A track runner, starting from rest, reaches a velocity of 9.3 m/s [fwd] in 3.9 s. Determine the runner's average acceleration.
5. The Renault Espace is a production car that can go from rest to 26.7 m/s with an incredibly large average acceleration of magnitude $9.52\ m/s^2$.
   (a) How long does the Espace take to achieve its speed of 26.7 m/s?
   (b) What is this speed in kilometres per hour?
   (c) Show that the equation you applied in (a) is dimensionally correct.
6. The fastest of all fishes is the sailfish. If a sailfish accelerates at a rate of 14 (km/h)/s [fwd] for 4.7 s from its initial velocity of 42 km/h [fwd], what is its final velocity?
7. An arrow strikes a target in an archery tournament. The arrow undergoes an average acceleration of $1.37 \times 10^3\ m/s^2$ [W] in 3.12×10^{-2} s, then stops. Determine the velocity of the arrow when it hits the target.

Graphing Motion with Constant Acceleration

A speedboat accelerates uniformly from rest for 8.0 s, with a displacement of 128 m [E] over that time interval. **Table 1** contains the position-time data from the starting position. **Figure 3** is the corresponding position-time graph.

Table 1 Position-Time Data

t (s)	$\vec{d}$ (m [E])
0	0
2.0	8.0
4.0	32
6.0	72
8.0	128

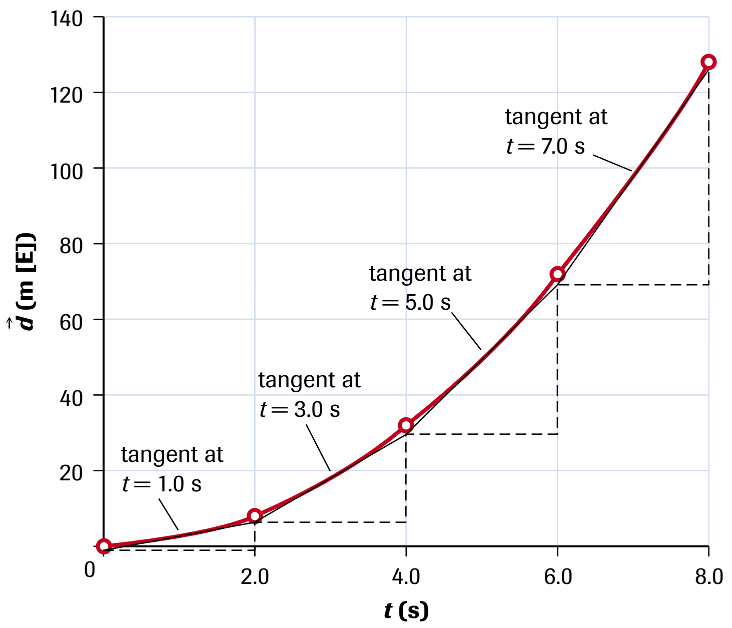

Figure 3
On this position-time graph of the boat's motion, the tangents at four different instants yield the instantaneous velocities at those instants. (The slope calculations are not shown here.)

Recall from Section 1.1 that the slope of the line at a particular instant on a position-time graph yields the instantaneous velocity. Since the slope continuously changes, several values are needed to determine how the velocity changes with time. One way to find the slope is to apply the "tangent technique" in which tangents to the curve are drawn at several instants and their slopes are calculated. **Table 2** shows the instantaneous velocities as determined from the slopes; **Figure 4** shows the corresponding velocity-time graph.

Table 2 Velocity-Time Data

t (s)	$\vec{v}$ (m/s [E])
0	0
1.0	4.0
3.0	12
5.0	20
7.0	28

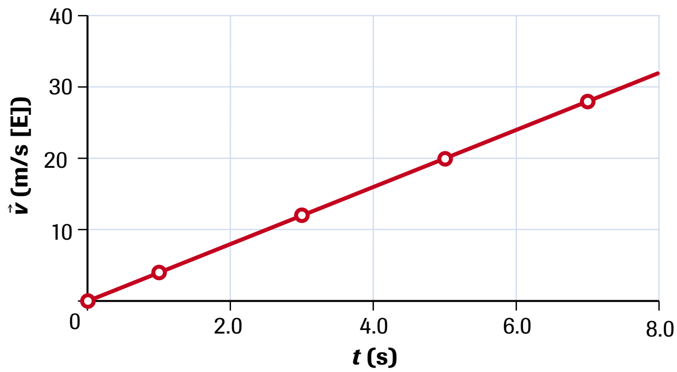

Figure 4
The velocity-time graph of motion with constant acceleration is a straight line. How would you find the instantaneous acceleration, average acceleration, and displacement of the boat from this graph?

The acceleration can be determined from the slope of the line on the velocity-time graph, which is $\frac{\Delta \vec{v}}{\Delta t}$. In this example, the slope, and thus the acceleration, is 4.0 m/s^2 [E]. **Figure 5** is the corresponding acceleration-time graph.

