1.1 Speed and Velocity in One and Two Dimensions

Visitors to an amusement park, such as the one in **Figure 1**, experience a variety of motions. Some people may walk at a constant speed in a straight line. Others, descending on a vertical drop ride, plummet at a very high speed before slowing down and stopping. All these people undergo motion in one dimension, or *linear motion*. Linear motion can be in the horizontal plane (walking on a straight track on level ground, for example), in the vertical plane (taking the vertical drop ride), or on an inclined plane (walking up a ramp). Linear motion can also involve changing direction by 180°, for example, in going up and then down, or moving east and then west on level ground.

Figure 1
How many different types of motion can you identify at this amusement park?

Visitors to an amusement park also experience motion in two and three dimensions. Riders on a merry-go-round experience two-dimensional motion in the horizontal plane. Riders on a Ferris wheel experience two-dimensional motion in the vertical plane. Riders on a roller coaster experience motion in three dimensions: up and down, left and right, around curves, as well as twisting and rotating.

The study of motion is called **kinematics**. To begin kinematics, we will explore simple motions in one and two dimensions, like those in **Figure 2**. Later, we will apply what we have learned to more complex types of motion.

kinematics the study of motion

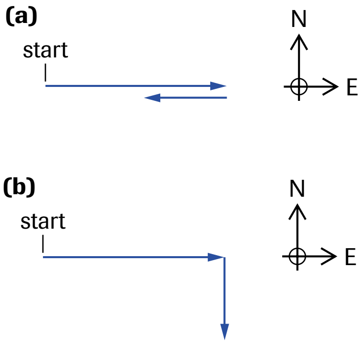


Figure 2
(a) A dog running on level ground 24 m eastward and then 11 m westward undergoes motion in one dimension.
(b) A dog running on level ground 24 m eastward and then 11 m southward undergoes motion in two dimensions.

Speed and Other Scalar Quantities

Consider the speed limits posted on roads and highways near where you live. In a school zone, the limit may be posted as 40 km/h, while 100 km/h may be permitted on a highway. The unit km/h (kilometres per hour) reminds you that speed is a distance divided by a time. Speed, distance, and time are examples of **scalar quantities**, which have magnitude (or size) but no direction.

scalar quantity quantity that has magnitude but no direction

In car racing, the starting lineup is determined by qualifying time trials in which the average speeds of the drivers are compared. Each driver must cover the same distance around the track, and the driver with the shortest time gets the pole (or first) position. During some parts of the qualifying trials, other drivers may have achieved a higher **instantaneous speed**, or speed at a particular instant. But the winner has the greatest

instantaneous speed speed at a particular instant

average speed, or total distance travelled divided by total time of travel. The equation for average speed is

$$v_{av} = \frac{d}{\Delta t}$$

where d is the total distance travelled in a total time Δt.

average speed (v_{av}) total distance of travel divided by total time of travel

LEARNING TIP

Scalar Quantities

"Scalar" stems from the Latin *scalae*, which means "steps" or "ladder rungs," and suggests a magnitude or value. Scalars can be positive, zero, or negative.

SAMPLE problem 1

At the Molson Indy race in Toronto, Ontario, a driver covers a single-lap distance of 2.90 km at an average speed of 1.50×10^2 km/h. Determine

(a) the average speed in metres per second

(b) the time in seconds to complete the lap

Solution

(a) To convert units, we multiply by factors that are equivalent to 1. We know that

1 km = 1000 m and 1 h = 3600 s

$$\therefore 1.50 \times 10^2 \text{ km/h} = 1.50 \times 10^2 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 41.7 \text{ m/s}$$

The average speed is 41.7 m/s.

(b) $v_{av} = 41.7$ m/s

$d = 2.90$ km $= 2.90 \times 10^3$ m

$\Delta t = ?$

Rearranging the equation $v_{av} = \frac{d}{\Delta t}$ to isolate Δt, we have

$$\Delta t = \frac{d}{v_{av}}$$

$$= \frac{2.90 \times 10^3 \text{ m}}{41.7 \text{ m/s}}$$

$$\Delta t = 69.6 \text{ s}$$

The time to complete the lap is 69.6 s. (Refer to the Learning Tip, Significant Digits and Rounding.)

LEARNING TIP

The Average Speed Equation

In the equation $v_{av} = \frac{d}{\Delta t}$, the symbol v comes from the word velocity (a vector quantity) and the subscript "av" indicates average. The Greek letter Δ (delta) denotes the change in a quantity, in this case time. The symbol t usually represents a time at which an event occurs, and Δt represents the time between events, or the elapsed time.

LEARNING TIP

Significant Digits and Rounding

In all sample problems in this text—take a close look at Sample Problem 1—the answers have been rounded off to the appropriate number of significant digits. Take special care in answering a question with two or more parts. For example, when working on a two-part problem, keep the intermediate (excess-precision) answer for part (a) in your calculator so that you can use it to solve part (b) without rounding error. Appendix A reviews the rules for significant digits and rounding-off.

Practice

Understanding Concepts

1. For each of the following motions, state whether the number of dimensions is one, two, or three:
   (a) A tennis ball falls vertically downward after you release it from rest in your hand.
   (b) A tennis ball falls vertically downward after you release it from rest in your hand, hits the court, and bounces directly upward.
   (c) A basketball flies in a direct arc into the hoop.
   (d) A baseball pitcher tosses a curveball toward the batter.
   (e) A passenger on a Ferris wheel travels in a circle around the centre of the wheel.
   (f) A roller coaster moves around the track.

