Worksheet 1:
GPS Activities

Below are four activities that you can do with an intermediate science class to explore the ideas presented in the video. Each activity is independent of the others and takes 10 to 15 minutes. You can choose to do any number of them.

1. DISCUSSION QUESTIONS

Discuss the following questions with a partner and write down your answers.

What is the GPS?
Who uses the GPS?
How does the GPS work?

2. RESEARCHING APPLICATIONS OF THE GPS

After watching the video, form small groups and make a list of five different uses of the GPS. Next, create a three-column chart as shown below. Write a brief description of each GPS use from your list. Then describe a few of the advantages GPS offers over traditional methods. For example:

USE	DESCRIPTION	ADVANTAGES
Farming - spraying pesticide on crop fields	Farmers use the GPS to more accurately spray chemicals on their crops.	The GPS allows farmers to be much more efficient and accurate because: i) chemicals are sprayed exactly where they are needed. ii) no area is sprayed twice. iii) the amount of chemicals deposited on the crops is accurate as both location and speed of travel are known precisely.

3. NUMBER CRUNCHING

The video mentions a lot of numbers. The following questions are designed to help give meaning to these numbers.

a) GPS satellites orbit at a height of 20 200 km above Earth's surface. The radius of Earth is 6370 km. Break into small groups and draw large scale diagrams of the GPS orbits and Earth.

b) GPS satellites move at a speed of 14 000 km/h. How does this compare to a car? A jet? Two times faster? Twenty times? Two hundred times? Two thousand times?

c) The speed of light is 300 000 000 m/s. How long does it take for a radio signal, traveling at the speed of light, to get from a GPS satellite to Earth's surface?

d) How long does it take light to travel the length of a 30 cm ruler?

4. EXPLORING HOW THE GPS LOCATES OBJECTS

BACKGROUND

The Global Positioning System is a navigational tool that locates your position by sending distance information from GPS satellites to GPS receivers.

OVERVIEW

You will be given some information about how far you are from three cities and your goal is to find your location.

MATERIALS

Compasses, maps of Canada

INSTRUCTIONS FOR STUDENTS

1. You are located 975 km from Vancouver. Using a compass, draw a circle on the map that is centred on Vancouver and has a radius of 975 km.
2. You are also located 2716 km from Toronto. Draw a circle on the map that is centred on Toronto and has a radius of 2716 km.
3. You are also located 3028 km from Montreal. Draw a circle on the map that is centred on Montreal and has a radius of 3028 km.
4. The three circles should intersect at a single point. Mark their intersection with a cross. This is the city at which you are located. Write down the name of this city.

DISCUSSION

1. If you only knew how far you were from Vancouver, at which points on the map could you be located? Label these points on the map.
2. If you only knew how far you were from Vancouver *and Toronto*, at which points on the map could you be located? Label these points on the map.
3. The GPS uses information from *four* satellites. What extra information does using this many satellites provide?

Worksheet 3:
Energy in Satellites

Useful equations:

$$E_K = \frac{1}{2}mv^2 \qquad E_G = -\frac{GMm}{r} \qquad W = \vec{F}\cdot\vec{d} \qquad v = \frac{d}{t} \qquad G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/(kg^2)}$$

$$c = 2.997\ 924\ 58 \times 10^{8}\ \mathrm{m/s} \qquad m_{earth} = 5.97 \times 10^{24}\ \mathrm{kg} \qquad r_{earth} = 6.37 \times 10^{6}\ \mathrm{m}$$

QUESTION 1

Each GPS satellite orbits at a distance of 2.66×10^7 m from Earth's centre at a speed of 3.874×10^3 m/s. Each satellite has a mass of 2.0×10^3 kg.

a) What is the kinetic energy of a GPS satellite?
b) What is the gravitational potential energy of a GPS satellite?
c) What is the total energy of a GPS satellite?

QUESTION 2

Each GPS satellite is launched from Earth's surface by rocket. How much work must the rocket do on the satellite so that it reaches the height of its orbit?