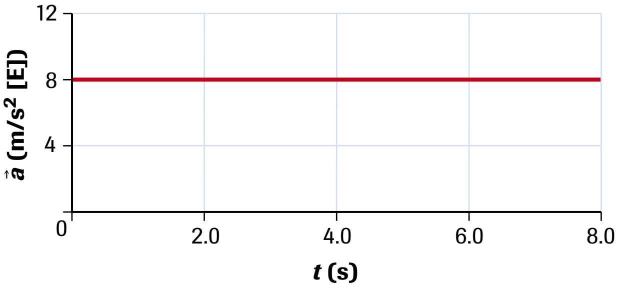

Figure 5
The acceleration-time graph of motion with constant acceleration is a straight horizontal line. How would you find the change in velocity over a given time interval from this graph?

What additional information can we determine from the velocity-time and acceleration-time graphs? As you discovered earlier, the area under the line on a velocity-time graph indicates the change in position (or the displacement) over the time interval for which the area is found. Similarly, the area under the line on an acceleration-time graph indicates the change in velocity over the time interval for which the area is calculated.

SAMPLE problem 3

Figure 6 is the acceleration-time graph of a car accelerating through its first three gears. Assume that the initial velocity is zero.

(a) Use the information in the graph to determine the final velocity in each gear. Draw the corresponding velocity-time graph.

(b) From the velocity-time graph, determine the car's displacement from its initial position after 5.0 s.

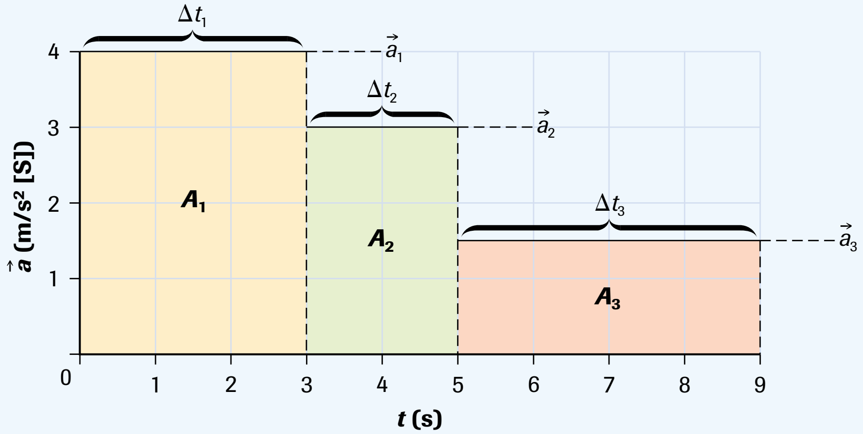

Figure 6
Acceleration-time graph

Solution

(a) The area beneath each segment of the acceleration-time plot is the change in velocity during that time interval.

$$A_1 = \vec{a}_1 \Delta t_1 = (4.0\ \text{m/s}^2\ [\text{S}])(3.0\ \text{s})$$
$$A_1 = 12\ \text{m/s [S]}$$

$$A_2 = \vec{a}_2 \Delta t_2 = (3.0\ \text{m/s}^2\ [\text{S}])(2.0\ \text{s})$$
$$A_2 = 6.0\ \text{m/s [S]}$$

$$A_3 = \vec{a}_3 \Delta t_3 = (1.5\ \text{m/s}^2\ [\text{S}])(4.0\ \text{s})$$
$$A_3 = 6.0\ \text{m/s [S]}$$

$$A_{total} = A_1 + A_2 + A_3 = 12\ \text{m/s} + 6.0\ \text{m/s} + 6.0\ \text{m/s}$$
$$A_{total} = 24\ \text{m/s}$$

The initial velocity is $\vec{v}_1 = 0.0$ m/s. The final velocity in first gear is $\vec{v}_2 = 12$ m/s [S], in second gear is $\vec{v}_3 = 18$ m/s [S], and in third gear is $\vec{v}_4 = 24$ m/s [S].

Figure 7 is the corresponding velocity-time graph.

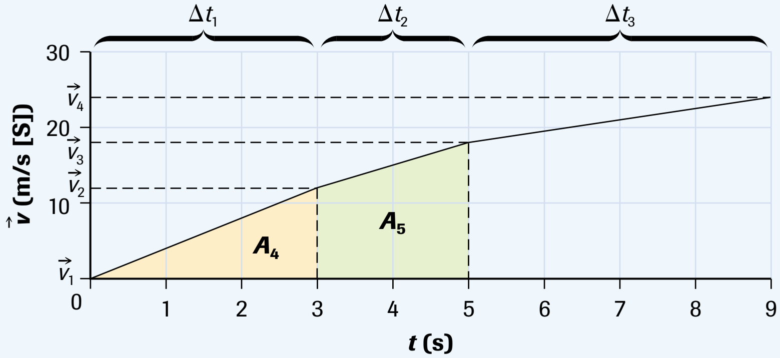

Figure 7
Velocity-time graph

(b) The area beneath each line on the velocity-time graph yields the change in position during that time interval.