2. Which of the following measurements are scalar quantities?
(a) the force exerted by an elevator cable
(b) the reading on a car's odometer
(c) the gravitational force of Earth on you
(d) the number of physics students in your class
(e) your age

3. Does a car's speedometer indicate average speed or instantaneous speed? Is the indicated quantity a scalar or a vector? Explain.

4. In the Indianapolis 500 auto race, drivers must complete 200 laps of a 4.02-km (2.50-mile) track. Calculate and compare the average speeds in kilometres per hour, given the following times for 200 laps:
(a) 6.69 h (in 1911, the first year of the race)
(b) 3.32 h (in 1965)
(c) 2.69 h (in 1990, still a record more than a decade later)

5. A swimmer crosses a circular pool of diameter 16 m in 21 s.
(a) Determine the swimmer's average speed.
(b) How long would the swimmer take to swim, at the same average speed, around the edge of the pool?

6. Determine the total distance travelled in each case:
(a) Sound propagating at 342 m/s crosses a room in 3.54×10^{-2} s.
(b) Thirty-two scuba divers take turns riding an underwater tricycle at an average speed of 1.74 km/h for 60.0 h. (Express your answer both in kilometres and in metres.)

Answers

4. (a) 1.20×10^2 km/h
   (b) 2.42×10^2 km/h
   (c) 2.99×10^2 km/h
5. (a) 0.76 m/s
   (b) 66 s
6. (a) 12.1 m
   (b) 104 km; 1.04×10^5 m

DID YOU KNOW?

The Origin of "Vector"
Vector comes from the Latin word *vector*, which has as one of its meanings "carrier"—implying something being carried from one place to another in a certain direction. In biology, a vector is a carrier of disease.

Velocity and Other Vector Quantities

Many of the quantities we measure involve a direction. A **vector quantity** is a quantity with a magnitude and direction. Position, displacement, and velocity are common vector quantities in kinematics. In this text, we indicate a vector quantity algebraically by a symbol containing an arrow with the direction stated in square brackets. Examples of directions and their specifications are east [E], upward [up], downward [down], and forward [fwd].

Position, $\vec{d}$, is the directed distance of an object from a reference point. **Displacement**, $\Delta\vec{d}$, is the change of position—that is, the final position minus the initial position. In **Figure 3**, a cyclist initially located 338 m west of an intersection moves to a new position 223 m west of the same intersection.

vector quantity quantity that has both magnitude and direction

position ($\vec{d}$) the distance and direction of an object from a reference point

displacement ($\Delta\vec{d}$) change in position of an object in a given direction

LEARNING TIP

The Magnitude of a Vector
The symbol $|\;|$ surrounding a vector represents the magnitude of the vector. For example, $|\Delta\vec{d}|$ represents the distance, or magnitude, without indicating the direction of the displacement; thus it is a scalar quantity. For example, if $\Delta\vec{d}$ is 15 m [E], then $|\Delta\vec{d}|$ is 15 m.

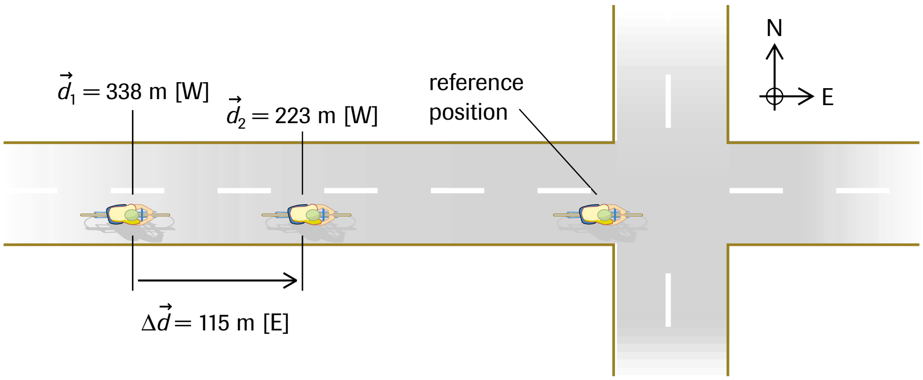


Figure 3
In moving from position $\vec{d}_1$ to $\vec{d}_2$, the cyclist undergoes a displacement $\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$.

We can determine the cyclist's displacement as follows:

$$\begin{aligned}\Delta\vec{d} &= \vec{d}_2 - \vec{d}_1 \\ &= 223 \text{ m [W]} - 338 \text{ m [W]} \\ &= -115 \text{ m [W]} \\ \Delta\vec{d} &= 115 \text{ m [E]}\end{aligned}$$

The quantities "−115 m [W]" and "115 m [E]" denote the same vector.

A fundamental vector quantity involving position and time is **velocity**, or rate of change of position. The velocity at any particular instant is called the **instantaneous velocity**, $\vec{v}$. If the velocity is constant (so that the moving body travels at an unchanging speed in an unchanging direction), the position is said to change uniformly with time, resulting in *uniform motion.*

The **average velocity**, $\vec{v}_{av}$, of a motion is the change of position divided by the time interval for that change. From this definition, we can write the equation

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

where $\Delta\vec{d}$ is the displacement (or change of position) and Δt is the time interval. For motion with constant velocity, the average velocity is equal to the instantaneous velocity at any time.

SAMPLE problem 2

The cyclist in **Figure 3** takes 25.1 s to cover the displacement of 115 m [E] from $\vec{d}_1$ to $\vec{d}_2$.

(a) Calculate the cyclist's average velocity.

(b) If the cyclist maintains the same average velocity for 1.00 h, what is the total displacement?