QUESTION 3

The GPS calculates the distance *d* from a GPS satellite to a receiver by multiplying the speed *c* of a GPS signal by the time Δt the signal takes to travel from satellite to receiver:

$$d = c\ \Delta t$$

Let $\Delta t = 0.068\ 503\ 387$ s

a) Calculate *d* using i) all of the digits in Δt and c and ii) rounding off Δt and *c* to three significant figures.
b) What is the difference between your answers to parts i) and ii)?
c) Use your answers to a) and b) to explain why the GPS needs to use atomic clocks accurate to at least 10^{-9} s, instead of regular quartz clocks accurate to only 10^{-6} s.

QUESTION 4

Einstein's theories of special relativity and general relativity have opposing effects on time in the GPS. Einstein's theory of *special* relativity states that the clocks inside GPS satellites run slower than a stationary clock on Earth by 8.3×10^{-11} s per second. This is due to the speed of the satellites. Einstein's theory of *general* relativity says the satellite clocks also run faster than those on Earth by 5.2×10^{-10} s per second because Earth's gravity is weaker at the satellites' altitude.

a) How much slower does a GPS clock run each day due to special relativity? How much faster does it run each day due to general relativity?
b) GPS satellites emit signals that travel at the speed of light *c*. Any timing error in the GPS translates into a distance error equal to:

$$\text{Distance Error} = c\ \Delta t_{err}$$

Calculate the daily distance error from both special and general relativity.
c) Calculate the difference between these two distances to find the overall distance error per day from relativity.

Worksheet 4:
Relativity

Useful equations:

$v = d/t$ $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/(kg^2)}$ $c = 2.997\,924\,58 \times 10^{8}\ \mathrm{m/s}$ $\Delta t' = \Delta t\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$

QUESTION 1

GPS satellites send time signals to GPS receivers.
A receiver gets a signal that reads t_1 = 9:00:27.723 119 038 (i.e. 9 am and 27.723 119 038 seconds). The signal is received at t_2 = 9:00:27.790 249 045 according to the receiver.

a) How long did it take the signal to travel from the satellite to the receiver?
b) How far is the receiver from the satellite?

QUESTION 2

In special relativity, the relationship between the time elapsed for a GPS satellite clock and a clock on Earth is

$$\Delta t' = \Delta t\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

where Δ*t* is the time elapsed according to the satellite clock, Δ*t*' is the time elapsed on Earth and *v* is the satellite's speed relative to Earth.

GPS satellites move at $v = 3.874 \times 10^3$ m/s. When *v* is much less than *c*, the relationship is well approximated by

$$\Delta t' \approx \Delta t\left(1+\frac{v^2}{2c^2}\right)$$

where the term $v^2/(2c^2)$ is the rate at which a GPS clock runs slowly from the perspective of someone on Earth.

a) Calculate $v^2/(2c^2)$ for a GPS satellite.
b) How slowly does someone on Earth see a GPS satellite running over the course of a day?

QUESTION 3

In addition to the timing error in the GPS from special relativity, GPS satellite clocks run faster by 5.2×10^{-10} s per second due to effects from *general* relativity.

a) How large a timing error does this correspond to over the course of a day?
b) How large is the daily distance error from the combined effects of special and general relativity?

QUESTION 4

A friend says "*The theory of relativity is just that, a theory. It's not real, unlike Newton's laws of motion which are laws.*" Is your friend correct? Explain why or why not using the information you have learned about the effects of relativity on the GPS.

Worksheet 5:
General Relativity

1. A plastic water bottle has a small hole in the cap. It is turned upside down and dropped with the hole uncovered. What happens to the water as the bottle falls?

   a) It pours out at the same rate as when the bottle is stationary.
   b) It pours out more slowly than when the bottle is stationary.
   c) It pours out faster than when the bottle is stationary.
   d) It stays in the bottle.

2. A plastic water bottle has a small hole in the cap and is turned upside down. It is thrown upwards with the hole uncovered. What happens to the water while the bottle is in the air?

   a) It pours out on both the way up and down.
   b) It stays in the bottle.
   c) It pours out only on the way up.
   d) It pours out only on the way down.