$$A_4 = \frac{1}{2}\left(\vec{v}_2 - \vec{v}_1\right)(\Delta t_1)$$
$$= \frac{1}{2}(12 \text{ m/s [S]})(3.0 \text{ s})$$
$$A_4 = 18 \text{ m [S]}$$

$$A_5 = \left(\vec{v}_2\right)(\Delta t_2) + \frac{1}{2}\left(\vec{v}_3 - \vec{v}_2\right)(\Delta t_2)$$
$$= (12 \text{ m/s [S]})(2.0 \text{ s}) + \frac{1}{2}(18 \text{ m/s [S]} - 12 \text{ m/s [S]})(2.0 \text{ s})$$
$$A_5 = 30 \text{ m [S]}$$

(Area A_5 could also be found by using the equation for the area of a trapezoid.)
The car's displacement after 5.0 s is 18 m [S] + 30 m [S] = 48 m [S].

TRY THIS activity — Graphing Motion with Acceleration

The cart in **Figure 8** is given a brief push so that it rolls up an inclined plane, stops, then rolls back toward the bottom of the plane. A motion sensor is located at the bottom of the plane to plot position-time, velocity-time, and acceleration-time graphs. For the motion that occurs *after* the pushing force is no longer in contact with the cart, sketch the shapes of the $\vec{d}$-t, $\vec{v}$-t, and $\vec{a}$-t graphs for these situations:

(a) upward is positive
(b) downward is positive

Observe the motion and the corresponding graphs, and compare your predictions to the actual results.

Catch the cart on its downward motion before it reaches the motion sensor.

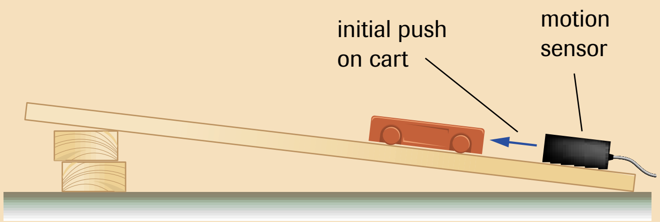

Figure 8
A motion sensor allows you to check predicted graphs.

Practice

Understanding Concepts

8. Describe how to determine
  (a) average acceleration from a velocity-time graph
  (b) change in velocity from an acceleration-time graph

9. Describe the motion depicted in each graph in **Figure 9**.

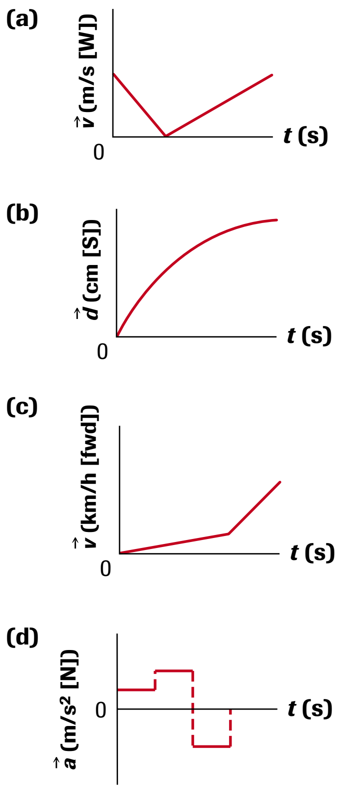

Figure 9
For question 9

Answer

12. 132 m [S]

10. **Table 3** summarizes observations of a crawling baby experiencing constant acceleration over several successive 2.0-s intervals.
(a) Draw a velocity-time graph for this motion.
(b) Use the information on your velocity-time graph to draw the corresponding acceleration-time graph.

Table 3 Data for Question 10

t (s)	0.0	2.0	4.0	6.0	8.0	10	12
$\vec{v}$ (cm/s [E])	10	15	20	15	10	5.0	0.0

11. **Figure 10** is the acceleration-time graph of a football lineman being pushed, from an initial velocity of zero, by other players. Draw the corresponding velocity-time graph.

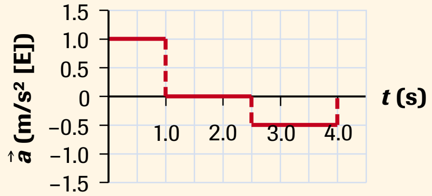

Figure 10
Acceleration-time graph

12. Determine the car's displacement after 9.0 s from the velocity-time graph in **Figure 7**.

Making Connections

13. The acceleration-time graphs in **Figures 6**, **9(b)**, and **10** represent idealized situations of constant acceleration.
(a) What does "idealized" mean here?
(b) Suggest one advantage of presenting idealized, rather than real-life, examples in a text of fundamental physics theory.
(c) Redraw the graph in **Figure 6** to suggest more accurately the real-life motion of an accelerating car.

