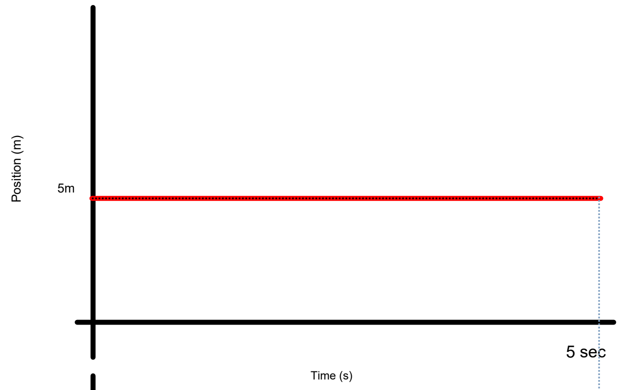
Sec. 1.2 - Acceleration in One Dimension

Learning Goals:

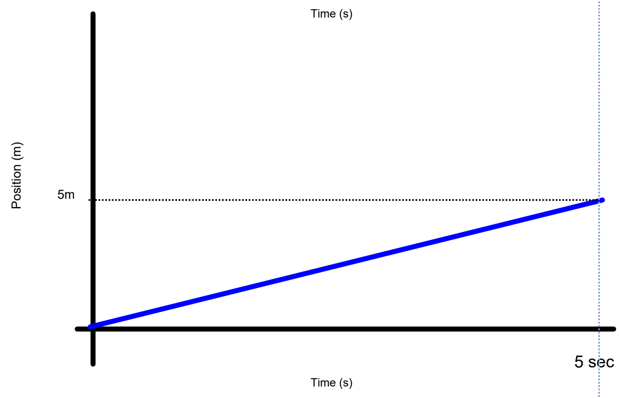
- acceleration in one direction
- acceleration graphs
- 5 Kinematic Equations

Types of Motion - Recap

Position - Time Graphs

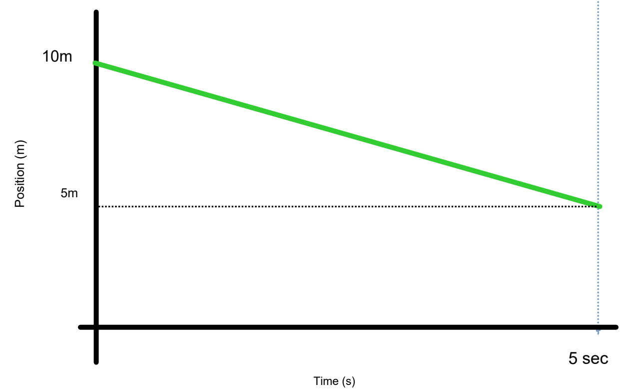


- Object starts 5 m from the reference point;
-
- position does not change over time, therefore object is NOT moving (rate of change of 0 m/s, the line has a slope of Zero)



- Object starts 0 m from the reference point;
-
- position **is** changing over time (increasing - moving AWAY from ref. pt.) in a linear or constant manner, therefore the object is moving at a constraint velocity.

Vel = 5m / 5sec
Vel = 1m/s

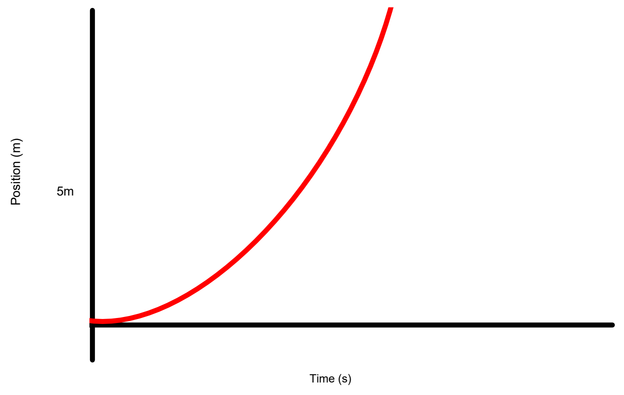


- Object starts 10 m from the reference point;
-
- position **is** changing over time (decreasing - moving TOWARDS the ref. pt) in a linear or constant manner, therefore the object is moving at a constraint velocity.

Vel = -5m / 5sec
Vel = -1 m/s

Type of Motion - Non Uniform

- Object starts 0 m from the reference point;
-
- position is changing over time, therefore object is moving AWAY from the ref. pt
- graph is NOT linear, therefore the rate of change is NOT constant
- therefore NON uniform velocity,which means the object is accelerating



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Velocity is the change in position over time. ie. 5 m/s, 30 km/hr, 45 mi/hr

Acceleration is the change in velocity over time. ie. 3 m/s per second, 5 km/hr per second.

If the time units for acceleration are the same, we tend to write the time values squared.

For example, 4 m/s per second would be written as 4 m/s^2. (4 m s^{-2})

When an object is accelerating, the velocity of the object changes over time.

