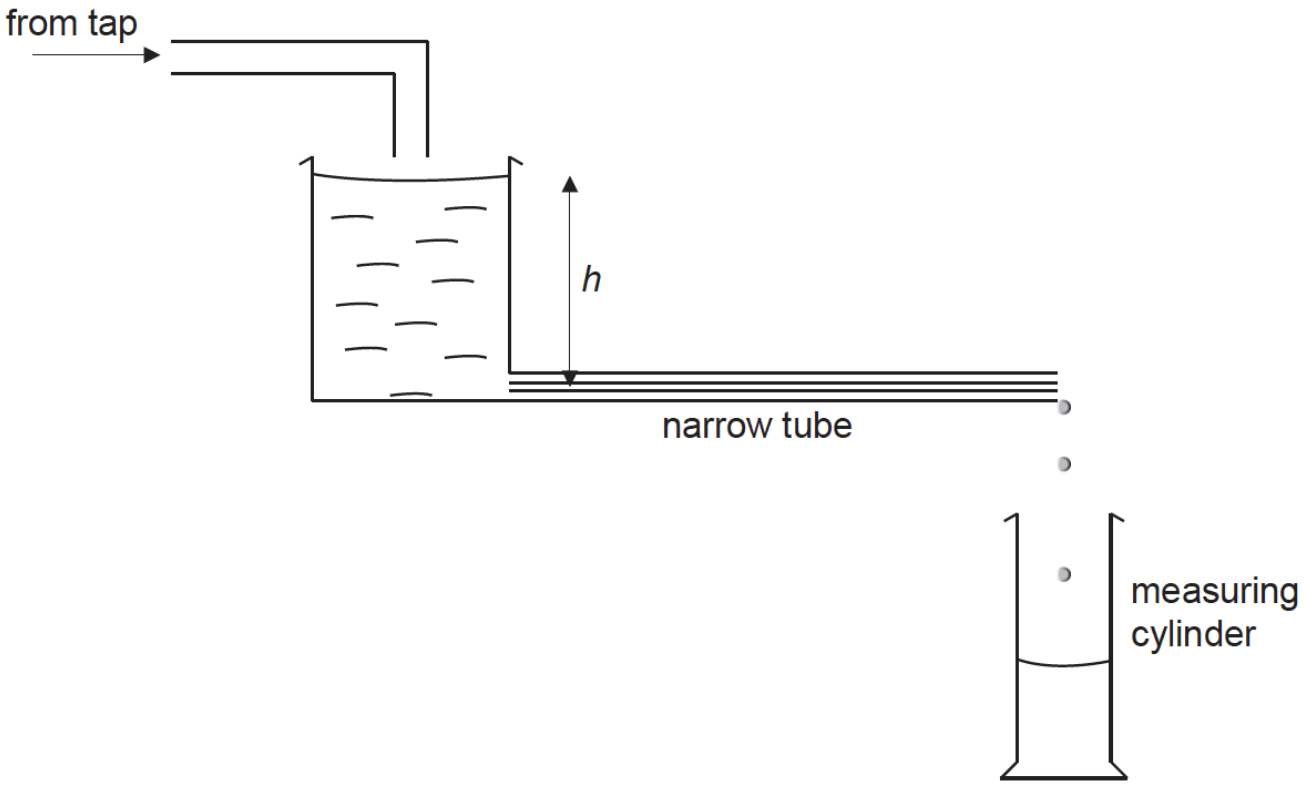
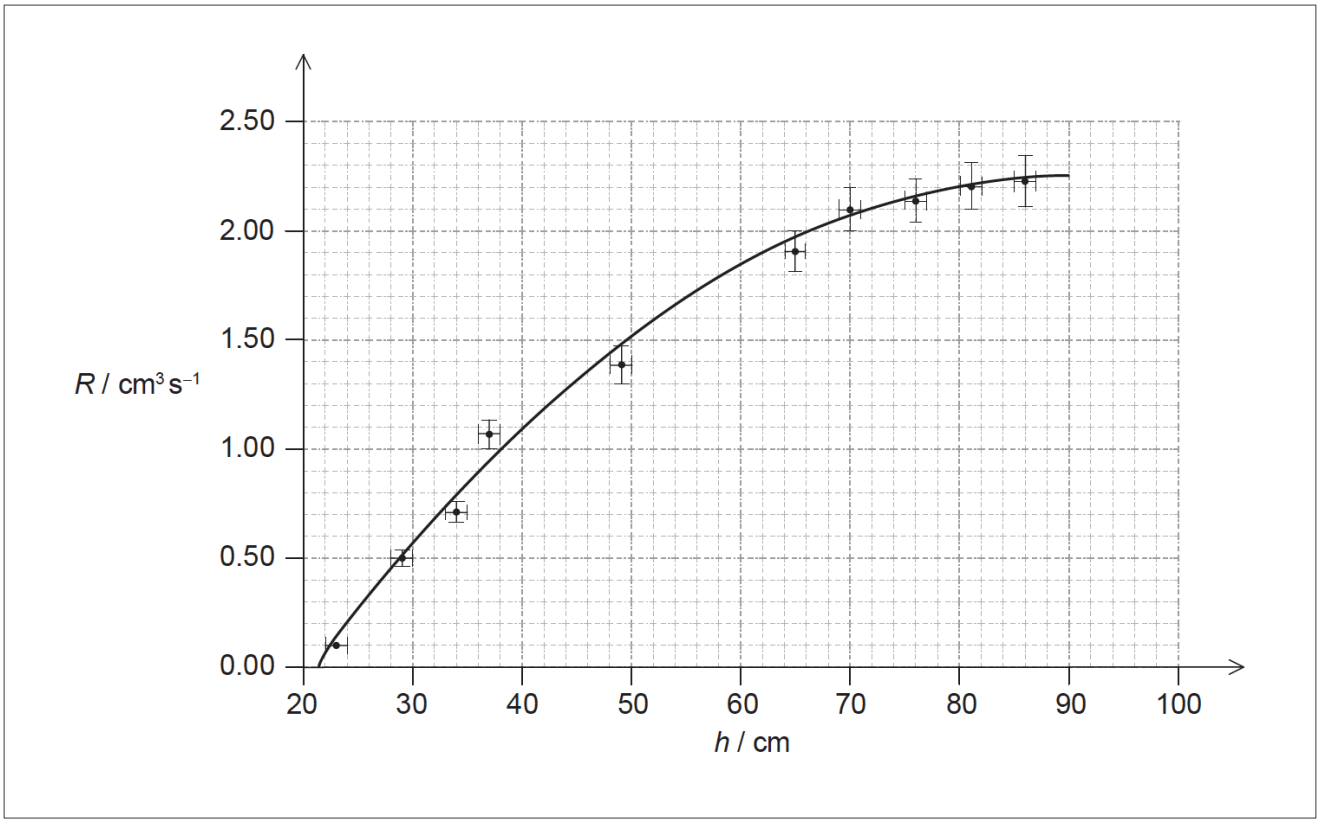
Measurement-practice-extended *[76 marks]*