(c) If the cyclist turns around at $\vec{d}_2$ and travels to position $\vec{d}_3 = 565$ m [W] in 72.5 s, what is the average velocity for the entire motion?

Solution

(a) $\Delta\vec{d} = 115$ m [E]

$\Delta t = 25.1$ s

$\vec{v}_{av} = ?$

$$\begin{aligned}\vec{v}_{av} &= \frac{\Delta\vec{d}}{\Delta t} \\ &= \frac{115 \text{ m [E]}}{25.1 \text{ s}} \\ \vec{v}_{av} &= 4.58 \text{ m/s [E]}\end{aligned}$$

The cyclist's average velocity is 4.58 m/s [E].

(b) $\Delta t = 1.00 \text{ h} = 3600 \text{ s}$

$\vec{v}_{av} = 4.58$ m/s [E]

$\Delta\vec{d} = ?$

$$\begin{aligned}\Delta\vec{d} &= \vec{v}_{av}\Delta t \\ &= (4.58 \text{ m/s [E]})(3600 \text{ s}) \\ \Delta\vec{d} &= 1.65 \times 10^4 \text{ m [E] or } 16.5 \text{ km [E]}\end{aligned}$$

The total displacement is 16.5 km [E].

DID YOU KNOW?

Comparing Displacements

The displacement from Quebec City to Montreal is 250 km [41° S of W]. The displacement from Baltimore, Maryland, to Charlottesville, Virginia, is 250 km [41° S of W]. Since both displacements have the same magnitude (250 km) and direction [41° S of W], they are the same vectors. Vectors with the same magnitude and direction are identical, even though their starting positions can be different.

velocity ($\vec{v}$) the rate of change of position

instantaneous velocity velocity at a particular instant

average velocity ($\vec{v}_{av}$) change of position divided by the time interval for that change

LEARNING TIP

Properties of Vectors

A vector divided by a scalar, as in the equation $\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$, is a vector. Multiplying a vector by a scalar also yields a vector. Appendix A discusses vector arithmetic.

LEARNING TIP

Unit and Dimensional Analysis

Unit analysis (using such units as metres, kilograms, and seconds) or dimensional analysis (using such dimensions as length, mass, and time) can be useful to ensure that both the left-hand side and the right-hand side of any equation have the same units or dimensions. Try this with the equation used to solve Sample Problem 2(b). If the units or dimensions are not the same, there must be an error in the equation. For details, see Appendix A.

DID YOU KNOW?

Other Direction Conventions

In navigation, directions are taken clockwise from due north. For example, a direction of 180° is due south, and a direction of 118° is equivalent to the direction [28° S of E]. In mathematics, angles are measured counterclockwise from the positive *x*-axis.

Answers

10. (a) 3.0×10^1 km/h
    (b) 3.0×10^1 km/h [E]
    (c) 0.0 km/h
11. 8.6 m [fwd]
12. 7.6×10^2 h; 32 d

Figure 4
A typical windsock

(c) $\Delta\vec{d} = \vec{d}_3 - \vec{d}_1$

$= 565$ m [W] $- 338$ m [W]

$\Delta\vec{d} = 227$ m [W]

$\Delta t = 25.1 \text{ s} + 72.5 \text{ s} = 97.6 \text{ s}$

$\vec{v}_{av} = ?$

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} = \frac{227 \text{ m [W]}}{97.6 \text{ s}}$$

$\vec{v}_{av} = 2.33$ m/s [W]

The average velocity is 2.33 m/s [W].
(Can you see why this average velocity is so much less than the average velocity in (a)?)

Practice

Understanding Concepts

7. Give specific examples of three different types of vector quantities that you have experienced today.

8. (a) Is it possible for the total distance travelled to equal the magnitude of the displacement? If "no," why not? If "yes," give an example.
(b) Is it possible for the total distance travelled to exceed the magnitude of the displacement? If "no," why not? If "yes," give an example.
(c) Is it possible for the magnitude of the displacement to exceed the total distance travelled? If "no," why not? If "yes," give an example.

9. Can the average speed ever equal the magnitude of the average velocity? If "no," why not? If "yes," give an example.

10. A city bus leaves the terminal and travels, with a few stops, along a straight route that takes it 12 km [E] of its starting position in 24 min. In another 24 min, the bus turns around and retraces its path to the terminal.
(a) What is the average speed of the bus for the entire route?
(b) Calculate the average velocity of the bus from the terminal to the farthest position from the terminal.
(c) Find the average velocity of the bus for the entire route.
(d) Why are your answers for (b) and (c) different?

11. A truck driver, reacting quickly to an emergency, applies the brakes. During the driver's 0.32 s reaction time, the truck maintains a constant velocity of 27 m/s [fwd]. What is the displacement of the truck during the time the driver takes to react?

12. The Arctic tern holds the world record for bird migration distance. The tern migrates once a year from islands north of the Arctic Circle to the shores of Antarctica, a displacement of approximately 1.6×10^4 km [S]. (The route, astonishingly, lies mainly over water.) If a tern's average velocity during this trip is 21 km/h [S], how long does the journey take? (Answer both in hours and days.)

Applying Inquiry Skills

13. Small airports use windsocks, like the one in **Figure 4**.
(a) Does a windsock indicate a scalar quantity or a vector quantity? What is that quantity?
(b) Describe how you would set up an experiment to help you calibrate the windsock.

Position and Velocity Graphs

Graphing provides a useful way of studying motion. We begin by studying position-time and velocity-time graphs for bodies with constant velocity motion.