For example, cyclist is riding at 6 m/s when they start peddling faster and accelerate at 2 m/s^2 for 4 seconds.

What is their new velocity?

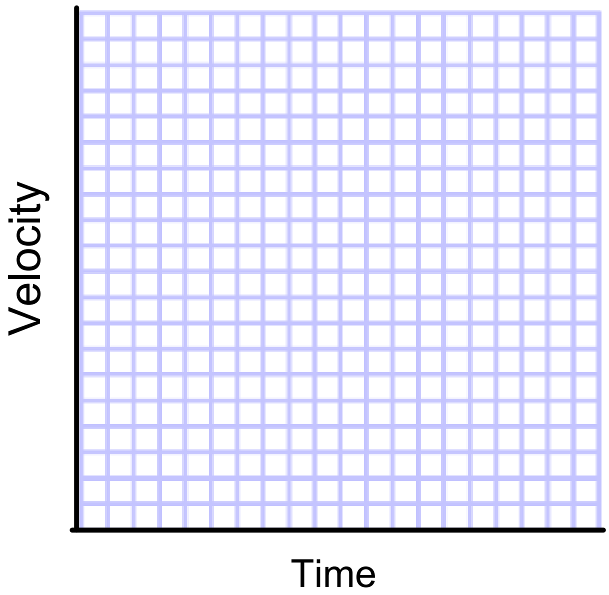
t = 0 v = 6 m/s

t = 1

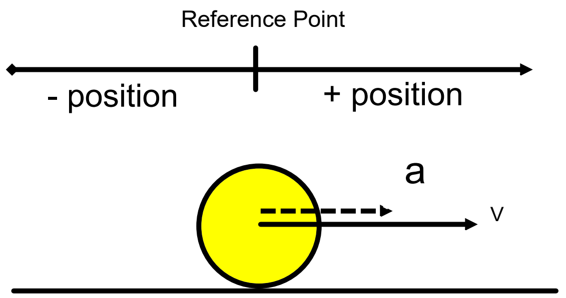
t= 2

t = 3

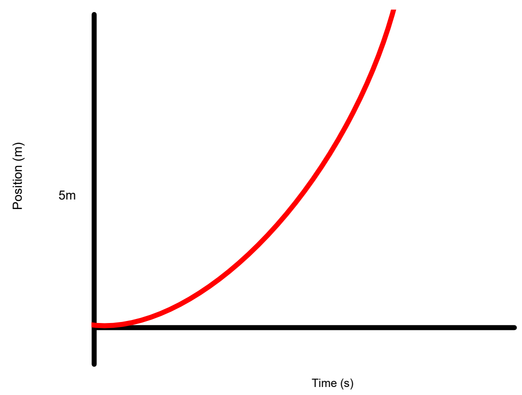
t = 4



Representing Velocity and Acceleration with Vector Diagrams.



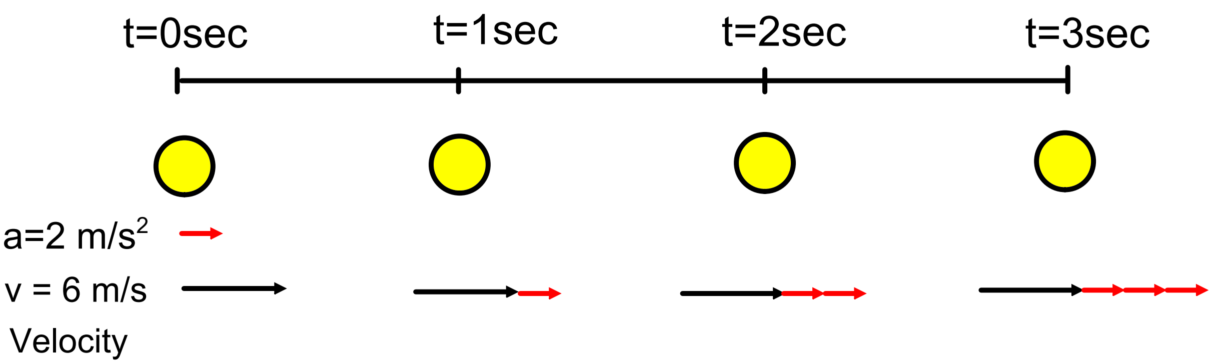
Draw "tangents" to help see the rate of change behaviour.



Case 1 - velocity and acceleration are BOTH in the same direction

Object will continue to travel in its current direction AND will increase in speed.

Object is moving AWAY from reference point in a positive direction.