14. Explore the capabilities of graphing tools in acceleration problems. You may have access to a graphing calculator and graphing software in such scientific packages as IDL®, Maple®, or Mathematica®. Your school may have a planimeter, a mechanical instrument for finding the area of paper under a plotted curve. You could begin by checking the answers to Sample Problem 3.

Solving Constant Acceleration Problems

The defining equation for average acceleration, $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$, does not include displacement. You have seen that displacement can be found by determining the area under the line on a velocity-time graph. We can combine this observation with the defining equation for average acceleration to derive other equations useful in analyzing motion with constant acceleration. Remember that when the acceleration is constant, $\vec{a} = \vec{a}_{av}$, so we use the symbol $\vec{a}$ to represent the acceleration.

Figure 11 shows a velocity-time graph of constant acceleration with initial velocity $\vec{v}_i$. The area beneath the line is the area of a trapezoid, $\Delta\vec{d} = \dfrac{1}{2}(\vec{v}_f + \vec{v}_i)\Delta t$. This equation, without the variable $\vec{a}$, can be combined with the defining equation for average acceleration to derive three other equations, each of which involves four of the five possible variables that are associated with constant acceleration.

For example, to derive the equation in which Δt is eliminated, we omit the vector notation; this allows us to avoid the mathematical problem that would occur if we mul-

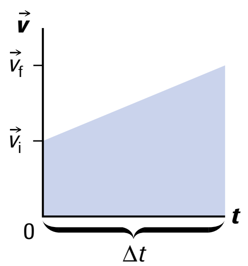

Figure 11
The shape of the figure under the line on this graph is a trapezoid, so the area under the line is the product of the average length of the two parallel sides, $\dfrac{\vec{v}_i + \vec{v}_f}{2}$, and the perpendicular distance between them, Δt.

tiplied two vectors. We can now rearrange the defining equation to get Δt, which we can substitute to solve for Δd:

$$a = \frac{v_f - v_i}{\Delta t}$$

$$\Delta t = \frac{v_f - v_i}{a}$$

$$\Delta d = \frac{1}{2}(v_f + v_i)\Delta t$$

$$= \frac{1}{2}(v_f + v_i)\left(\frac{v_f - v_i}{a}\right)$$

$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$

$$2a\Delta d = v_f^2 - v_i^2$$

Therefore $v_f^2 = v_i^2 + 2a\Delta d$.

In a similar way, substitution can be used to derive the final two equations in which $\vec{v}_f$ and $\vec{v}_i$ are eliminated. The resulting five equations for constant acceleration are presented in **Table 4**. Applying dimensional analysis or unit analysis to the equations will allow you to check that the derivations and/or substitutions are valid.

Table 4 Constant Acceleration Equations for Uniformly Accelerated Motion

Variables Involved	General Equation	Variable Eliminated
$\vec{a}, \vec{v}_f, \vec{v}_i, \Delta t$	$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$	$\Delta\vec{d}$
$\Delta\vec{d}, \vec{v}_i, \vec{a}, \Delta t$	$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$	$\vec{v}_f$
$\Delta\vec{d}, \vec{v}_i, \vec{v}_f, \Delta t$	$\Delta\vec{d} = \vec{v}_{av}\Delta t$ or $\Delta\vec{d} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$	$\vec{a}$
$\vec{v}_f, \vec{v}_i, \vec{a}, \Delta\vec{d}$	$v_f^2 = v_i^2 + 2a\Delta d$	Δt
$\Delta\vec{d}, \vec{v}_f, \Delta t, \vec{a}$	$\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$	$\vec{v}_i$

SAMPLE problem 4

A motorcyclist, travelling initially at 12 m/s [W], changes gears and speeds up for 3.5 s with a constant acceleration of 5.1 m/s^2 [W]. What is the motorcyclist's displacement over this time interval?

Solution

$\vec{v}_i$ = 12 m/s [W] $\qquad \Delta t$ = 3.5 s

$\vec{a}$ = 5.1 m/s^2 [W] $\qquad \Delta\vec{d}$ = ?

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$= (12\text{ m/s [W]})(3.5\text{ s}) + \frac{1}{2}(5.1\text{ m/s}^2\text{ [W]})(3.5\text{ s})^2$$

$$\Delta\vec{d} = 73\text{ m [W]}$$

The motorcyclist's displacement is 73 m [W].

SAMPLE problem 5

A rocket launched vertically from rest reaches a velocity of 6.3×10^2 m/s [up] at an altitude of 4.7 km above the launch pad. Determine the rocket's acceleration, which is assumed constant, during this motion.