Consider a marathon runner moving along a straight road with a constant velocity of 5.5 m/s [S] for 3.0 min. At the start of the motion (i.e., at $t = 0$), the initial position is $\vec{d} = 0$. The corresponding position-time data are shown in **Table 1**. The corresponding position-time graph is shown in **Figure 5**. Notice that for constant velocity motion, the position-time graph is a straight line.

Since the line on the position-time graph has a constant slope, we can calculate the slope since it is the ratio of the change in the quantity on the vertical axis to the corresponding change in the quantity on the horizontal axis. Thus, the slope of the line on the position-time graph from $t = 0.0$ s to $t = 180$ s is

$$m = \frac{\Delta\vec{d}}{\Delta t}$$

$$= \frac{990 \text{ m [S]} - 0 \text{ m}}{180 \text{ s} - 0 \text{ s}}$$

$$m = 5.5 \text{ m/s [S]}$$

This value would be the same no matter which part of the line we used to calculate the slope. It is apparent that for constant velocity motion, the average velocity is equal to the instantaneous velocity at any particular time and that both are equal to the slope of the line on the position-time graph.

Figure 6 is the corresponding velocity-time graph of the runner's motion. A practice question will ask you to show that the area under the plot (the shaded area) represents the displacement, in other words, represents $\Delta\vec{d}$ over the time interval that the area covers.

Table 1 Position-Time Data

Time t (s)	Position $\vec{d}$ (m [S])
0	0
60	330
120	660
180	990

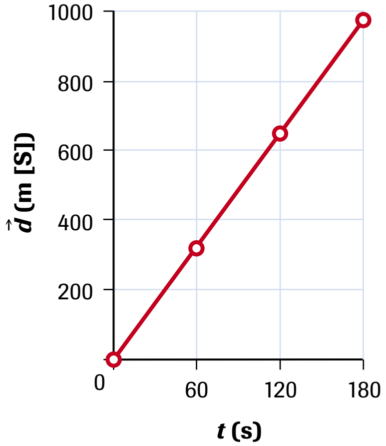


Figure 5
Position-time graph of the runner's motion

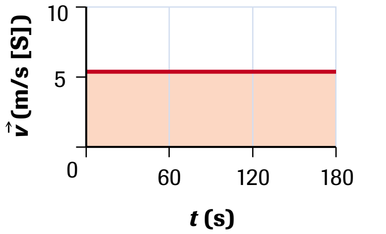


Figure 6
Velocity-time graph of the runner's motion

SAMPLE problem 3

Describe the motion represented by the position-time graph shown in **Figure 7**, and sketch the corresponding velocity-time graph.

Solution

The slope of the line is constant and it is negative. This means that the velocity is constant in the easterly direction. The initial position is away from the origin and the object is moving toward the origin. The velocity-time graph can be either negative west or positive east as shown in **Figure 8**.

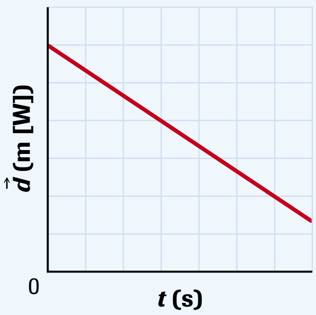


Figure 7
Position-time graph

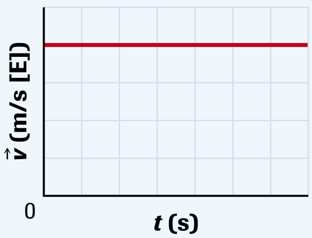


Figure 8
Velocity-time graph

Table 2 Position-Time Data

Time t (s)	Position $\vec{d}$ (m [fwd])
0	0
2.0	4.0
4.0	16
6.0	36
8.0	64

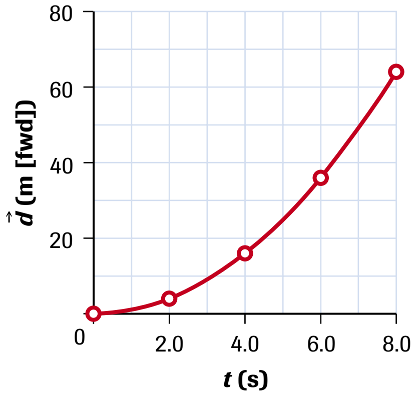


Figure 9
Position-time graph for changing instantaneous velocity. The average velocity between any two times can be found by applying the equation $\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$, but a different approach must be used to find the instantaneous velocity.

Now let's turn to graphs of motion with changing instantaneous velocity. This type of motion, often called *nonuniform motion*, involves a change of direction, a change of speed, or both.

Consider, for example, a car starting from rest and speeding up, as in **Table 2** and **Figure 9**.

Since the slope of the line on the position-time graph is gradually increasing, the velocity is also gradually increasing. To find the slope of a curved line at a particular instant, we draw a straight line touching—but not cutting—the curve at that point. This straight line is called a **tangent** to the curve. The slope of the tangent to a curve on a position-time graph is the instantaneous velocity.