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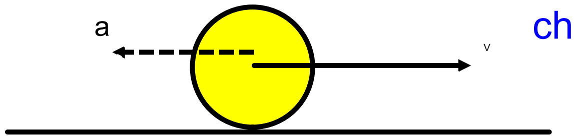

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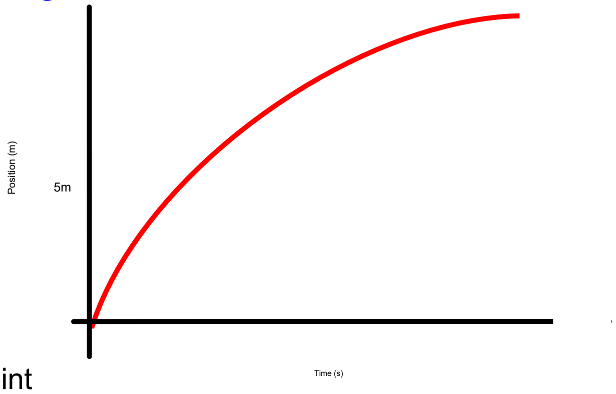
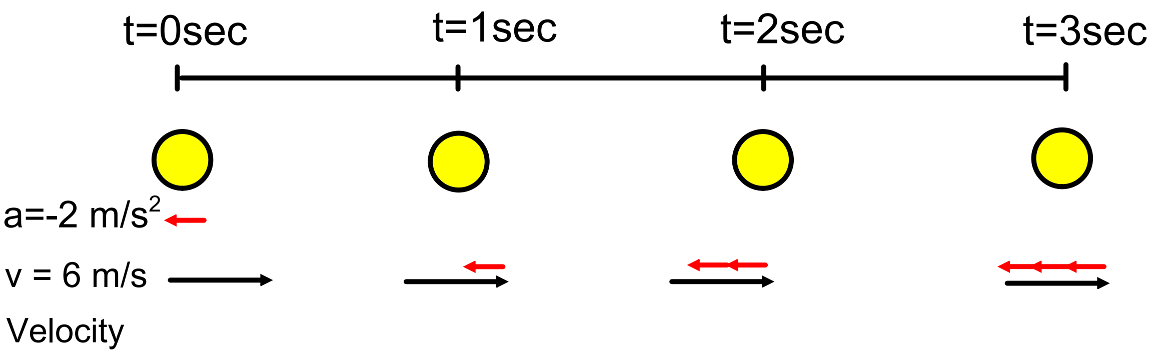


Case 2 - velocity and acceleration are in the OPPOSITE directions

Object will continue to travel in its current direction BUT will decrease in speed.

Object is moving away from the reference point in a positive direction.



Position - Time Graphs

A position - time graph plots the position of a moving object with respect to time.

The rate of change of a Position - Time graph is the VELOCITY.

Velocity - Time Graphs

A velocity - time graph plots the velocity of a moving object with respect to time.

The rate of change of a Velocity - Time graph is the acceleration.

$$a_{ave} = \frac{\Delta v}{\Delta t} \qquad a_{ave} = \frac{v_f - v_i}{\Delta t} \qquad a_{ave} = \frac{v_f - v_i}{t_f - t_i}$$

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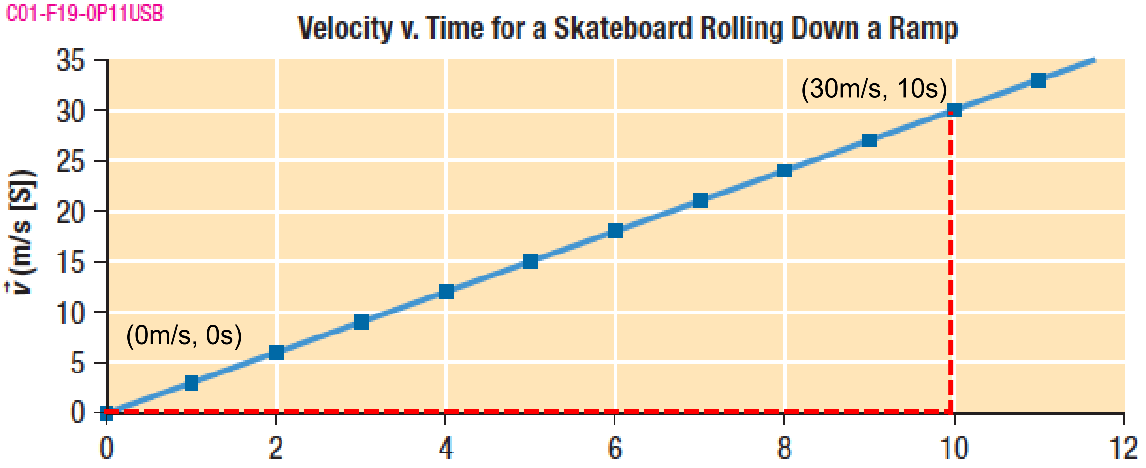

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Using the Velocity - Time Graph to determine the Displacement on an Object