Solution

$\vec{v}_i = 0$ m/s　　　$\Delta\vec{d} = 4.7$ km [up] $= 4.7 \times 10^3$ m [up]

$\vec{v}_f = 6.3 \times 10^2$ m/s [up]　　　$\vec{a} = ?$

We choose [up] as the positive direction. Since Δt is not given, we will use the equation

$$
\begin{aligned}
v_f^2 &= v_i^2 + 2a\Delta d \\
v_f^2 &= 2a\Delta d \\
a &= \frac{v_f^2}{2\Delta d} \\
&= \frac{(6.3 \times 10^2 \text{ m/s})^2}{2(4.7 \times 10^3 \text{ m})} \\
a &= 42 \text{ m/s}^2
\end{aligned}
$$

Since a is positive, the acceleration is 42 m/s^2 [up].

SAMPLE problem 6

A curling rock sliding on ice undergoes a constant acceleration of 5.1 cm/s^2 [E] as it travels 28 m [W] from its initial position before coming to rest. Determine (a) the initial velocity and (b) the time of travel.

Solution

Figure 12 shows that the acceleration is opposite in direction to the motion of the rock and that the positive direction is chosen to be west.

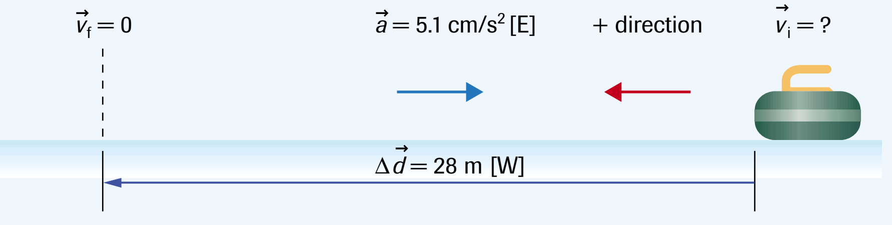

Figure 12
The situation for Sample Problem 6

(a) $\Delta\vec{d} = 28$ m [W]　　　$\vec{a} = 5.1$ cm/s^2 [E] $= 0.051$ m/s^2 [E] $= -0.051$ m/s^2 [W]

$\vec{v}_f = 0$ m/s　　　$\Delta t = ?$

$\vec{v}_i = ?$

$$
\begin{aligned}
v_f^2 &= v_i^2 + 2a\Delta d \\
0 &= v_i^2 + 2a\Delta d \\
v_i^2 &= -2a\Delta d \\
v_i &= \pm\sqrt{-2a\Delta d} \\
&= \pm\sqrt{-2(-0.051 \text{ m/s}^2)(28 \text{ m})} \\
v_i &= \pm 1.7 \text{ m/s}
\end{aligned}
$$

The initial velocity is $v_i = 1.7$ m/s [W].

(b) Any of the constant acceleration equations can be used to solve for Δt.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\Delta t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}}$$

$$= \frac{0 - 1.7 \text{ m/s [W]}}{-0.051 \text{ m/s}^2 \text{ [W]}}$$

$$\Delta t = 33 \text{ s}$$

The time interval over which the curling rock slows down and stops is 33 s.

Practice

Understanding Concepts

15. You know the initial velocity, the displacement, and the time interval for a certain constant acceleration motion. Which of the five standard equations would you use to find (a) acceleration and (b) final velocity?

16. Show that the constant acceleration equation from which Δt has been eliminated is dimensionally correct.

17. Rearrange the constant acceleration equation from which average acceleration has been eliminated to isolate (a) Δt and (b) $\vec{v}_f$.

18. Starting with the defining equation for constant acceleration and the equation for displacement in terms of average velocity, derive the constant acceleration equation
- (a) from which final velocity has been eliminated
- (b) from which initial velocity has been eliminated

19. A badminton shuttle, or "birdie," is struck, giving it a horizontal velocity of 73 m/s [W]. Air resistance causes a constant acceleration of 18 m/s^2 [E]. Determine its velocity after 1.6 s.

20. A baseball travelling horizontally at 41 m/s [S] is hit by a baseball bat, causing its velocity to become 47 m/s [N]. The ball is in contact with the bat for 1.9 ms, and undergoes constant acceleration during this interval. What is that acceleration?

21. Upon leaving the starting block, a sprinter undergoes a constant acceleration of 2.3 m/s^2 [fwd] for 3.6 s. Determine the sprinter's (a) displacement and (b) final velocity.

22. An electron travelling at 7.72×10^7 m/s [E] enters a force field that reduces its velocity to 2.46×10^7 m/s [E]. The acceleration is constant. The displacement during the acceleration is 0.478 m [E]. Determine
- (a) the electron's acceleration
- (b) the time interval over which the acceleration occurs

Applying Inquiry Skills

23. Describe how you would perform an experiment to determine the acceleration of a book sliding to a stop on a lab bench or the floor. Which variables will you measure and how will you calculate the acceleration? If possible, perform the experiment.

Making Connections

24. Reaction time can be crucial in avoiding a car accident. You are driving at 75.0 km/h [N] when you notice a stalled vehicle 48.0 m directly ahead of you. You apply the brakes, coming to a stop just in time to avoid a collision. Your brakes provided a constant acceleration of 4.80 m/s^2 [S]. What was your reaction time?