This question is about the flow of liquids.

A student carries out an experiment to investigate how the rate of flow R of water through a narrow tube varies with the pressure difference across the tube. The pressure difference is proportional to the height h shown in the diagram. The student measures h in cm with a metre ruler. R is obtained by measuring the volume of water collected in a measuring cylinder in a time of 100s.



1a. The student enters the data on a spreadsheet and produces the graph and trend line shown below. *[5 marks]*



The data point for $h = 57$cm, $R = 1.70\text{cm}^3\text{s}^{-1}$ has not been shown on the graph. The student estimates the uncertainties in all values of h to be ±1cm and the uncertainties in the values of R to be ±5%.

(i) On the graph, draw the missing data point.

(ii) On the graph, draw the vertical error bar for this data point.

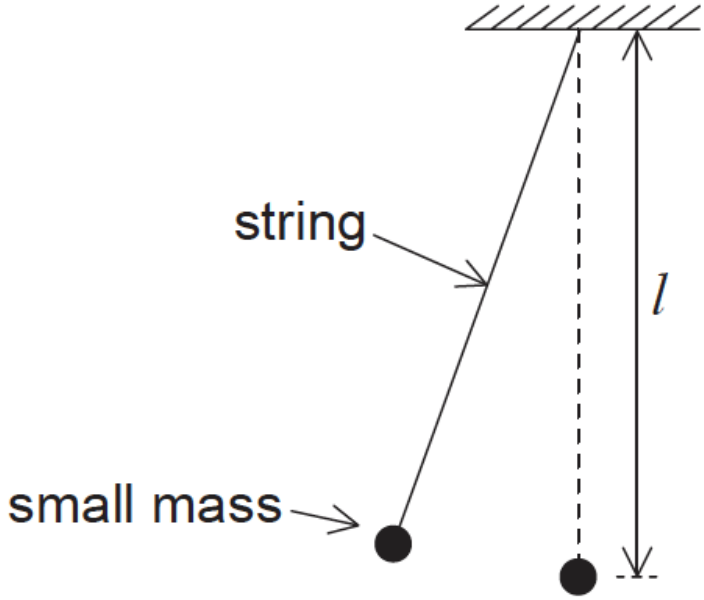
(iii) Comment on why the trend line is not a perfect match for the data.

(iv) Explain why the student's estimate of a 5% uncertainty in all values for R is unlikely to be correct.

1b. The student estimates that the uncertainty in timing 100s is ±1s. Using the data on the graph, deduce the absolute uncertainty in the volume of water collected when $R = 2.1\text{cm}^3\text{s}^{-1}$. *[4 marks]*

Data analysis question.

A simple pendulum of length *l* consists of a small mass attached to the end of a light string.



The time *T* taken for the mass to swing through one cycle is given by

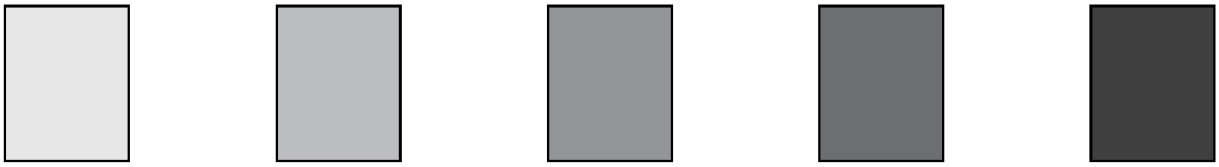
$$T = 2\pi\sqrt{\frac{l}{g}}$$

where *g* is the acceleration due to gravity.

2. A student measures *T* for one length *l* to determine the value of *g*. Time $T = 1.9s \pm 0.1s$ and length $l = 0.880m \pm 0.001m$. Calculate the fractional uncertainty in *g*. *[2 marks]*

Data analysis question.