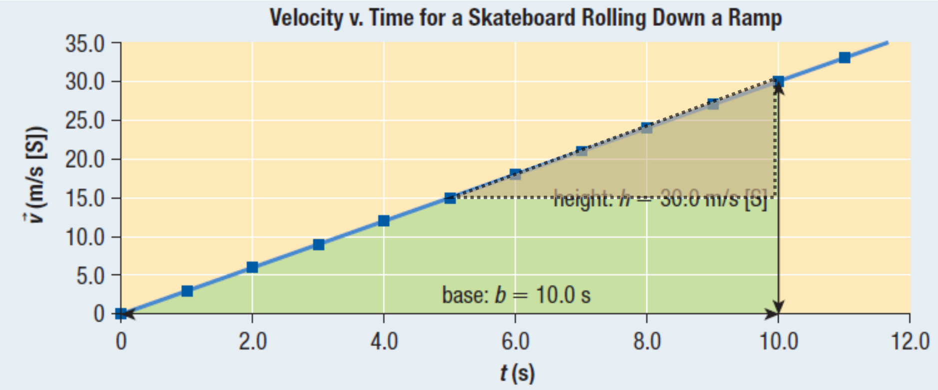

Figure 3 Velocity–time graph showing the area underneath the line

If an object moved at 4 m/s for 5 seconds, it would travel a distance of 20m.

The object above has different velocities at different times throughout its motion. We determine the distance travelled by:
(i) finding the average velocity of the object over the given period of time, or
(ii) we can find the area underneath the given graph.

What is the distance travelled in 10 sec?

(i) Average Velocity - rearrange the triangle to create a rectangle and multiply the velocity by the time.

(ii) Find the area under the velocity graph (area of a triangle)

NOTE: finding the distance travelled by using the AREA under the curve only works with Velocity - Time graphs, NOT position - time graphs.

Practice

What is the displacement represented by the graph in Figure 4 over the time interval from 0 s to 10.0 s?

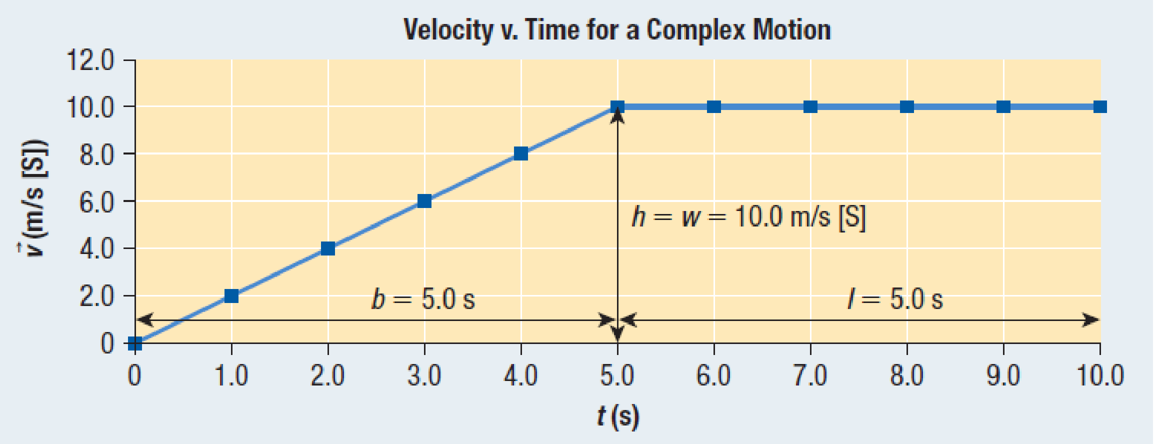

Figure 4 Velocity–time graph showing more complex motion

Acceleration - Time Graphs

Acceleration - Time graphs describe the acceleration of an object with respect to time.

The graph to the right, represents uniform acceleration of an object at 4 m/s^2.

The AREA under the graph represents the change in velocity of an object over a given interval.

For the given example, for the first 5 seconds, the change in velocity is given by: $4\ m/s^2 \times 5\ s = 20\ m/s$

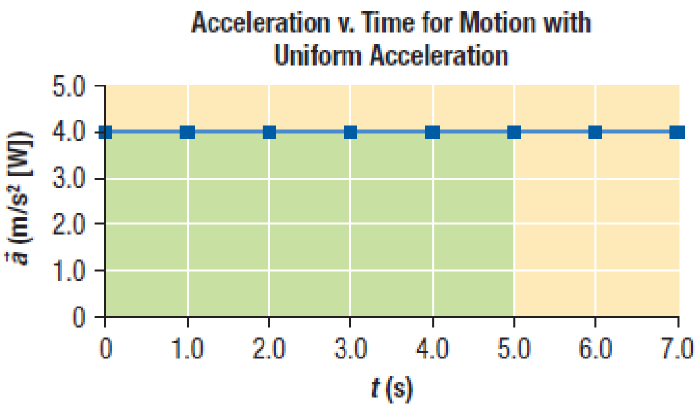

Figure 2 Acceleration–time graph showing motion with uniform acceleration

Note: we do not know the initial velocity of the object, so all we can determine is the CHANGE in velocity.