Answers

19. 44 m/s [W]
20. 4.6×10^4 m/s^2 [N]
21. (a) 15 m [fwd]
    (b) 8.3 m/s [fwd]
22. (a) 5.60×10^{15} m/s^2 [W]
    (b) 9.39×10^{-9} s
24. 0.13 s

Acceleration in Two Dimensions

Acceleration in two dimensions occurs when the velocity of an object moving in a plane undergoes a change in magnitude, or a change in direction, or a simultaneous change in both magnitude and direction. In **Figure 13**, a parks worker is pushing a lawn mower on a level lawn at a constant speed of 1.8 m/s around a kidney-shaped flowerbed. Is the mower accelerating? Yes: the mower's velocity keeps changing in direction, even though it does not change in magnitude.

The equation for average acceleration introduced for one-dimensional motion also applies to two-dimensional motion. Thus,

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

It is important to remember that $\vec{v}_f - \vec{v}_i$ is a vector subtraction. The equation can also be applied to the components of vectors. Thus,

$$a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{fx} - v_{ix}}{\Delta t} \quad \text{and} \quad a_{av,y} = \frac{\Delta v_y}{\Delta t} = \frac{v_{fy} - v_{iy}}{\Delta t}$$

where, for example, v_{fy} represents the y-component of the final velocity.

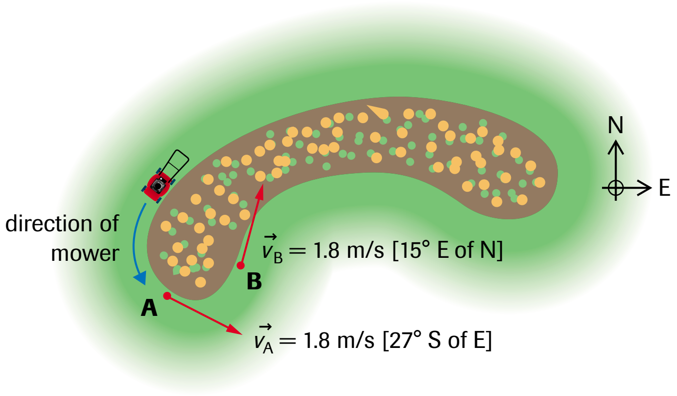

Figure 13
As the lawn mower follows the edge of the flowerbed at constant speed, it is accelerating: its direction of motion keeps changing.

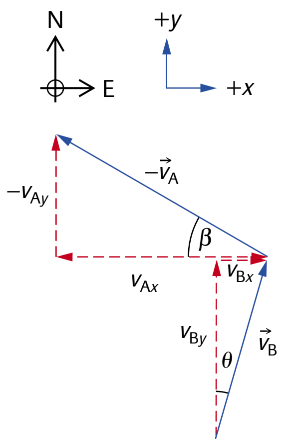

Figure 14
Determining the direction of the change in the velocity vector

SAMPLE problem 7

The lawn mower in **Figure 13** takes 4.5 s to travel from A to B. What is its average acceleration?

Solution

$\vec{v}_A$ = 1.8 m/s [27° S of E] Δt = 4.5 s

$\vec{v}_B$ = 1.8 m/s [15° E of N] $\vec{a}_{av}$ = ?

We begin by finding $\Delta\vec{v}$, which is needed in the equation for average acceleration. In this case, we choose to work with vector components, although other methods could be used (such as the sine and cosine laws). The vector subtraction, $\Delta\vec{v} = \vec{v}_B + (-\vec{v}_A)$, is shown in **Figure 14**. Taking components:

$$\begin{aligned}\Delta v_x &= v_{Bx} + (-v_{Ax}) \\ &= v_B \sin\theta + (-v_A \cos\beta) \\ &= 1.8\text{ m/s }(\sin 15°) - 1.8\text{ m/s }(\cos 27°) \\ \Delta v_x &= -1.1\text{ m/s}\end{aligned}$$

and

$$\begin{aligned}\Delta v_y &= v_{By} + (-v_{Ay}) \\ &= v_B \cos\theta + (-v_A \sin\beta) \\ &= 1.8 \text{ m/s } (\cos 15°) + 1.8 \text{ m/s } (\sin 27°) \\ \Delta v_y &= +2.6 \text{ m/s}\end{aligned}$$

Using the law of Pythagoras,

$$\begin{aligned}|\Delta\vec{v}|^2 &= |\Delta v_x|^2 + |\Delta v_y|^2 \\ |\Delta\vec{v}|^2 &= (1.1 \text{ m/s})^2 + (2.6 \text{ m/s})^2 \\ |\Delta\vec{v}| &= 2.8 \text{ m/s}\end{aligned}$$

We now find the direction of the vector as shown in **Figure 15**:

$$\begin{aligned}\phi &= \tan^{-1}\frac{1.1 \text{ m/s}}{2.6 \text{ m/s}} \\ \phi &= 24°\end{aligned}$$

The direction is [24° W of N].