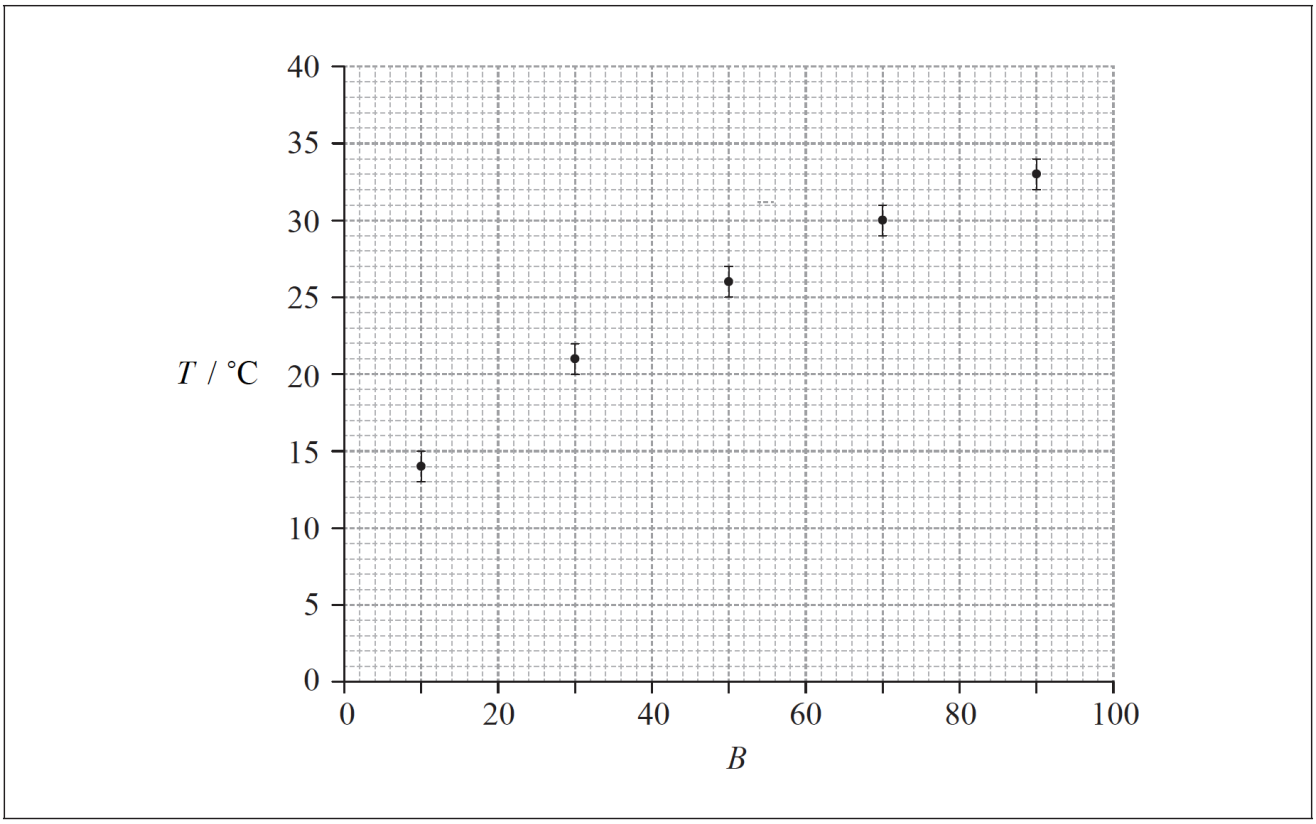
Connie and Sophie investigate the effect of colour on heat absorption. They make grey paint by mixing black and white paint in different ratios. Five identical tin cans are painted in five different shades of grey.



10% black paint 30% black paint 50% black paint 70% black paint 90% black paint

Connie and Sophie put an equal amount of water at the same initial temperature into each can. They leave the cans under a heat lamp at equal distances from the lamp. They measure the temperature increase of the water, T, in each can after one hour.

3a. Connie suggests that T is proportional to B, where B is the percentage of black in the paint. To test this hypothesis, she plots a graph of T against B, as shown on the axes below. The uncertainty in T is shown and the uncertainty in B is negligible. *[6 marks]*



(i) State the value of the absolute uncertainty in T.

(ii) Comment on the fractional uncertainty for the measurement of T for B=10 and the measurement of T for B=90.

(iii) On the graph opposite, draw a best-fit line for the data.