Moving from Acceleration - Time Graph to Position - Time Graph Using Data

Find the area under the curve for each of the first 5 seconds.

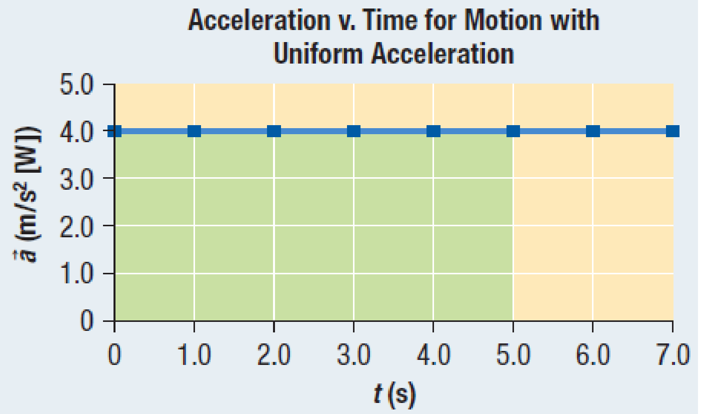

Figure 4 Using an acceleration–time graph to create other motion graphs

Table 1 Calculating the Velocity at Various Time Points in Figure 4

Time t (s)	Acceleration $\vec{a}$ (m/s² [W])	Equation $\vec{v} = (\Delta\vec{a})(\Delta t)$	Velocity $\vec{v}$ (m/s) [W]
0	4.0	$\vec{v} = \left(4.0\frac{m}{s^{\not 2}}[W]\right)(0\ \not s)$	0
1.0	4.0	$\vec{v} = \left(4.0\frac{m}{s^{\not 2}}[W]\right)(1.0\ \not s)$	4.0
2.0	4.0	$\vec{v} = \left(4.0\frac{m}{s^{\not 2}}[W]\right)(2.0\ \not s)$	8.0
3.0	4.0	$\vec{v} = \left(4.0\frac{m}{s^{\not 2}}[W]\right)(3.0\ \not s)$	12
4.0	4.0	$\vec{v} = \left(4.0\frac{m}{s^{\not 2}}[W]\right)(4.0\ \not s)$	16
5.0	4.0	$\vec{v} = \left(4.0\frac{m}{s^{\not 2}}[W]\right)(5.0\ \not s)$	20.0

Plot the velocities

t(s)	vel
0	0
1	4
2	8
3	12
4	16
5	20

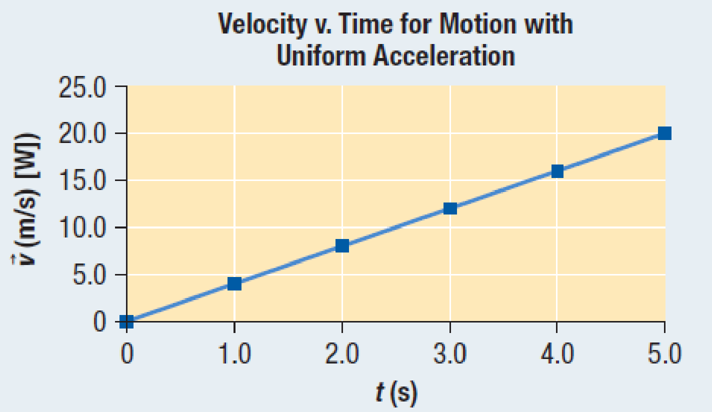

Figure 5 Velocity–time solution graph

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Use the velocities and the Velocity - Time graph to determine the Position Values