3. A cup of water is on a tray. The tray is swung rapidly in a horizontal circle. The water stays in the cup and the cup stays on the tray because there is a large acceleration

   a) inwards which resembles a large gravitational field outwards.
   b) outwards which resembles a large gravitational field outwards.
   c) inwards which resembles a large gravitational field inwards.
   d) outwards which resembles a large gravitational field inwards.

4. Figure 1 shows the positions of a rocket every 100 ns, going from left to right. The rocket is moving up at a constant velocity. Bob is at the rear and sends pulses of light towards Alice in the nose of the rocket every 100 ns. How often does Alice receive the pulses?

   a) every 100 ns
   b) more frequently than every 100 ns
   c) less frequently than every 100 ns

5. Figure 2 shows the positions of the same rocket when it is accelerating upwards. How often does Alice receive the pulses now?

   a) every 100 ns
   b) more frequently than every 100 ns
   c) less frequently than every 100 ns

6. The rocket is stationary in a constant gravitational field. How often does Alice receive the pulses in this situation?

   a) every 100 ns
   b) more frequently than every 100 ns
   c) less frequently than every 100 ns

7. General relativity states that the ratio of the times elapsed for Alice and Bob is $t_A / t_B = 1 - (g\Delta h)/c^2$, where $g = 9.8$ m/s^2, Δh is the rocket's height and c is the speed of light. In 1960, an experiment was done to test this relationship. Pulses of light were sent from the top floor of a building to the basement, a distance of 20 m. How large is the relativistic effect $(g\Delta h)/c^2$ in this case?

   a) 2×10^{-9}
   b) 2×10^{-12}
   c) 2×10^{-15}
   c) 2×10^{-18}

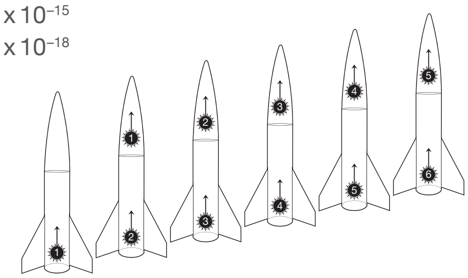


Figure 1 Rocket moving with constant velocity

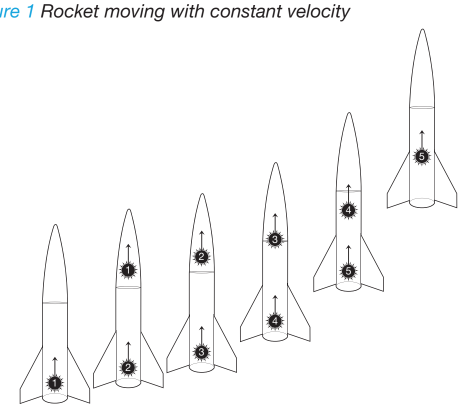
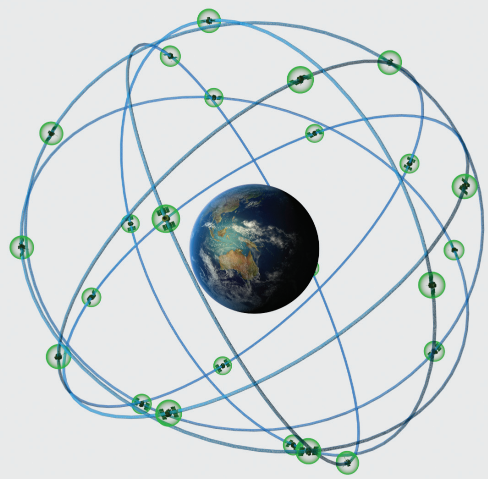


Figure 2 Rocket moving with constant acceleration.

What is the Global Positioning System?



The Global Positioning System (GPS) is a network of more than 30 satellites operated by the US Air Force that broadcasts signals accessible to anyone with a GPS receiver. The satellites are orbiting Earth at an altitude of 20 200 km and a speed of 3874 m/s. The satellites follow one of six orbital planes inclined to the equator by 55° and separated around the equator by 60°. This pattern ensures that at least four satellites are visible from any point on Earth's surface at any one time.