To calculate the average acceleration:

$$\begin{aligned}\vec{a}_{av} &= \frac{\Delta\vec{v}}{\Delta t} \\ &= \frac{2.8 \text{ m/s [24° W of N]}}{4.5 \text{ s}} \\ \vec{a}_{av} &= 0.62 \text{ m/s}^2 \text{ [24° W of N]}\end{aligned}$$

The average acceleration is 0.62 m/s^2 [24° W of N].

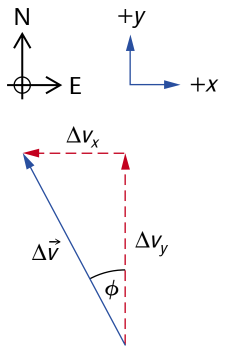

Figure 15
The velocities and their components for Sample Problem 7

Practice

Understanding Concepts

25. A car with a velocity of 25 m/s [E] changes its velocity to 25 m/s [S] in 15 s. Calculate the car's average acceleration.

26. A watercraft with an initial velocity of 6.4 m/s [E] undergoes an average acceleration of 2.0 m/s^2 [S] for 2.5 s. What is the final velocity of the watercraft?

27. A hockey puck rebounds from a board as shown in **Figure 16**. The puck is in contact with the board for 2.5 ms. Determine the average acceleration of the puck over the interval.

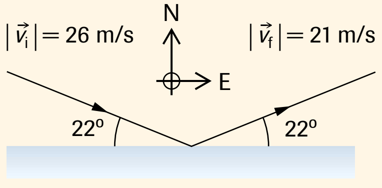

Figure 16
Motion of the puck

28. A passenger in a hot-air balloon throws a ball with an initial unknown velocity. The ball accelerates at 9.8 m/s^2 [down] for 2.0 s, at which time its instantaneous velocity is 24 m/s [45° below the horizontal]. Determine the ball's initial velocity.

29. At 3:00 P.M. a truck travelling on a winding highway has a velocity of 82.0 km/h [38.2° E of N]; at 3:15 P.M., it has a velocity of 82.0 km/h [12.7° S of E]. Assuming that $+x$ is east and $+y$ is north, determine the x- and y-components of the average acceleration during this time interval.

Answers

25. 2.4 m/s^2 [45° S of W]
26. 8.1 m/s [38° S of E]
27. 7.3×10^3 m/s^2 [7.5° N of W]
28. 17 m/s [10° above the horizontal]
29. $a_{av,x} = 9.0 \times 10^{-3}$ m/s^2; $a_{av,y} = -2.5 \times 10^{-2}$ m/s^2

SUMMARY Acceleration in One and Two Dimensions

- Average acceleration is the average rate of change of velocity.
- Instantaneous acceleration is the acceleration at a particular instant.
- The tangent technique can be used to determine the instantaneous velocity on a position-time graph of accelerated motion.
- The slope of the line on a velocity-time graph indicates the acceleration.
- The area under the line on an acceleration-time graph indicates the change in velocity.
- There are five variables involved in the mathematical analysis of motion with constant acceleration and there are five equations, each of which has four of the five variables.
- In two-dimensional motion, the average acceleration is found by using the vector subtraction $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$ divided by the time interval Δt.

Section 1.2 Questions

Understanding Concepts

1. State the condition under which instantaneous acceleration and average acceleration are equal.
2. Is it possible to have a northward velocity with westward acceleration? If "no," explain why not. If "yes," give an example.
3. A supersonic aircraft flying from London, England, to New York City changes its velocity from 1.65×10^3 km/h [W] to 1.12×10^3 km/h [W] as it prepares for landing. This change takes 345 s. Determine the average acceleration of the aircraft (a) in kilometres per hour per second and (b) in metres per second squared.
4. (a) Sketch a velocity-time graph, over an interval of 4.0 s, for a car moving in one dimension with increasing speed and decreasing acceleration.
   (b) Show how to determine the instantaneous acceleration at $t = 2.0$ s on this graph.
5. **Table 5** gives position-time data for a person undergoing constant acceleration from rest.
   (a) Draw the corresponding velocity-time and acceleration-time graphs.
   (b) Use at least one constant acceleration equation to check the final calculation of acceleration in (a).