t(s)	vel
0	0
1	4
2	8
3	12
4	16
5	20

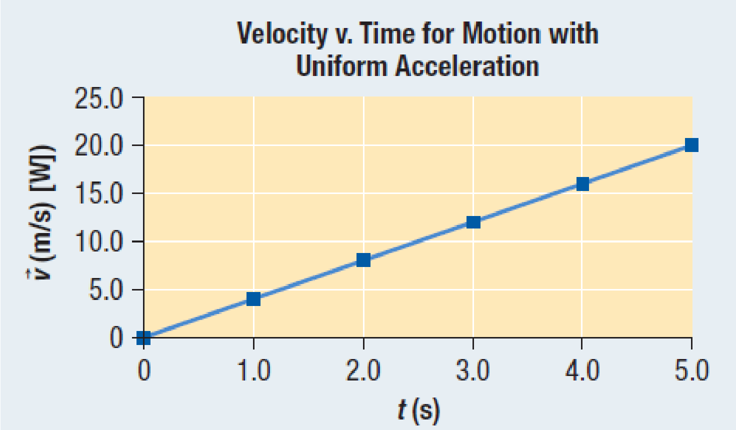


Figure 5 Velocity–time solution graph

$$A = \frac{1}{2}bh$$

t(s)	position
0	0
1	2
2	8
3	18
4	32
5	50

$$A = \frac{1}{2}(0)(4)$$

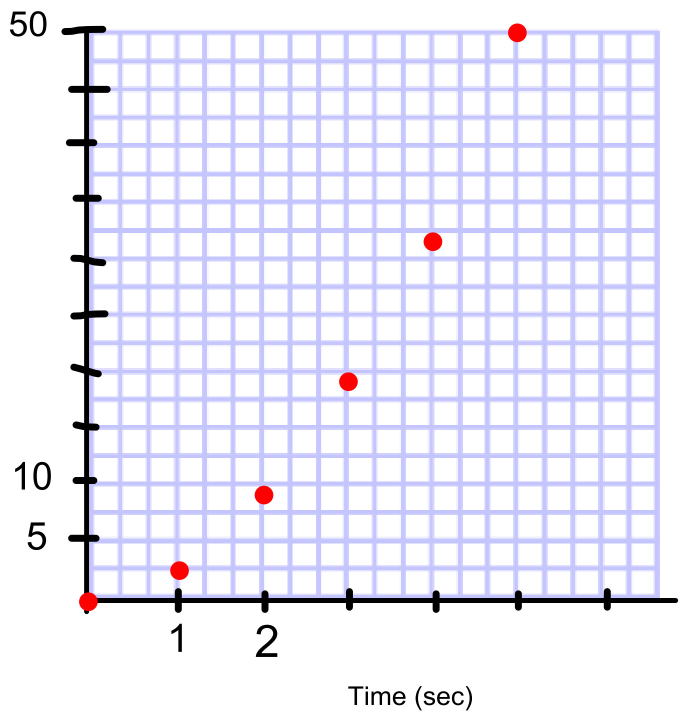
$$A = \frac{1}{2}(1)(4)$$

$$A = \frac{1}{2}(2)(8)$$

$$A = \frac{1}{2}(3)(12)$$

$$A = \frac{1}{2}(4)(16)$$

$$A = \frac{1}{2}(5)(20)$$



Summary of Relationship Amongst the Graphs

Position - Time Graph

$s = -4.9t^2 + 98t + 1.5$

Second degree power

Velocity - Time Graph

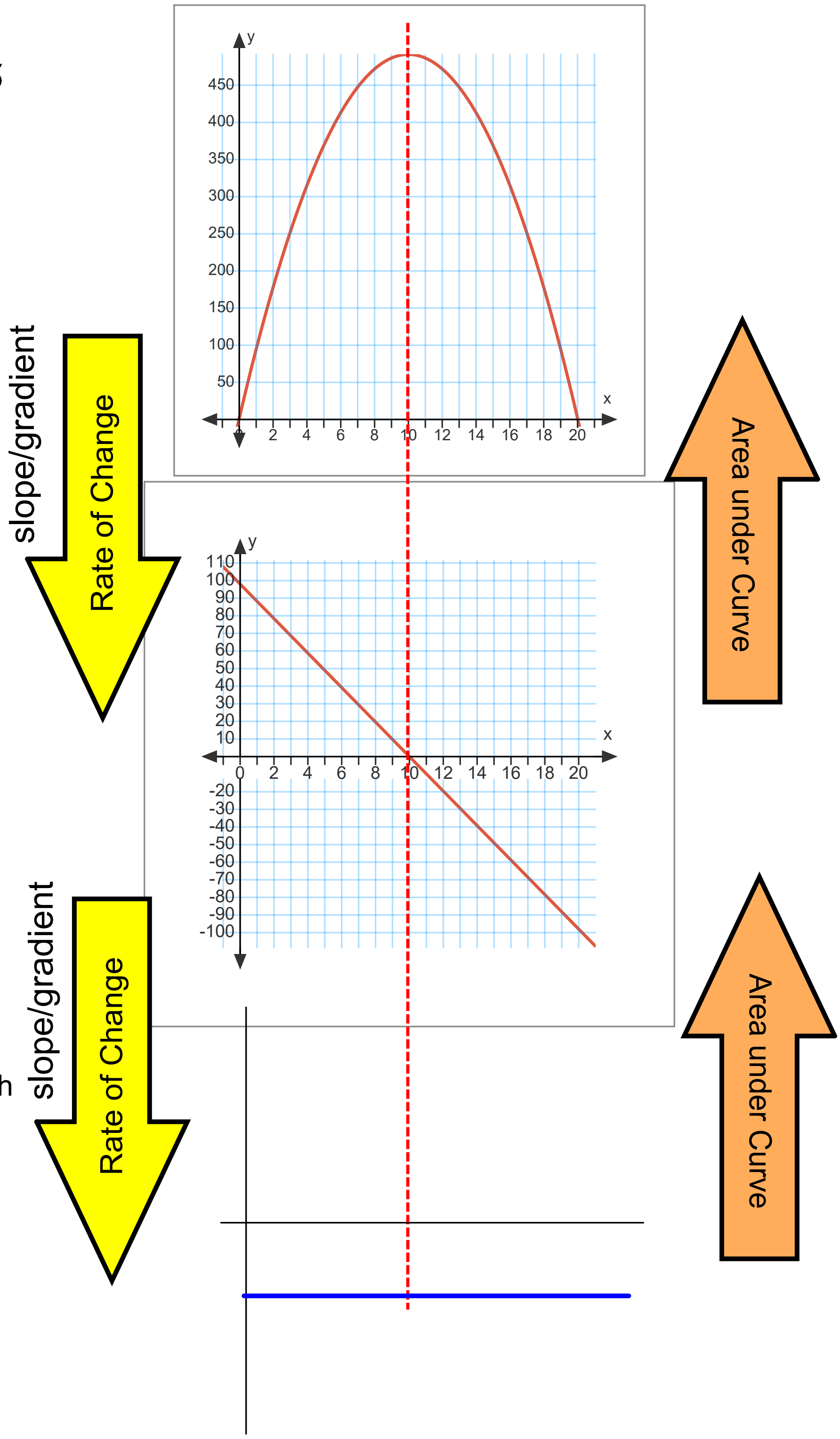
$v = -9.8t + 98$

First degree power

Acceleration - Time Graph

$a = -9.8$

Constant

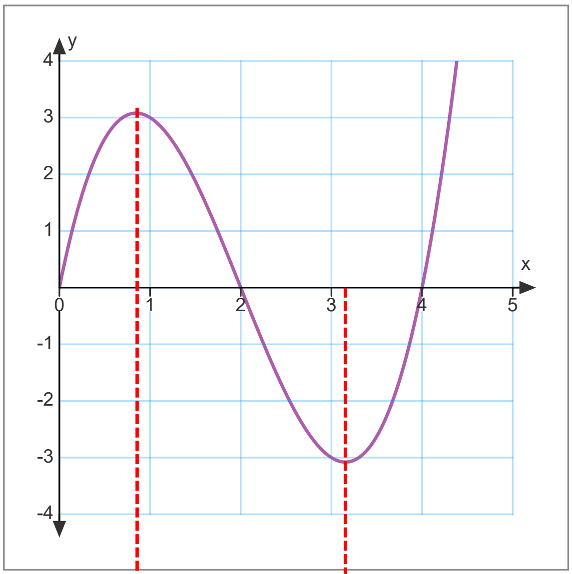


Moving from Graph to Graph

Position - Time Graph

$s=(t)(t-2)(t-4)$

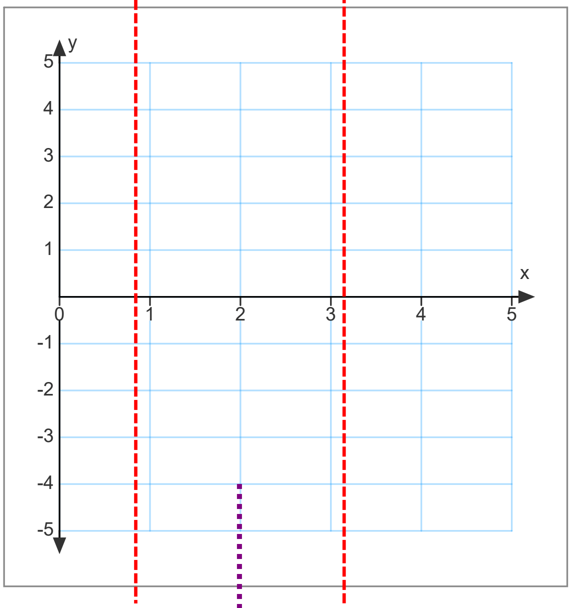
Third degree power



Velocity - Time Graph

$v=3t^2-12t+8$

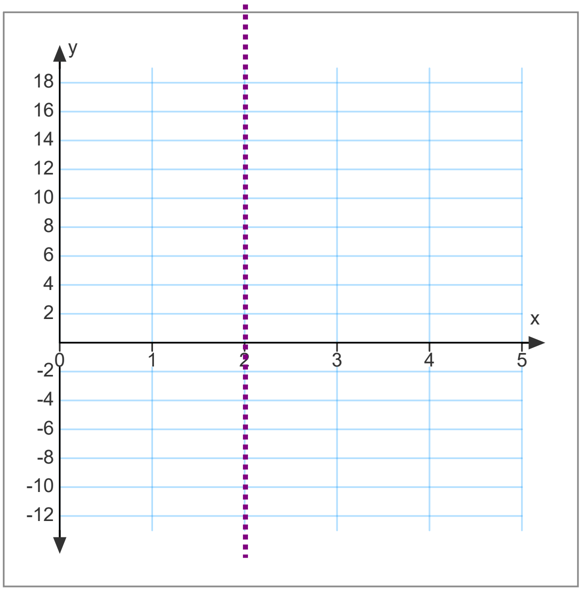
First degree power



Acceleration - Time Graph

$a=6t-12$

Constant



A Displacement Equation for Uniformly Accelerated Motion

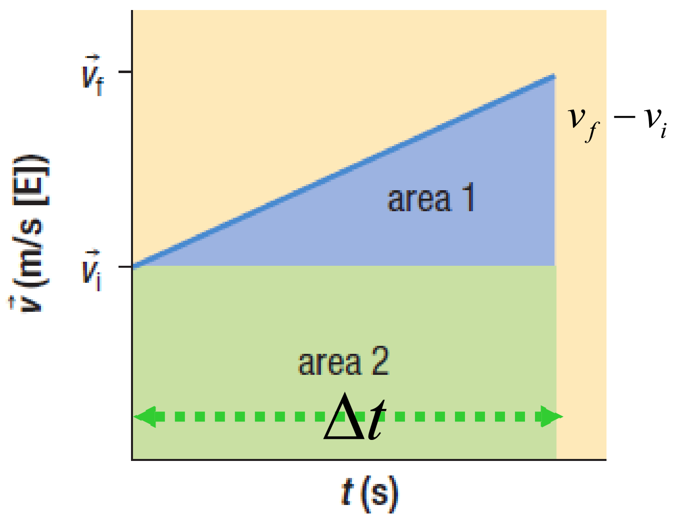


Figure 1 A velocity–time graph for an object undergoing uniform acceleration