The master control station for the GPS is at the Schriever Air Force Base in Colorado Springs, USA. This station, along with several other bases around the world, tracks the satellites and provides regular updates to the information in the radio signals broadcast by the satellites.

GPS receivers are essentially sophisticated radios tuned to 1575.42 MHz, which decipher and compare the signals from several satellites to determine where the receiver is located.

The original purpose for the GPS was to provide a navigational aid for the US military. As the system developed it became evident that it would also be very useful for civilian purposes. In 1983, the accidental shooting down of a commercial passenger plane, which had strayed from its scheduled path by mistake into restricted air space over the Soviet Union, prompted then US President Ronald Reagan to make GPS available to everyone. Since the system became fully functional in 1995, thousands of applications have been developed.

GPS is not just used for navigation. It has become a standard tool for surveyors, builders, and farmers. When combined with a transmitter it is a powerful tracking device used by hospitals, police, and wildlife biologists. GPS signals can also be used to generate very precise timestamps, which are used by cell-phone networks, financial institutions, and computer companies to establish highly accurate transaction times. The overall economic impact of the GPS runs into billions of dollars annually.

How Does the GPS Work?

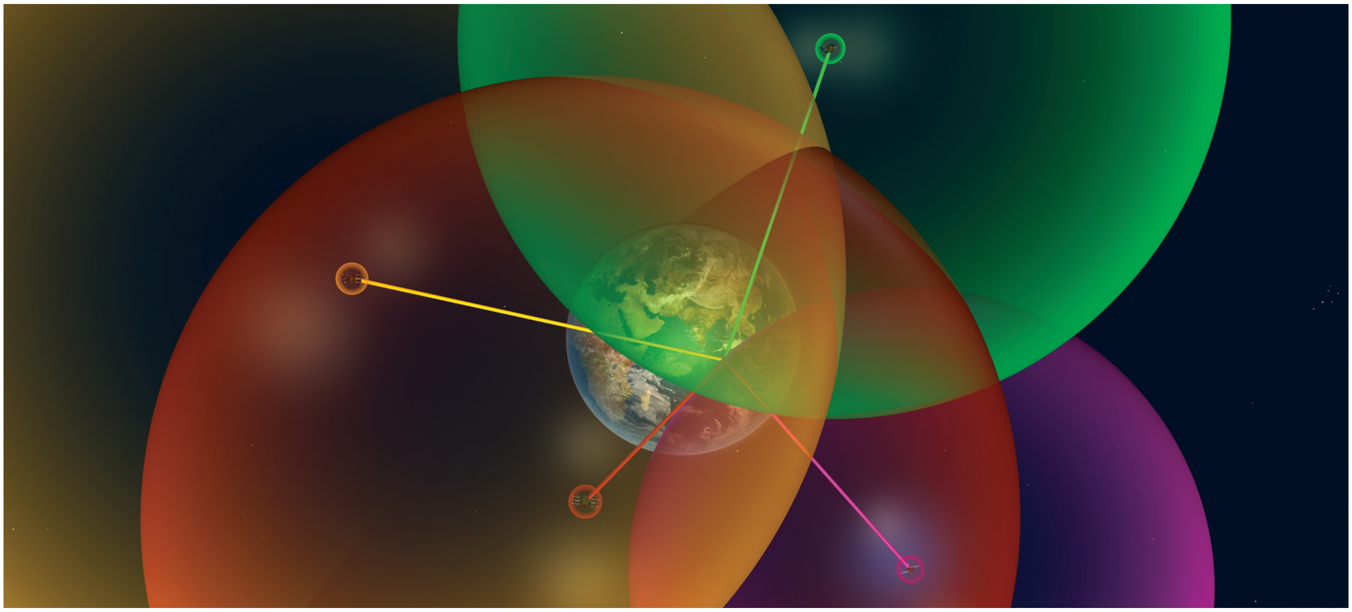


Figure 4

Each GPS satellite broadcasts a continuous signal containing information about where the satellite is (ephemeris), what time it is (a timestamp), and the general health of the system (almanac). The receiver uses this signal to estimate how far away it is from the transmitting satellite by multiplying the signal's travel time by the speed at which the signals travel — the speed of light. The receiver is located on the surface of an imaginary sphere centred on the satellite (see Figure 4). The receiver repeats this process for three more satellites to produce four overlapping spheres. Using a geometrical method called *trilateration* the receiver determines its location as the unique point where the surfaces of the four imaginary spheres intersect (see Figure 4). Three spheres are needed to determine the location, and the fourth signal is needed to establish the precise time used in the calculations. With four signals the receiver is able to determine its position within a few metres.