Table 5 Position-Time Data

t (s)	0	0.2	0.4	0.6	0.8
$\vec{d}$ (m [W])	0	0.26	1.04	2.34	4.16

6. Describe the motion in each graph in **Figure 17**.

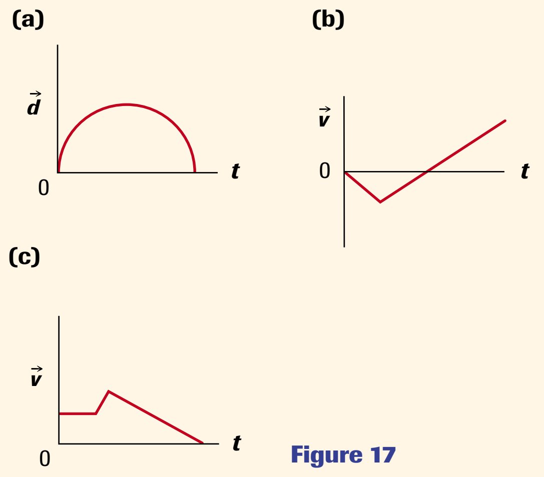

Figure 17

7. A car, travelling initially at 26 m/s [E], slows down with a constant average acceleration of magnitude 5.5 m/s^2. Determine its velocity after 2.6 s.
8. The maximum braking acceleration of a certain car is constant and of magnitude 9.7 m/s^2. If the car comes to rest 2.9 s after the driver applies the brakes, determine its initial speed.

9. Use the information from the velocity-time graph in **Figure 18** to generate the corresponding position-time and acceleration-time graphs.

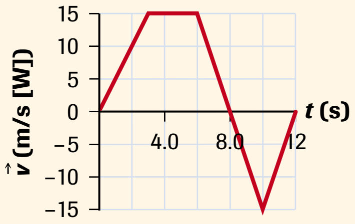

Figure 18

10. A ski jumper, starting from rest, skis down a straight slope for 3.4 s with a constant acceleration of 4.4 m/s^2 [fwd]. At the end of 3.4 s, determine the jumper's (a) final velocity and (b) displacement.

11. An electron is accelerated uniformly from rest to a velocity of 2.0×10^7 m/s [E] over the displacement 0.10 m [E].
(a) What is the (constant) acceleration of the electron?
(b) How long does the electron take to reach its final velocity?

12. During a 29.4-s interval, the velocity of a rocket changes from 204 m/s [fwd] to 508 m/s [fwd]. Assuming constant acceleration, determine the displacement of the rocket during this time interval.

13. A bullet leaves the muzzle of a rifle with a velocity of 4.2×10^2 m/s [fwd]. The rifle barrel is 0.56 m long. The acceleration imparted by the gunpowder gases is uniform as long as the bullet is in the barrel.
(a) What is the average velocity of the bullet in the barrel?
(b) Over what time interval does the uniform acceleration occur?

14. A car (C) and a van (V) are stopped beside each other at a red light. When the light turns green, the vehicles accelerate with the motion depicted in **Figure 19**.

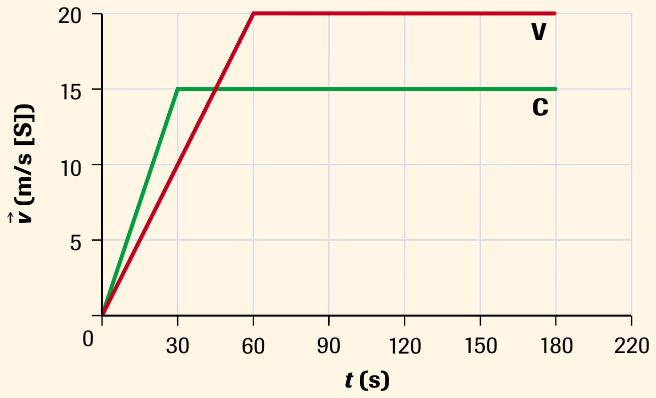

Figure 19
Velocity-time graph of the motions of two vehicles

(a) At what instant after the light turns green do C and V have the same velocity?
(b) At what instant after the light turns green does V overtake C? (*Hint:* Their displacements must be equal at that instant.)
(c) Determine the displacement from the intersection when V overtakes C.

15. A bird takes 8.5 s to fly from position A to position B along the path shown in **Figure 20**. Determine the average acceleration.

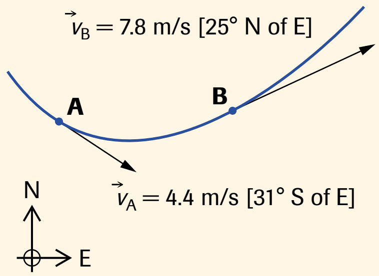

Figure 20

16. A helicopter travelling horizontally at 155 km/h [E] executes a gradual turn, and after 56.5 s flies at 118 km/h [S]. What is the helicopter's average acceleration in kilometres per hour per second?

Applying Inquiry Skills

17. Predict the average acceleration in each of the following situations. State what measurements and calculations you would use to test your predictions.
(a) A bullet travelling with a speed of 175 m/s is brought to rest by a wooden plank.
(b) A test car travelling at 88 km/h is brought to rest by a crash barrier consisting of sand-filled barrels.

Making Connections

18. The fastest time for the women's 100-m dash in a certain track-and-field competition is 11.0 s, whereas the fastest time for the four-woman 100-m relay is 42.7 s. Why would it be wrong to conclude that each of the four women in the relay can run a 100-m dash in less than 11.0 s? (*Hint:* Consider acceleration.)