Pressure, Density, Stokes Law and More Forces!

Learning Goal: see above, lots to do.

Density

Density is the amount of mass an object has per unit volume.

ie. kg/m^3, g/cm^3, lbs/ft^3

Material	Density (g/cm^3)
Water	1.00
Air	0.0012
Ice	0.92
Aluminum	2.70
Steel	7.85
Copper	8.96
Lead	11.34
Gold	19.32
Mercury	13.56
Concrete	2.40

Water is one to know:

$1000 kg/m^3$ $1\ g/cm^3$ $62.4\ lbs/ft^3$

Volumes and Formulas

Shape	Volume Formula
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Rectangular Prism	$V = l \times w \times h$

- r = radius
- h = height
- l = length
- w = width

Sphere focus

$$V = \frac{4}{3}\pi r^3$$

$d = 2r$

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$r = d/2$

$$V = \frac{4}{3}\pi\left(\frac{d^3}{8}\right)$$

$$V = \frac{\pi}{6}d^3$$

Mass as a Product of Density

m mass => kg

ρ density => kg/m^3

V volume => m^3

$$m = \rho \cdot V$$

Weight (force-gravity) as a Product of Density

$$F = \rho \cdot V \cdot g$$

Allows for the mass and force to be written in terms of the physical dimensions of the object versus using a fixed size.

Pressure - Force over an Area

Pressure is defined as the force exerted per unit area. It quantifies how much force is applied to a specific area, and is commonly expressed in units such as pascals (Pa), atmospheres (atm), or pounds per square inch (psi). Mathematically, pressure (P) can be represented by the formula:

$$P = \frac{F}{A}$$

where:

- F = force applied,
- A = area over which the force is distributed.

In essence, pressure describes how concentrated a force is across a given surface.

Metric => $P = \frac{N}{m^2} = Pa$ Pascal

kPa (kiloPascal)

Imperial => $P = \frac{lb}{in^2} = psi$

Atmospheric Pressure

Just like the water in a pool causes pressure on your ear drums at the bottom, the column of air above you causes pressure at the surface of the Earth. This is how you breath, you inflate because of this pressure.

The standard atmospheric pressure is commonly defined as:

- Pascals (Pa): $101,325\,\mathrm{Pa}$
- Atmospheres (atm): $1\,\mathrm{atm}$
- Millimeters of Mercury (mmHg): $760\,\mathrm{mmHg}$
- Torr: $760\,\mathrm{Torr}$
- Pounds per square inch (psi): $14.7\,\mathrm{psi}$
- Kilopascals (kPa): $101.325\,\mathrm{kPa}$

These values represent the pressure at sea level under standard conditions.

High Pressure Systems:

- **Characteristics:** Often associated with sinking air and clear skies.
- **Weather Effects:** Generally bring fair weather, light winds, and stable atmospheric conditions. Clouds are minimal, and precipitation is unlikely.
- **Temperature**: Can lead to warmer conditions during the day due to sunlight but cooler temperatures at night due to radiational cooling.

Low Pressure Systems:

- **Characteristics:** Associated with rising air, leading to cooling and condensation.
- **Weather Effects:** Typically bring cloudy skies, precipitation (rain, snow, etc.), and stronger winds. These systems are often linked to storms and unsettled weather.
- **Temperature**: Can cause cooler conditions, especially with cloud cover and rain.

In summary, high pressure generally means nice weather, while low pressure is often linked to storms and inclement weather.

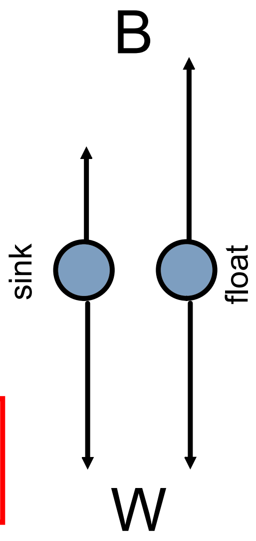
Buoyancy Overview:

Buoyancy is the upward force exerted by a fluid (liquid or gas) that opposes the weight of an object submerged in it. This force allows objects to float or sink in a fluid based on their density relative to the fluid.

Key Concepts:

- **Archimedes' Principle**: States that an object submerged in a fluid experiences a buoyant force equal to the weight of the fluid it displaces. This principle helps explain why some objects float while others sink.