To achieve this degree of accuracy, the GPS must use very precise information. The speed of light used in the calculations is c = 299 792 458 m/s and the timestamps must be accurate to within 20 to 30 ns. To generate these timestamps, each GPS satellite contains several atomic clocks that generate very precise, highly stable measurements of time. Atomic clocks use the oscillation of electrons rather than a pendulum or a piece of quartz to keep track of time. The standard clock uses cesium-133, which has one valence electron that can be excited so that it undergoes a transition with a very specific energy and frequency. This transition is used to produce a resonant vibration at 9 192 631 770 Hz which is extremely sensitive to variations in frequency and thus produces a highly accurate measurement of time. This resonant frequency is now used as the global basis for the definition of a second.

The incredible precision required by the GPS makes it very sensitive to error. The scientists and engineers who operate the system have to take into account many sources of error. The atomic clocks have to be adjusted to compensate for relativity (both special and general relativity). The slight variations in the satellite orbits caused by the tiny gravitational pulls of the Sun and Moon have to be corrected regularly. The satellite signals travel through the various layers of the atmosphere, each with a slightly different index of refraction. The signals also bounce off objects such as mountains and buildings and can follow various paths before reaching the receiver. There are also rounding errors introduced by the receiver's processor during calculations. All of these errors must be taken into account to ensure that the GPS can locate objects to within a few metres, even though the satellites are orbiting over 20 000 km away.

Special Relativity
and the GPS

INTRODUCTION TO SPECIAL RELATIVITY

Special relativity is a theory of how motion affects measurements of length and time. Developed by Einstein in 1905, it is based on two postulates:

1. The speed of light in a vacuum is c = 2.997 924 58 x 10^8 m/s in all inertial reference frames. All observers in these frames measure light as travelling at this speed independent of their own speed relative to the light's source.
2. The laws of physics are the same in all inertial reference frames.

Everything in special relativity can be derived from these two statements.

TIME DILATION

One of the most significant consequences of the postulates is that "moving clocks run slow". Stated more accurately, special relativity says an observer in an inertial reference frame sees a clock that is moving relative to them as running slow. They observe that less time elapses according to such a clock than one in their own frame. This phenomenon is called *time dilation* and is governed by the following equation:

$$\Delta t' = \Delta t\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right) \qquad (1)$$

where $\Delta t'$ is the amount of time elapsed for the observer, Δt is the amount of time elapsed for the moving clock, v is the clock's speed relative to the observer, and c is the speed of light.

Equation 1 shows that the observer sees the clock as slowed by an amount that depends on the clock's relative speed v. Since the factor containing v appears frequently in special relativity, Equation 1 is written as

$$\Delta t' = \gamma \Delta t \qquad (2)$$

where

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (3)$$

The GPS satellites move at 3.874 km/s relative to Earth, a speed that is 0.0013% of the speed of light. Calculating γ from Equation 3, we get γ =1.000 000 000 0 83. This means that for every second we observe as passing for a GPS satellite clock, we observe 1.000 000 000 0 83 s passing on Earth's surface. This represents a slowing down of time by 8.3 x 10^{-11} s per second. Although this deviation is extremely small, it has a critical impact on the operation of the GPS.