$$\begin{aligned}\Delta\vec{d} &= A_{\text{triangle}} + A_{\text{rectangle}} \\ &= \frac{1}{2}bh + lw \\ &= \frac{1}{2}\Delta t(\vec{v}_f - \vec{v}_i) + \Delta t\vec{v}_i \\ &= \frac{1}{2}\vec{v}_f\Delta t - \frac{1}{2}\vec{v}_i\Delta t + \vec{v}_i\Delta t \\ &= \frac{1}{2}\vec{v}_f\Delta t + \frac{1}{2}\vec{v}_i\Delta t\end{aligned}$$

$$\Delta\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t \text{ (Equation 1)}$$

Additional Motion Equations

Consider the defining equation for acceleration: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_1}{\Delta t}$

If we rearrange this equation to solve for final velocity ($\vec{v}_f$), we get Equation 2:

$$\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t \text{ (Equation 2)}$$

You may use Equation 2 in problems that do not directly involve displacement.

If we substitute the expression $v_i + \vec{a}_{av}\Delta t$ from Equation 2 into Equation 1, we get

$$\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t \text{ (Equation 2)}$$

$$\Delta\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t \text{ (Equation 1)}$$

$$= \frac{1}{2}(\vec{v}_i + \vec{a}_{av}\Delta t + \vec{v}_i)\Delta t$$

$$= \frac{1}{2}(2\vec{v}_i + \vec{a}_{av}\Delta t)\Delta t$$

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}_{av}\Delta t^2 \text{ (Equation 3)}$$

The Five Key Equations of Accelerated Motion