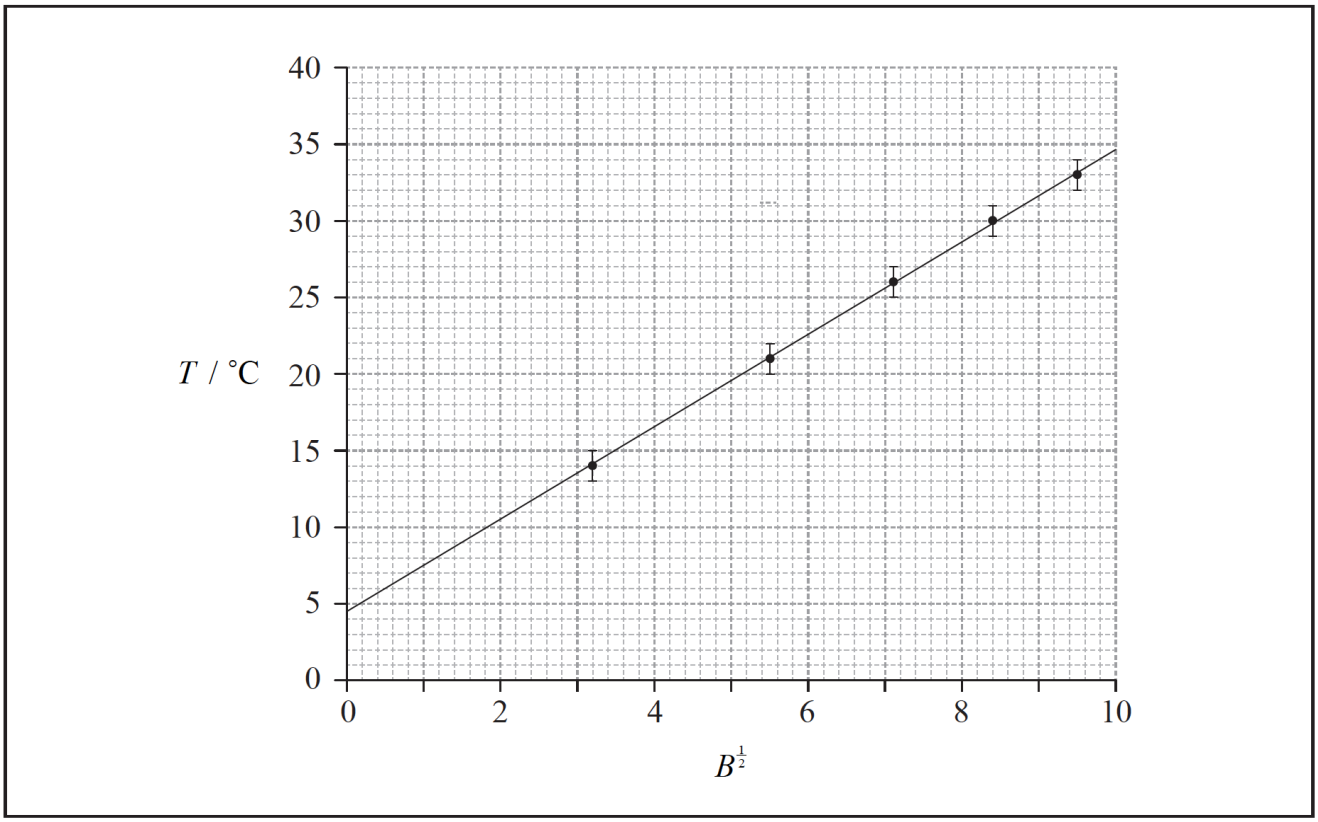
(iv) Outline why the data do not support the hypothesis that T is proportional to B.

3b. Sophie suggests that the relationship between T and B is of the form *[5 marks]*

$$T = kB^{\frac{1}{2}} + c$$

where k and c are constants.

To test whether or not the data support this relationship, a graph of T against $B^{\frac{1}{2}}$ is plotted as shown below. The uncertainty in T is shown and the uncertainty in $B^{\frac{1}{2}}$ is negligible.



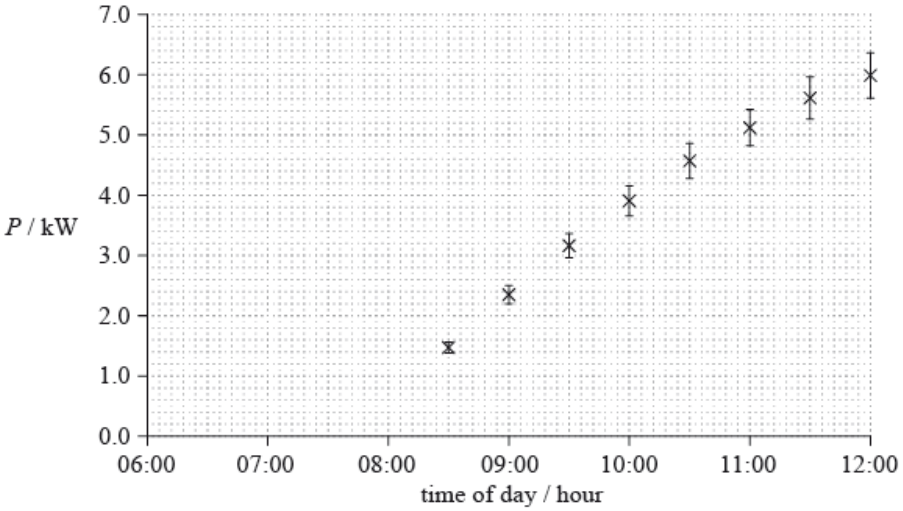
(i) Use the graph to determine the value of c with its uncertainty.

(ii) State the unit of k.

Data analysis question.

An array of photovoltaic cells is used to provide electrical energy for a house. When the array produces more power than is consumed in the house, the excess power is fed back into the mains electrical supply for use by other consumers.