Note that because the value of γ for GPS satellites is so close to one, most classroom calculators will return the answer '1'. *See the answer to Worksheet 4, Question 2 on page 18 for information on how to get around this issue.*

Because GPS satellite clocks run slow by 8.3 x 10^{-11} s per second in our reference frame, they gradually fall behind the clocks in the GPS receivers. Over the course of a day, the amount they fall behind is (8.3 x 10^{-11} s)(24)(60)(60) = 7.2 μs

If this difference was not corrected for, GPS satellite clocks would become unsynchronized from GPS receiver clocks. This would mean that the receivers would measure the travel time Δt for signals inaccurately. In turn, this would result in errors in GPS distance measurements equal to

$d = c\,\Delta t$ = (2.997 295 x 10^8 m/s) (7.2 x 10^{-6} s) = 2.1 km

So, the effect of time dilation would cause an error of more than 2 km per day, something that would render the GPS useless.

FREQUENTLY ASKED QUESTIONS

Q - At different positions in its orbit, a GPS satellite will have differing speeds relative to different GPS receivers. Given this, do we need to adjust the speed used in the equation for time dilation to account for this variation?

A - In principle, we do need to use a different value for *v* in Equation 1 depending on the precise speed of a given satellite relative to a particular receiver. However, the speed of the satellites (3874 m/s) is much larger than the speed of a GPS receiver as it moves with Earth's rotation (465 m/s at the equator). Differences in the values of the relative speed between a satellite and a receiver result in variations in the amount of time dilation of just 1% at most and so are insignificant for the current accuracy of the GPS.

Q - GPS satellites are in orbit and so are accelerating. They are not in inertial reference frames. Similarly, GPS receivers are accelerating due to Earth's rotation and so are also not in inertial frames. Given this, how can we use special relativity, which primarily deals with inertial frames, to calculate the amount of time dilation?

A - The reason we can use this theory is that the acceleration of GPS receivers (0.034 m/s^2) is so small that we can ignore it. Over the course of one second, the acceleration changes each receiver's speed by just 0.034 m/s. For a receiver at the equator, this is just 0.007% of its speed due to Earth's rotation. So, the effect the acceleration has on the amount of time dilation is at most only about 0.007% of the total value per day of 7 μs. This corresponds to just 0.0005 μs, a negligible effect.

Approximating GPS receivers as being in inertial frames, a GPS satellite moves at a speed of 3874 m/s relative to this frame. At each moment in time, it has an instantaneous velocity of 3874 m/s along its orbit.

Imagine a second object with the same velocity but which is not accelerating (see Figure 5). This object is in an inertial frame and so, using Equation 1, we can calculate that we see its clock running slow by 8.3 x 10^{-11} s per second. The GPS satellite shares the same instantaneous motion and so we will also see its clock running slow by the same amount. In the next instant, the satellite clock shares the same motion as a third object moving at 3874 km/s in a slightly different inertial frame. So, its clock runs slow by the same amount as in the previous instant.

Continuing this process over the satellite's entire orbit, we find that the satellite's clock runs slow by 8.3 x 10^{-11} s per second throughout its orbit. We can use special relativity at each instant of the satellite's motion and then add up all of the amounts of time dilation to calculate the total amount. Even though the satellite is accelerating, by comparing it to other objects in inertial frames moving at the same instantaneous speeds, we can use special relativity to determine how slowly its clock runs.

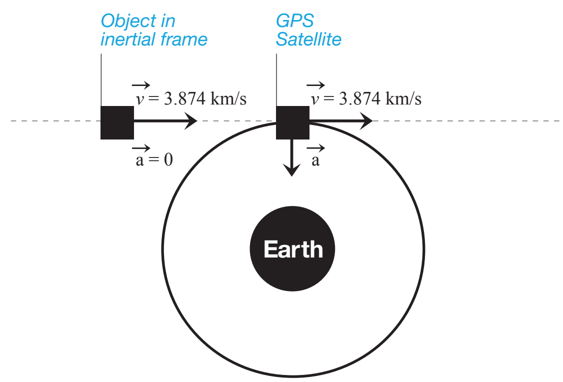


Figure 5

General Relativity and the GPS

WHY DOES GRAVITY SLOW DOWN TIME?

One of the key ideas in the video is that gravity slows down time. That is, clocks in a stronger gravititional field run slower than clocks in weaker gravitational field. For the GPS, this means that the clocks on Earth run slower than those in the satellites by 45 μs per day. At first glance, gravity and time seem to be two unrelated phenomena. Why would gravity affect time? Einstein discovered this connection by considering frames of reference.