Forces Involved:



1. **Weight of the Object (W)**: The downward force due to gravity, calculated as:

$$W = mg$$

where m is the mass of the object and g is the acceleration due to gravity.

2. **Buoyant Force (B)**: The upward force exerted by the fluid, calculated as:

$$B = \rho_{fluid} \cdot V_{displaced} \cdot g$$

$$F_b = \rho_f \cdot V \cdot g$$

where:

- ρ_{fluid} = density of the fluid,
- $V_{displaced}$ = volume of fluid displaced by the object,
- g = acceleration due to gravity.

Conditions for Floating and Sinking:

- **Float**: An object will float if its weight is less than or equal to the buoyant force ($W \leq B$).
- **Sink**: An object will sink if its weight is greater than the buoyant force ($W > B$).

Summary:

Buoyancy explains how objects behave in fluids, with the key forces being the weight of the object and the buoyant force determined by the displaced fluid. The balance between these forces dictates whether an object sinks or floats.

Ex. A large floating mat is used to play on the lake surface. The mat is 2m x 4m x 0.1m

What is the largest mass that can be placed on the mat before it sinks. The mass of the mat is 20 kg.

Solution

Find V of mat as if it is completely submerged.

Find buoyancy force associated with mat.

Find the Weight of the mat and objects on the mat (20 + m).

Set up an Fnet statement.

Previously we looked at the forces on a sky diver. One of those forces we called a Drag Force. We will now amend that name to include Viscous Force.

Viscous Force Overview:

Viscous force is the resistance exerted by a fluid against the motion of an object moving through it or the internal flow of the fluid itself. This force arises due to the fluid's viscosity, which is a measure of a fluid's resistance to deformation or flow.

Key Concepts:

- **Viscosity** (η): A measure of a fluid's resistance to flow. It describes how "thick" or "sticky" a fluid is. Higher viscosity means greater resistance.

Formula:

For a Newtonian fluid (where viscosity is constant), the viscous force (F_v) can be expressed using the formula:

$$F_v = \eta \cdot A \cdot \frac{v}{d}$$

where:

- F_v = viscous force (N),
- η = dynamic viscosity (Pa·s or N·s/m^2),
- A = cross-sectional area of the object moving through the fluid (m^2),
- v = velocity of the object relative to the fluid (m/s),
- d = characteristic length or distance over which the fluid flows (m). (length of the object)

Units:

- **Dynamic Viscosity** (η): Measured in pascal-seconds (Pa·s) or poise (1 poise = 0.1 Pa·s).
- **Viscous Force** (F_v): Measured in newtons (N).

Summary:

Viscous force is the resistance encountered by an object moving through a fluid, determined by the fluid's viscosity, the object's velocity, and the area it presents to the fluid. The formula quantifies this force, and viscosity is a key property influencing fluid behavior.

Stokes' Law Overview:

Stokes' Law describes the motion of a spherical object moving through a viscous fluid. It quantifies the drag force acting on the object due to the viscosity of the fluid. This law is applicable for low Reynolds numbers, where the flow is laminar and the effects of turbulence are negligible.

Formula:

The drag force (F_d) acting on a spherical object moving at a constant velocity (v) through a fluid is given by:

$$F_d = 6\pi\eta r v$$

$$F_v = \eta \cdot A \cdot \left(\frac{V}{d}\right)$$

$$F_v = \eta\left(\frac{A}{d}\right) \cdot v$$

$$SA = 4\pi r^2$$

$$F_v = \eta\left(\frac{4\pi r^2}{r}\right) \cdot v$$

$$F_v = \eta(4\pi r) \cdot v$$

where:

- F_d = drag force (N),
- η = dynamic viscosity of the fluid (Pa·s or N·s/m²),
- r = radius of the sphere (m),
- v = velocity of the sphere relative to the fluid (m/s).

Units:

- **Drag Force** (F_d): Measured in newtons (N).
- **Dynamic Viscosity** (η): Measured in pascal-seconds (Pa·s).
- **Radius** (r): Measured in meters (m).
- **Velocity** (v): Measured in meters per second (m/s).

Summary:

Stokes' Law provides a formula to calculate the drag force on a sphere moving through a viscous fluid, emphasizing the relationship between the object's size, velocity, and the fluid's viscosity. It is fundamental in understanding the behavior of small particles in fluids.