Figure 10 shows the tangent drawn at 2.0 s for the motion of the car. The broken lines in the diagram show the average velocities between $t = 2.0$ s and later times. For example, from $t = 2.0$ s to $t = 8.0$ s, $\Delta t = 6.0$ s and the average velocity is the slope of line A. From $t = 2.0$ s to $t = 6.0$ s, $\Delta t = 4.0$ s and the average velocity is the slope of line B, and so on. Notice that as Δt becomes smaller, the slopes of the lines get closer to the slope of the tangent at $t = 2.0$ s (i.e., they get closer to the instantaneous velocity, $\vec{v}$). Thus, we can define the instantaneous velocity as

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{d}}{\Delta t}$$

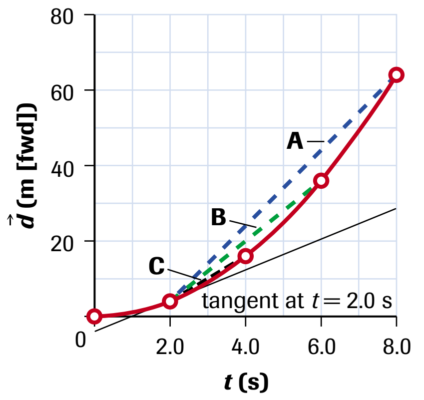


Figure 10
The slopes of lines A, B, and C represent the average velocities at times beyond 2.0 s. As these times become smaller, the slopes get closer to the slope of the tangent at $t = 2.0$ s.

tangent a straight line that touches a curve at a single point and has the same slope as the curve at that point

LEARNING TIP

Calculus Notation
In the notation used in calculus, the "Δ" symbols are replaced by "d" symbols to represent infinitesimal or very small quantities. Thus, the equation for instantaneous velocity is $\vec{v} = \frac{d\vec{d}}{dt}$.

TRY THIS activity — **Graphing Linear Motion**

Sketch the position-time and velocity-time graphs that you think correspond to each of the following situations. After discussing your graphs with your group, use a motion sensor connected to graphing software to check your predictions. Comment on the accuracy of your predictions.

(a) A person walks away from the sensor at a constant velocity for 5 or 6 steps.
(b) A person walks directly toward the sensor at a constant velocity from a distance of about 4.0 m.
(c) A person walks directly toward the sensor at a constant velocity from 3.0 m away. The person stops for a few seconds. Finally, the person walks directly back toward the origin at a constant, but slower velocity.
(d) A person walks halfway from the origin directly toward the sensor at a high constant velocity, stops briefly, walks the rest of the way at a low constant velocity, and then returns at a high constant velocity to the origin.

SAMPLE problem 4

Figure 11 is the position-time graph for a golf ball rolling along a straight trough which slopes downward from east to west. We arbitrarily choose one-dimensional coordinates on which the origin is at the western end of the trough.

(a) Describe the motion.

(b) Calculate the instantaneous velocity at t = 3.0 s.

(c) Determine the average velocity between 3.0 s and 6.0 s.

Solution

(a) The slope is zero at t = 0.0 s, then it becomes negative. Thus, the velocity starts off at zero and gradually increases in magnitude in the westerly direction. (Negative east is equivalent to positive west.) The object starts at a position east of the reference point or origin and then moves westward arriving at the origin 6.0 s later.

(b) The instantaneous velocity at t = 3.0 s is the slope of the tangent at that instant. Thus,

$$\vec{v} = \text{slope} = m = \frac{\Delta \vec{d}}{\Delta t}$$

$$= \frac{0.0\text{ m} - 24\text{ m [E]}}{8.0\text{ s} - 0.0\text{ s}}$$

$$= -3.0\text{ m/s [E]}$$

$$\vec{v} = 3.0\text{ m/s [W]}$$

The instantaneous velocity at 3.0 s is approximately 3.0 m/s [W]. (This answer is approximate because of the uncertainty of drawing the tangent.)

(c) We apply the equation for average velocity:

$$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$

$$= \frac{0.0\text{ m} - 15\text{ m [E]}}{6.0\text{ s} - 3.0\text{ s}}$$

$$= -5.0\text{ m/s [E]}$$

$$\vec{v}_{av} = 5.0\text{ m/s [W]}$$

The average velocity between 3.0 s and 6.0 s is 5.0 m/s [W].

Practice

Understanding Concepts

14. Describe the motion depicted in each of the graphs in **Figure 12**.

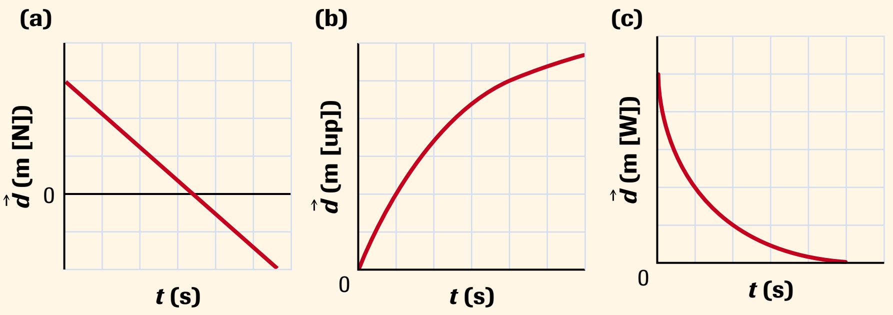


Figure 12

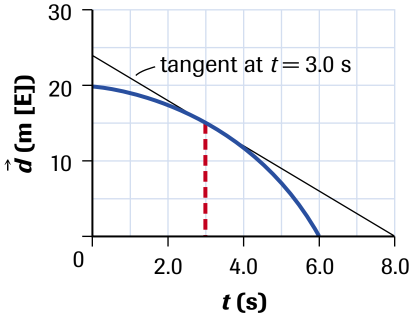


Figure 11
Position-time graph for Sample Problem 4

LEARNING TIP

The Image of a Tangent Line
A plane mirror can be used to draw a tangent to a curved line. Place the mirror as perpendicular as possible to the line at the point desired. Adjust the angle of the mirror so that the real curve merges smoothly with its image in the mirror, which allows the mirror to be perpendicular to the curved line at that point. Draw a line perpendicular to the mirror to obtain the tangent to the curve.