In 1905, Einstein published his theory of special relativity which deals with frames of reference moving at constant velocity. It took him another ten years to extend this theory so it could handle accelerating frames and gravity—the theory of general relativity. His first breakthrough was the equivalence principle, which he described as "the happiest thought of my life." This principle says that the laws of physics in a reference frame in freefall are equivalent to those in a frame with no gravity.

This can be seen in Figure 6. In Figure 6a the frame is far from Earth or any other large mass. In Figure 6b the frame is in freefall. In both frames of reference Alice feels weightless and her ball appears to her not to fall down. The equivalence principle applies to all situations where a frame is freely falling under gravity. An object in orbit is also in freefall and this is why the astronauts on the International Space Station feel weightless even though the gravitational field on the station is more than 9 N/kg.

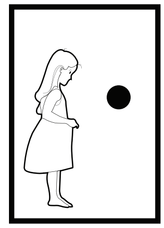


Figure 6a Alice and the ball are weightless because there are no large masses nearby.

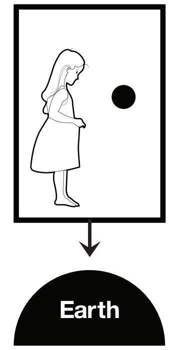


Figure 6b Alice feels weightless and the ball appears to her not to fall

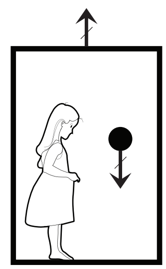


Figure 7a Alice feels heavy and the ball appears to fall down because the frame is accelerating up.

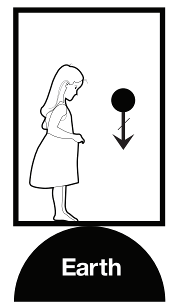


Figure 7b Alice feels heavy and the ball appears to fall down because of the Earth's gravity

Einstein next considered what it would be like in a frame accelerating in the absence of a gravitational force. In Figure 7a, Alice is once again way out in space far from Earth or any other large mass, but this time she is in a frame accelerating at 9.8 m/s^2 relative to her original frame in Figure 6a. It feels like being on Earth's surface (Figure 7b). The ball appears to fall down and she feels a normal force pushing up from the floor.

The equivalence principle tells us that any phenomenon that occurs in a frame with constant acceleration also happens in a frame with a constant gravitational field. Einstein used this to predict that gravity slows time.

Consider two people, Alice and Bob, in a rocket. Bob is at the rear of the rocket and Alice is at the front. Bob sends Alice signals every 100 ns, which is about how long it takes light to travel 30 m. If the rocket moves with a constant velocity (see Figure 8) Alice receives each pulse at constant intervals and so receives the pulses 100 ns apart. However, if the rocket is accelerating upwards (see Figure 9) the flight time of each pulse is longer because Alice is moving away from successive pulses at a greater speed.

Because Alice receives the pulses less frequently when the rocket accelerates, the equivalence principle says she will also receive them less frequently when the rocket is parked in a gravitational field. Once again, Bob is sending the pulses every 100 ns and Alice receives them separated by more than that. Alice infers that Bob's time is running slower than hers. Conversely, if Alice sends pulses 100 ns apart to Bob, he will receive them more frequently and he will say that her time is running fast. This result is very different from the time dilation of special relativity where if Alice and Bob are moving relative to each either, they both say that the other's time is passing more slowly. This is one place where general relativity is easier to comprehend than special relativity.

