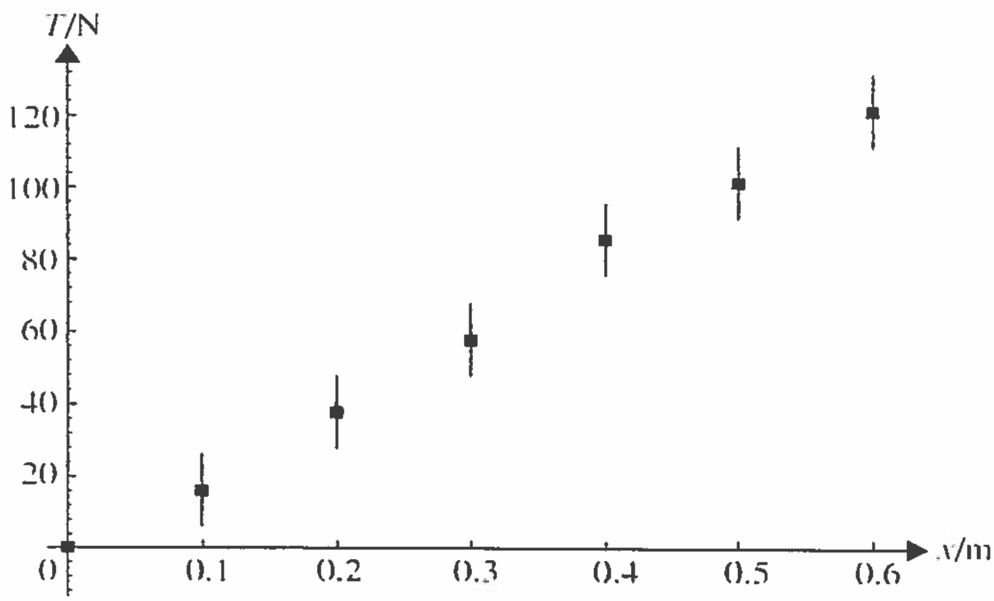
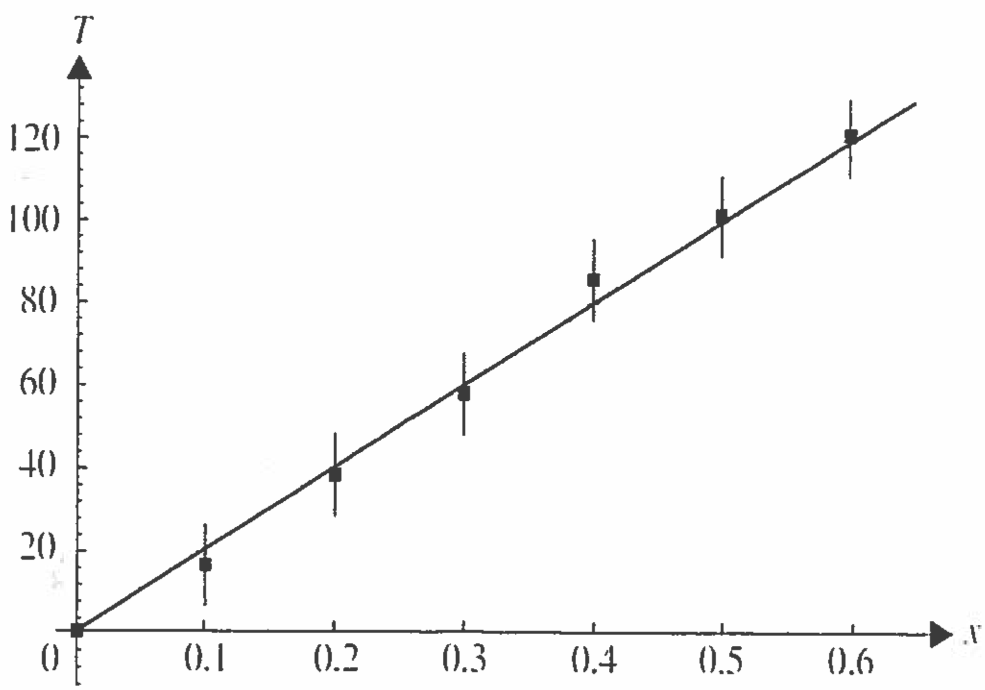
Uncertainties in Slopes and Intercepts

A) Line of best fit:
If we have reason to suspect that the data points we have plotted fall on a straight line, we must draw the line of best fit through the points. This means using a straight edge and choosing that line which goes through as many of the points as possible in such a way that the distances between the line and the points on one side of the line are, on the average, the same as the distances between the line and points on the other side of the line.
Thus, suppose that in an experiment to verify Hooke's law, data was collected and plotted as shown if figure 1 below. The experimenter included uncertainty bars representing ± 10 N in the values (i.e. the uncertainty bar is 20 N in length).