LEARNING TIP

Limitations of Calculator Use
Calculators provide answers very quickly, but you should always think about the answers they provide. Inverse trig functions, such as $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$, provide an example of the limitations of calculator use. In the range of 0° to 360°, there are two angles with the same sine, cosine, or tangent. For example, sin 85° = sin 95° = 0.966, and cos 30° = cos 330° = 0.866. Thus, you must decide on how to interpret answers given by a calculator.

Answers

16. 4.5 m [N]
17. 7 m/s [E]; 0 m/s; 7 m/s [W]; 13 m/s [W]; 7 m/s [W]

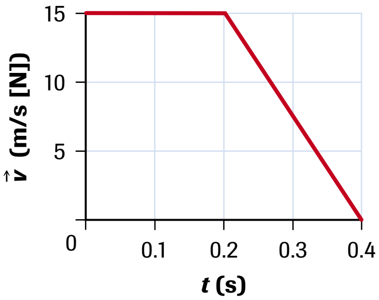


Figure 14
Velocity-time graph for question 16

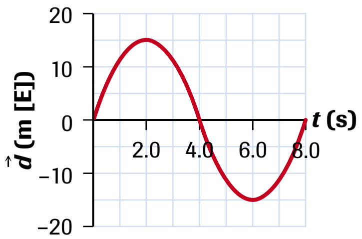
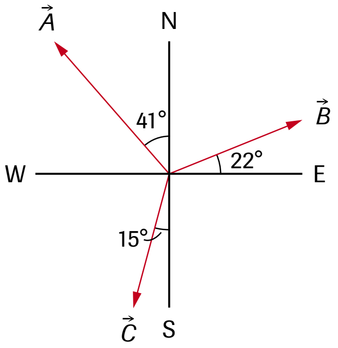


Figure 15
Position-time graph for question 17



directions of vectors:
$\vec{A}$ [41° W of N]
$\vec{B}$ [22° N of E]
$\vec{C}$ [15° W of S]

Figure 16
Notation for specifying the directions of vectors

15. Use the information on the graphs in **Figure 13** to generate the corresponding velocity-time graphs.

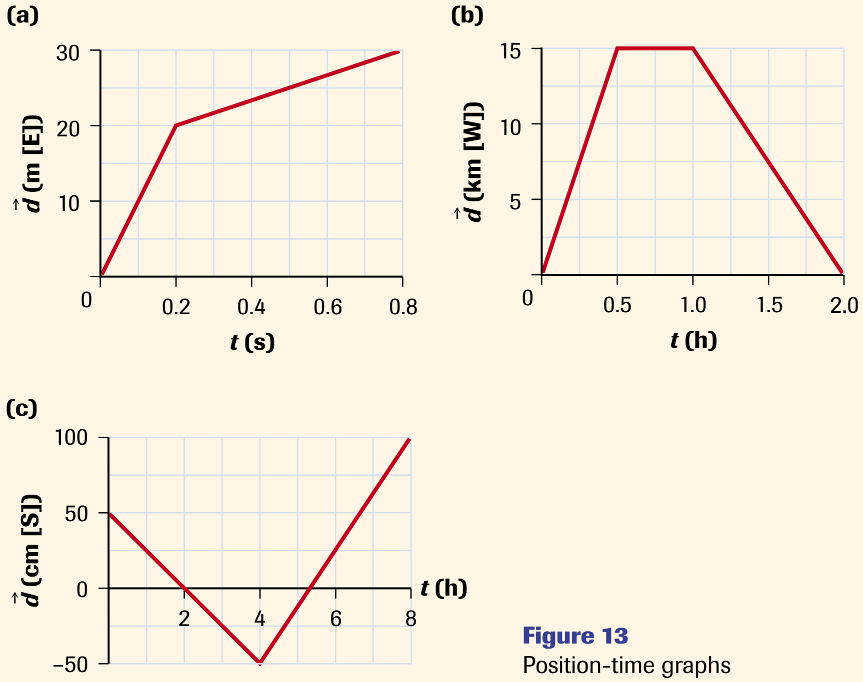


Figure 13
Position-time graphs

16. Determine the area between the line and the horizontal axis on the velocity-time graph in **Figure 14**. What does that area represent? (*Hint:* Include units in your area calculation.)

17. Redraw the position-time graph in **Figure 15** in your notebook and determine the (approximate) instantaneous velocities at t = 1.0 s, 2.0 s, 3.0 s, 4.0 s, and 5.0 s.

Displacement and Velocity in Two Dimensions

As you are driving north on a highway in level country, you come to a bridge closed for repairs. Your destination is across the bridge on the north side of the river. Using a map, you discover a road that goes eastward, then northward across the river, then westward to your destination. The concepts of displacement, velocity, and time interval help you analyze the alternative route as a vector problem in the horizontal plane. You can also analyze motion in a vertical plane (as when a football moves in the absence of a crosswind) or in a plane at an angle to the horizontal (such as a ski hill) in the same way.

In the horizontal plane, the four compass points—east, north, west, and south—indicate direction. If the displacement or the velocity is at an angle between any two compass points, the direction must be specified in some unambiguous way. In this text, the direction of a vector will be indicated using the angle measured from one of the compass points (**Figure 16**).

The defining equations for displacement ($\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$), average velocity $\left(\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}\right)$, and instantaneous velocity $\left(\vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{d}}{\Delta t}\right)$ apply to motion in two dimensions. However, where the two-dimensional motion analyzed involves more than one displacement, as shown in **Figure 17**, $\Delta\vec{d}$ is the result of successive displacements ($\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \ldots$), and is called *total displacement*.