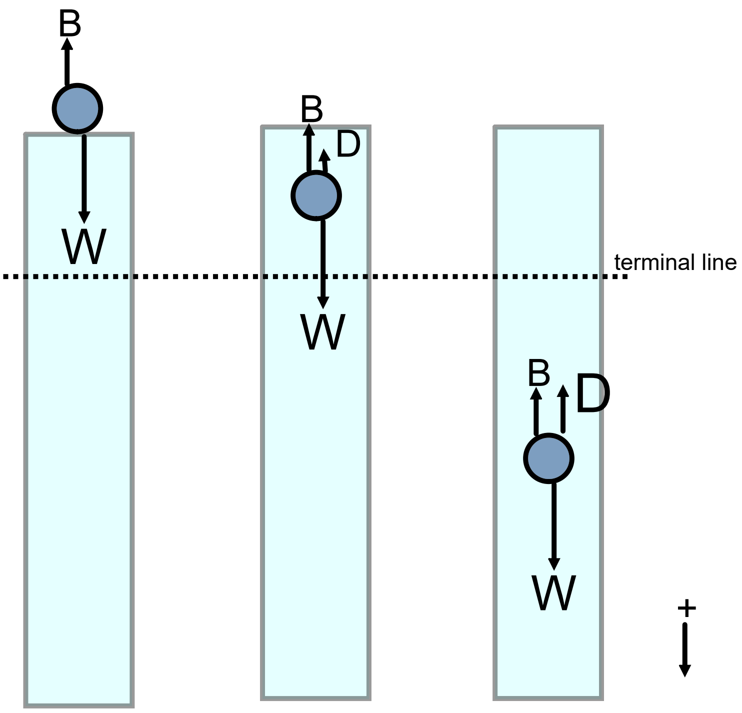
Here's a table of the dynamic viscosity of 10 common household substances at 20 degrees Celsius:

Substance	Dynamic Viscosity (mPa·s)
Water	1.0
Air	0.018
Olive Oil	81
Vegetable Oil	50-100
Honey	2,000-5,000
Motor Oil (SAE 30)	100-150
Corn Syrup	1,000-1,500
Milk	1.5-2.0
Ketchup	50-100
Syrup	1,000-3,000

Notes:

- **Dynamic viscosity** is measured in millipascal-seconds (mPa·s), where 1 mPa·s = 1 cP (centipoise).
- The values for some substances (like honey and corn syrup) can vary widely depending on their specific composition.

Free Body Diagram for a Sphere falling in a Fluid



$$F_{net} = W - B - D$$

(constant velocity) $$0 = m \cdot g - \rho_f \cdot V \cdot g - 6\pi\eta r v_t$$

$$0 = \rho_s \cdot V \cdot g - \rho_f \cdot V \cdot g - 6\pi\eta r v_t$$

$$6\pi\eta r v_t = (\rho_s - \rho_f) V \cdot g$$

$$v_t = \frac{(\rho_s - \rho_f) V \cdot g}{6\pi\eta r}$$

Worked example 14

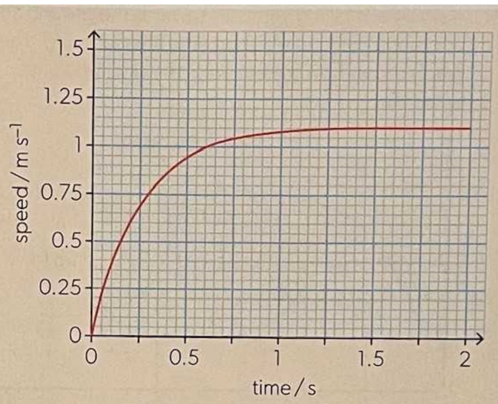
A ball of radius 8.0 mm and mass 1.3 g is released from rest from the bottom of a long vertical tube filled with oil. The ball rises towards the surface of the oil. The diagram shows how the vertical speed of the ball varies with time.

a. Estimate the initial acceleration of the ball.

b. Hence, calculate the magnitude of the buoyancy force on the ball.

c. Draw a free-body diagram for the ball at a time of 1.5 s.

d. Determine the coefficient of viscosity of the oil.