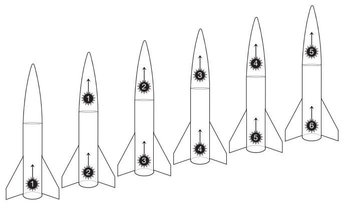


Figure 8 *Rocket moving with constant velocity*

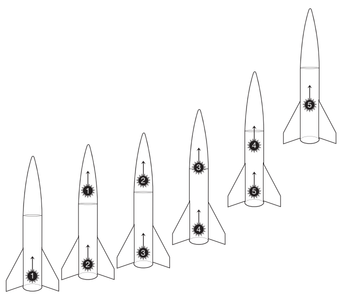


Figure 9 *Rocket moving with constant acceleration.*

This slowing down of time is known as gravitational time dilation and the ratio of the times in a gravitational field is given by

$$\frac{\Delta t_S}{\Delta t_R} = 1 + \frac{\Delta \Phi}{c^2}$$

where Δt_R is the time elapsed for the receiver, Δt_S is the time elapsed at the source, $\Delta \Phi$ is the gravitational potential difference between the source and the receiver, and c is the speed of light.

For Alice and Bob sitting in a rocket on the Earth, the gravitational potential difference, $\Delta \Phi$, can be approximated by $g\Delta h$, where g is 9.8 N/kg and Δh is their vertical separation. Thus the ratio of times for Alice (t_A) and Bob (t_B) will be given by

$$\frac{\Delta t_S}{\Delta t_R} = 1 + \frac{g \Delta h}{c^2}$$

For the GPS, the ratio of the satellite time t_{sat} and Earth-based reciver time t_E is given by

$$\frac{\Delta t_{\mathrm{sat}}}{\Delta t_{\mathrm{E}}} = 1 + \frac{\left(\frac{GM}{r_{\mathrm{E}}} - \frac{GM}{r_{\mathrm{sat}}}\right)}{c^2}$$

where r_E and r_{sat} are, respectively, the radius of Earth's surface and the satellite's orbit, $G = 6.67 \times 10^{-11}$ Nm^2/(kg^2) is the universal gravitational constant, and $M = 5.97 \times 10^{24}$ kg is Earth's mass.

Substituting the values $r_E = 6.4 \times 10^6$ m and $r_{sat} = 2.64 \times 10^7$ m into the equation, we get

$$\frac{\Delta t_{\mathrm{sat}}}{\Delta t_{\mathrm{E}}} = 1 + 5.2 \times 10^{-10}$$

So, due to gravitational time dilation, for every second that passes for a clock inside a GPS receiver on Earth, 1.000 000 000 52 s passes for a clock inside a GPS satellite. That is, the satellite clocks run faster by 5.2×10^{-10} seconds each second. Multiplying this result by the number of seconds per day ($24 \times 3600 = 86\,400$), we arrive at the result in the video that gravitational time dilation makes GPS satellite clocks run fast by $(86\,400)(5.2 \times 10^{-10}) = 45 \times 10^{-6}$ s per day.

ANOTHER WAY TO UNDERSTAND WHY GRAVITY SLOWS DOWN TIME

There is another very different way to derive gravitational time dilation. It involves the conservation of energy, special relativity, and quantum mechanics. Consider one of the photons sent up by Bob.

As the photon rises, there is an increase in gravitational potential energy. In order to conserve total energy, the photon must lose energy. Light always travels at speed c, so it can't lose energy by slowing down. A photon's energy is given by $E = hf$ (where h is Planck's constant, 6.626×10^{-34} J·s and f is the photon's frequency) and the only way the photon can lose energy is by decreasing its frequency. This decrease in frequency to conserve energy is exactly the same as the change derived from the equivalence principle.

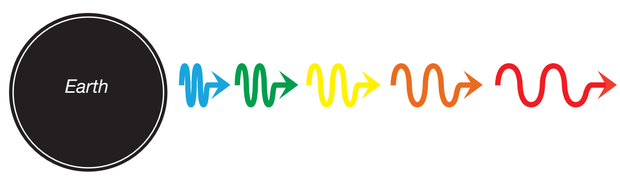


Figure 10 *The only way the photon can lose energy is by decreasing its frequency.*

MORE ABOUT GENERAL RELATIVITY

It is important to realize that the equivalence principle is only one part of the story behind general relativity. For the full story including black holes, gravitational lenses, and gravitational waves you also need the second key idea — that mass causes space and time (spacetime) to curve. However, the equivalence principle alone is responsible for around 99% of the 45 μs per day time difference between the receiver and satellite clocks in the GPS. This is because spacetime is only very slightly curved near the Earth.