SAMPLE problem 5

In 4.4 s, a chickadee flies in a horizontal plane from a fence post (P) to a bush (B) and then to a bird feeder (F), as shown in **Figure 18(a)**. Find the following:

(a) total distance travelled
(b) average speed
(c) total displacement
(d) average velocity

Solution

(a) The total distance travelled is a scalar quantity.

$d = 22\text{ m} + 11\text{ m} = 33\text{ m}$

(b) $d = 33\text{ m}$

$\Delta t = 4.4\text{ s}$

$v_{av} = ?$

$$v_{av} = \frac{d}{\Delta t} = \frac{33\text{ m}}{4.4\text{ s}}$$

$$v_{av} = 7.5\text{ m/s}$$

The average speed is 7.5 m/s.

(c) We will use the method of sine and cosine laws to solve this problem. (Alternatively, we could use the component technique or a vector scale diagram.) We apply the cosine law to find the magnitude of the displacement, $|\Delta\vec{d}|$. From **Figure 18(b)**, the angle B equals 119°.

$|\Delta\vec{d}_1| = 22\text{ m}$ $\quad\measuredangle B = 119°$

$|\Delta\vec{d}_2| = 11\text{ m}$ $\quad|\Delta\vec{d}| = ?$

Applying the cosine law:

$$|\Delta\vec{d}|^2 = |\Delta\vec{d}_1|^2 + |\Delta\vec{d}_2|^2 - 2|\Delta\vec{d}_1||\Delta\vec{d}_2|\cos B$$

$$|\Delta\vec{d}|^2 = (22\text{ m})^2 + (11\text{ m})^2 - 2(22\text{ m})(11\text{ m})(\cos 119°)$$

$$|\Delta\vec{d}| = 29\text{ m}$$

To determine the direction of the displacement, we use the sine law:

$$\frac{\sin P}{|\Delta\vec{d}_2|} = \frac{\sin B}{|\Delta\vec{d}|}$$

$$\sin P = \frac{|\Delta\vec{d}_2|\sin B}{|\Delta\vec{d}|}$$

$$\sin P = \frac{(11\text{ m})(\sin 119°)}{(29\text{ m})}$$

$$\measuredangle P = 19°$$

From the diagram, we see that the direction of the total displacement is $33° - 19° = 14°$ N of E. Therefore, the total displacement is 29 m [14° N of E].

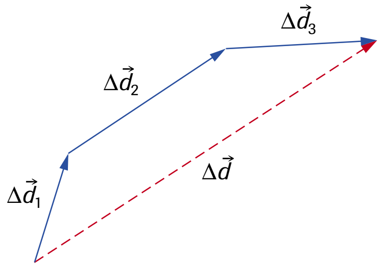


Figure 17
The total displacement is the vector sum of the individual displacements, $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$. Notice that the vectors are added head-to-tail, and the total vector faces from the initial position to the final position.

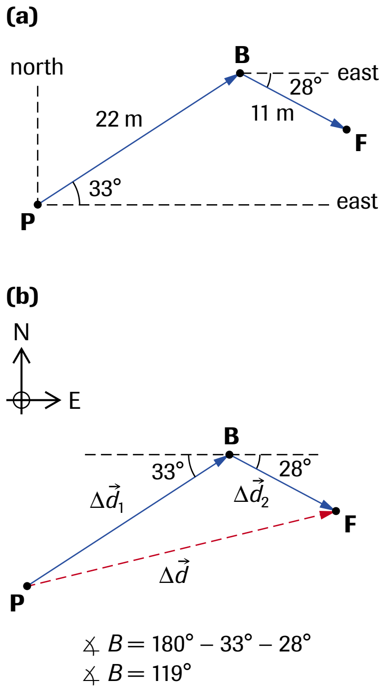


Figure 18
For Sample Problem 5
(a) The chickadee takes 4.4 s to complete the motion shown.
(b) Angle B is found to be 119°.

LEARNING TIP

Using Scientific Calculators
A warning about using scientific calculators: when first turned on, these calculators usually express angles in degrees (DEG). Pushing the appropriate key (e.g., DRG) will change the units to radians (RAD) or grads (GRA, where 90° = 100 grads). Only degrees will be used in this text.

(d) $\Delta\vec{d} = 29$ m [14° N of E]

$\Delta t = 4.4$ s

$\vec{v}_{av} = ?$

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

$$= \frac{29 \text{ m [14° N of E]}}{4.4 \text{ s}}$$

$$\vec{v}_{av} = 6.6 \text{ m/s [14° N of E]}$$

The average velocity is 6.6 m/s [14° N of E].

Practice

Understanding Concepts

18. Determine the vector sum of the displacements $\Delta\vec{d}_1 = 2.4$ m [32° S of W]; $\Delta\vec{d}_2 = 1.6$ m [S]; and $\Delta\vec{d}_3 = 4.9$ m [27° S of E].

19. Solve Sample Problem 5 using
(a) a vector scale diagram
(b) components (referring, if necessary, to Appendix A)

20. A skater on Ottawa's Rideau Canal travels in a straight line 8.5×10^2 m [25° N of E] and then 5.6×10^2 m in a straight line [21° E of N]. The entire motion takes 4.2 min.
(a) What is the skater's displacement?
(b) What are the skater's average speed and average velocity?