B) The experimenter the draws the line of best fit through the data points and obtains the line of best fit. The line of best fit will **ideally** pass through the error bars of all the points.



C) Having decided the line of best fit for a given set of data that are expected to result in a straight line, it is usually required to calculate the slope and the intercept of the line. However, since the data points are the results of measurements in an experiment, they are subject to uncertainties. To obtain the slope, we must use the line of best fit not the data points. We calculate the slope by taking two points on the line of best fit that are as far apart as possible and then apply the formula

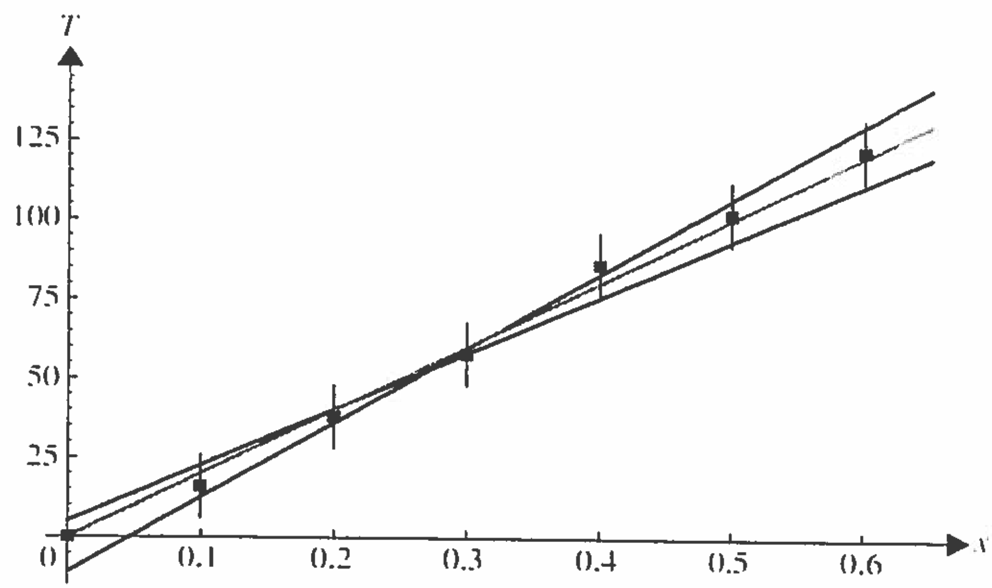
slope = rise / run = Δy / Δx

The slope of this line is $y = 120\text{ N} - 0 / 0.6\text{ m} - 0$

$y = 200\text{ N m}^{-1}$ this is the spring constant of the spring

What is the uncertainty in the slope and the intercept?

A simple way to estimate the uncertainties is by drawing two extreme straight lines as shown below and finding the slope of each. Both straight lines are made to pass through the midway point between the range of x values (in this case at x = 0.3 m). The first line is drawn so as to have the largest slope and still fit the data (this means it will pass through the extremes of the error bars). The second line is drawn to have the least slope and still fit the data points. The two slopes are then calculated.



Calculations give slopes of 235 N m^{-1} and 177 N m^{-1}; that is errors of + 35 N m^{-1} and -23 N m^{-1}. Taking the ***average*** of these gives 29 N m^{-1} - which to 1 sig fig = 30 N m^{-1}.
i.e. $m_{max} + m_{moin} / 2$

Therefore the spring constant is 200 ± 30 N m^{-1}

The same procedure allows us an estimate of the vertical intercept. The line of best fit gives an intercept of zero. But the line with the largest slope has an intercept of -11 N and the line with the least slope has an intercept of +5 N. The ***average*** of the intercepts is 11 + 5 / 2 = 8 N. Therefore the intercept is written as ,

y intercept is 0 ± 8 N

1. Daphne wanted to determine the electrical resistance of the substance. Daphne recorded the following data;

Voltage (V) ± 2 V	Current (A) ± 0.1 A
5.0	1.0
9.0	2.0
16	3.0
19	4.0
24	5.0

Plot a voltage **versus** current graph. Using the concepts of what a good graph should entail, and using appropriate error analysis, determine the resistance of the substance.
note - the error bars of the current are too small, therefore do not include the current error bars.