Table 1 shows the five key equations of accelerated motion. You should be able to solve any kinematics question by correctly choosing one of these five equations. You have seen how the first three are developed. We will leave the others to be developed as an exercise.

Table 1 The Five Key Equations of Accelerated Motion

	Equation	Variables found in equation	Variables not in equation
Equation 1	$\Delta\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$	$\Delta\vec{d}$, Δt, $\vec{v}_f$, $\vec{v}_i$	$\vec{a}_{av}$
Equation 2	$\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t$	$\vec{a}_{av}$, Δt, $\vec{v}_f$, $\vec{v}_i$	$\Delta\vec{d}$
Equation 3	$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}_{av}\Delta t^2$	$\Delta\vec{d}$, $\vec{a}_{av}$, Δt, $\vec{v}_i$	$\vec{v}_f$
Equation 4	$v_f^2 = v_i^2 + 2a_{av}\Delta d$	Δd, a_{av}, v_f, v_i	Δt
Equation 5	$\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}_{av}\Delta t^2$	$\Delta\vec{d}$, $\vec{a}_{av}$, Δt, $\vec{v}_f$	$\vec{v}_i$

(IB SUVAT equations)

s - displacement

u - initial velocity

v - final velocity

a - acceleration

t - time

Example

A sailboat accelerates uniformly from 6.0 m/s [N] to 8.0 m/s [N] at a rate of 0.50 m/s^2 [N]. What distance does the boat travel?

What is given?

What equation might be used?

Homework

Read Sec. 1.2 (up to page 27)

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