Answers

18. 5.6 m [24° E of S]

20. (a) 1.3×10^3 m [42° N of E]
(b) 5.6 m/s; 5.2 m/s [42° N of E]

LEARNING TIP

Adding Vectors
In applying the vector addition equation ($\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \ldots$) to two-dimensional motion, you can choose to add the displacement vectors by whichever of the methods summarized in Appendix A proves most convenient. The vector scale "head-to-tail" diagram method is excellent for visualizing and understanding the situation. However, this method is not as accurate as other methods. The component technique is accurate and can be used for any number of vectors, but it can be time-consuming. The method using the sine and cosine laws is accurate and fairly quick to use, but it is limited to the addition (or subtraction) of only two vectors.

SUMMARY Speed and Velocity in One and Two Dimensions

- A scalar quantity has magnitude but no direction.
- Average speed is the total distance travelled divided by the total time of travel.
- A vector quantity has both magnitude and direction.
- Position is the distance with a direction from some reference point.
- Displacement is the change of position.
- Velocity is the rate of change of position.
- Average velocity is change of position divided by the time interval for that change.
- Instantaneous velocity is the velocity at a particular instant.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- The slope of the line on a position-time graph indicates the velocity.
- The area under the line on a velocity-time graph indicates the change of position.
- In two-dimensional motion, the average velocity is the total displacement divided by the time interval for that displacement.

Section 1.1 Questions

Understanding Concepts

1. State whether each of the following is a scalar or a vector:
   (a) the magnitude of a vector quantity
   (b) a component of a vector quantity in some particular coordinate system
   (c) the mass you gained in the past 15 years
   (d) the product of a scalar and a vector
   (e) the area under the line and above the time axis on a velocity-time graph
2. Give a specific example for each of the following descriptions of a possible motion:
   (a) The velocity is constant.
   (b) The speed is constant, but the velocity is constantly changing.
   (c) The motion is in one dimension, and the total distance travelled exceeds the magnitude of the displacement.
   (d) The motion is in one dimension, the average speed is greater than zero, and the average velocity is zero.
   (e) The motion is in two dimensions, the average speed is greater than zero, and the average velocity is zero.
3. If two measurements have different dimensions, can they be added? multiplied? In each case, give an explanation if "no," an example if "yes."
4. Light travels in a vacuum at 3.00×10^8 m/s. Determine the time in seconds for each of the following:
   (a) Light travels from the Sun to Earth. The average radius of Earth's orbit around the Sun is 1.49×10^{11} m.
   (b) Laser light is sent from Earth, reflects off a mirror on the Moon, and returns to Earth. The average Earth-Moon distance is 3.84×10^5 km.
5. **Figure 19** shows the idealized motion of a car.
   (a) Determine the average speed between 4.0 s and 8.0 s; and between 0.0 s and 8.0 s.
   (b) Calculate the average velocity between 8.0 s and 9.0 s; between 12 s and 16 s; and between 0.0 s and 16 s.
   (c) Find the instantaneous speed at 6.0 s and 9.0 s.
   (d) Calculate the instantaneous velocity at 14 s.

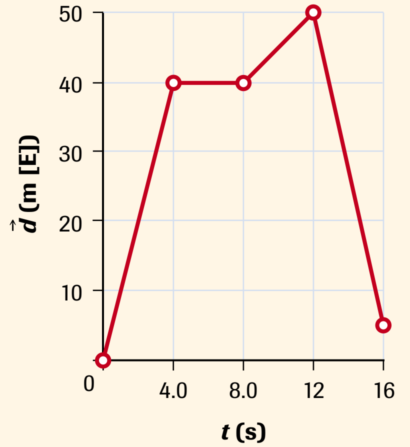


Figure 19
Position-time graph

6. What quantity can be calculated from a position-time graph to indicate the velocity of an object? How can that quantity be found if the line on the graph is curved?
7. Use the information in **Figure 20** to generate the corresponding position-time graph, assuming the position at time $t = 0$ is 8.0 m [E].

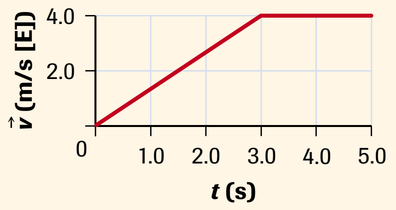


Figure 20
Velocity-time graph

8. In a total time of 2.0 min, a duck on a pond paddles 22 m [36° N of E] and then paddles another 65 m [25° E of S]. Determine the duck's
   (a) total distance travelled
   (b) average speed
   (c) total displacement
   (d) average velocity

Applying Inquiry Skills

9. (a) Review your work in Practice question 17, and use a plane mirror to determine how accurately you drew the tangents used to find the instantaneous velocities.
   (b) Describe how to draw tangents to curved lines as accurately as possible.

Making Connections

10. Research has shown that the average alcohol-free driver requires about 0.8 s to apply the brakes after seeing an emergency. **Figure 21** shows the approximate reaction times for drivers who have been drinking beer. Copy **Table 3** into your notebook, and use the data from the graph to determine the reaction distance (i.e., the distance travelled after seeing the emergency and before applying the brakes).

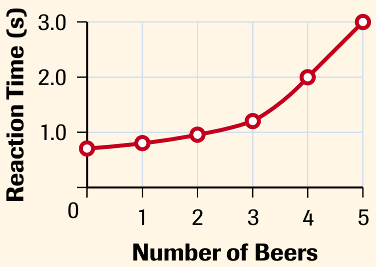


Figure 21
Effect of beer on reaction times for drivers

Table 3 Data for Question 10

Speed	Reaction Distance		
	no alcohol	***4 bottles***	***5 bottles***
17 m/s (60 km/h)	?	?	?
25 m/s (90 km/h)	?	?	?
33 m/s (120 km/h)	?	?	?