Solutions

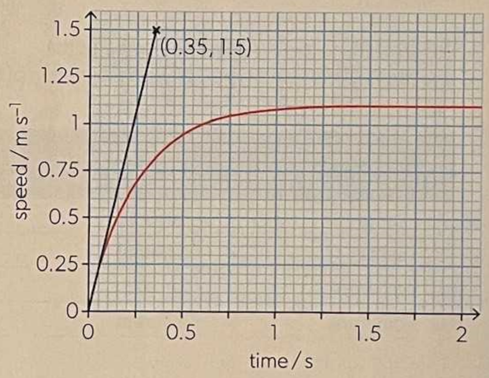
a. The initial acceleration is the gradient of the tangent to the speed-time graph at $t = 0$.

Acceleration $= \frac{1.5}{0.35} = 4.3\,\text{m}\,\text{s}^{-2}$.

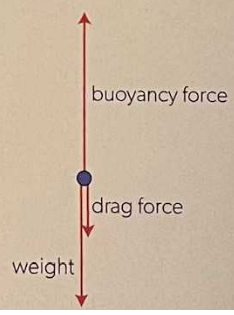
b. Initially, the only forces acting on the ball are the buoyancy force F_b and the ball's weight. The net force is the difference between the two and is related to the acceleration by Newton's second law.

$F_b - mg = ma$, so

$F_b = 1.3 \times 10^{-3}\,(9.8 + 4.3) = 1.8 \times 10^{-2}\,\text{N}$.



c. At $t = 1.5$ s the ball is moving upwards with a constant terminal speed and the viscous drag force acts downwards so that the net force on the ball is zero.



d. At terminal speed, the drag force is $F_d = F_b - mg = 1.3 \times 10^{-3} \times 4.3 = 5.6 \times 10^{-3}\,\text{N}$.

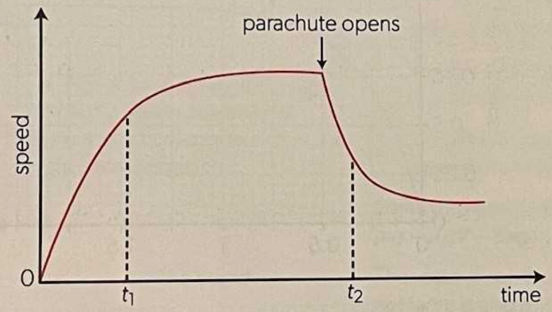
The drag force $F_d = 6\pi\eta r v$, where the terminal speed v is approximately $1.1\,\text{m}\,\text{s}^{-1}$.

Combining equations gives

$$\eta = \frac{5.6 \times 10^{-3}}{6\pi \times 8.0 \times 10^{-3} \times 1.1} = 3.4 \times 10^{-2}\,\text{Pa}\,\text{s}.$$

Practice questions

21. A skydiver is falling vertically towards the ground and opens the parachute a short time after jumping. The graph shows how the speed of the skydiver varies with time.



Which of the following correctly compares the directions of the velocity and of the acceleration of the skydiver at times t_1 and t_2?

	Directions of velocity at t_1 and t_2	Directions of acceleration at t_1 and t_2
A.	same	same
B.	same	different
C.	different	same
D.	different	different

22. Two balls of radius R and $2R$, made from the same type of steel, fall through a liquid that exerts a viscous drag force on the balls. The smaller ball reaches the terminal speed v. What is the terminal speed of the larger ball?

A. v B. $2v$ C. $4v$ D. $8v$

23. A ball of weight 1.2 N falls through a liquid at a constant speed. The density of the ball is 1.5 times greater than the density of the liquid. What is the magnitude of the drag force acting on the ball?

A. 0.4 N B. 0.6 N C. 0.8 N D. 1.2 N

24. A steel ball falls in a long vertical tube filled with vegetable oil.

a. Explain how the ball reaches a terminal speed.

The following data are given.

density of the vegetable oil = $920\ \text{kg m}^{-3}$
viscosity of the vegetable oil = 8.4×10^{-2} Pa s
density of steel = $8000\ \text{kg m}^{-3}$
radius of the ball = 2.0 mm

b. Calculate:

i. the weight of the ball

ii. the buoyancy force acting on the ball in the oil.

c. Determine the terminal speed of the ball in the oil, assuming that the ball is affected by a viscous drag force.