The graph shows how the power *P* produced by the array varies with the time of day. The error bars show the uncertainty in the power supplied. The uncertainty in the time is too small to be shown.

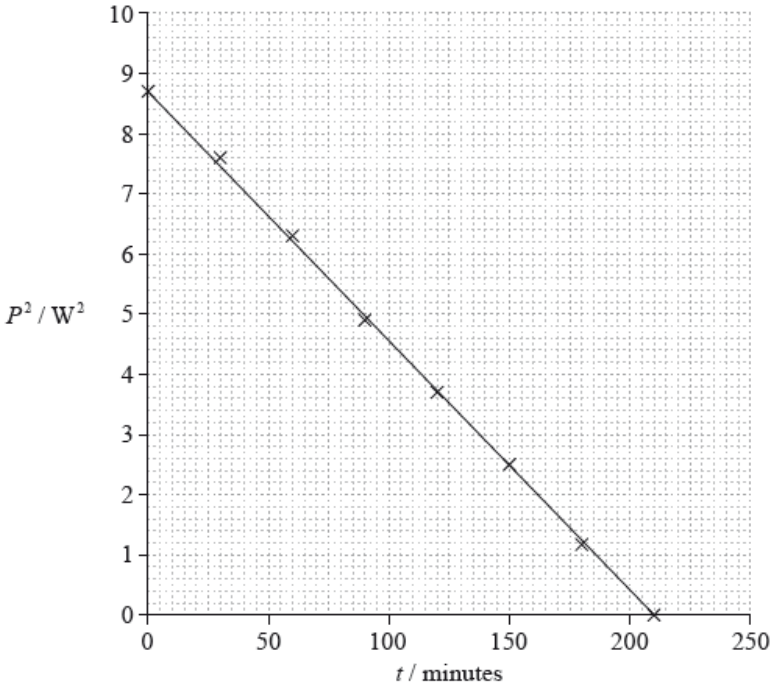


4a. Using the graph, estimate the time of day at which the array begins to generate energy. *[2 marks]*

4b. The average power consumed in the house between 08:00 and 12:00 is 2.0 kW. Determine the energy supplied by the array to the mains electrical supply between 08:00 and 12:00. *[3 marks]*

4c. The power *P* produced by the array is calculated from the generated emf *V* and the fixed resistance *R* of the array using the equation $\frac{V^2}{R}$. The uncertainty in the value of *R* is 2%. Calculate the percentage uncertainty in *V* at 12:00. *[3 marks]*

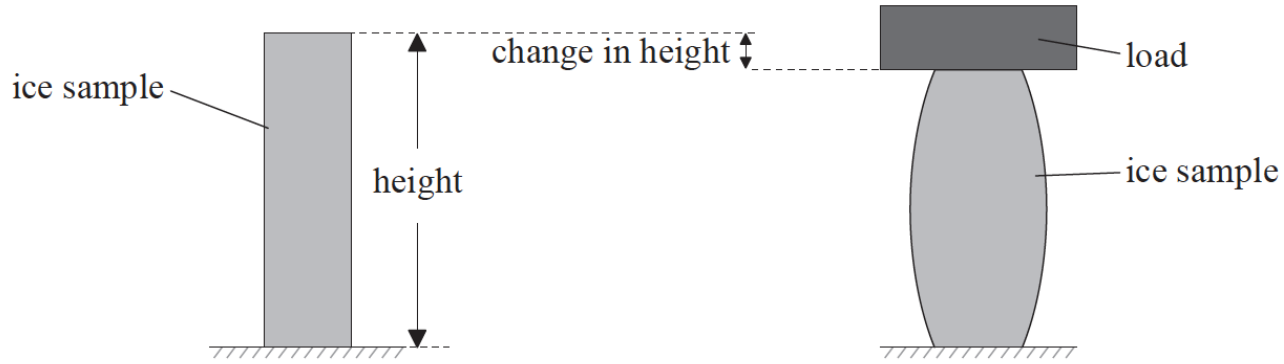
4d. Later that day a second set of data was collected starting at $t = 0$. The variation of P^2 with time t since the start of this second data collection is shown in the graph. *[3 marks]*



Using the graph, determine the relationship between P^2 and t.

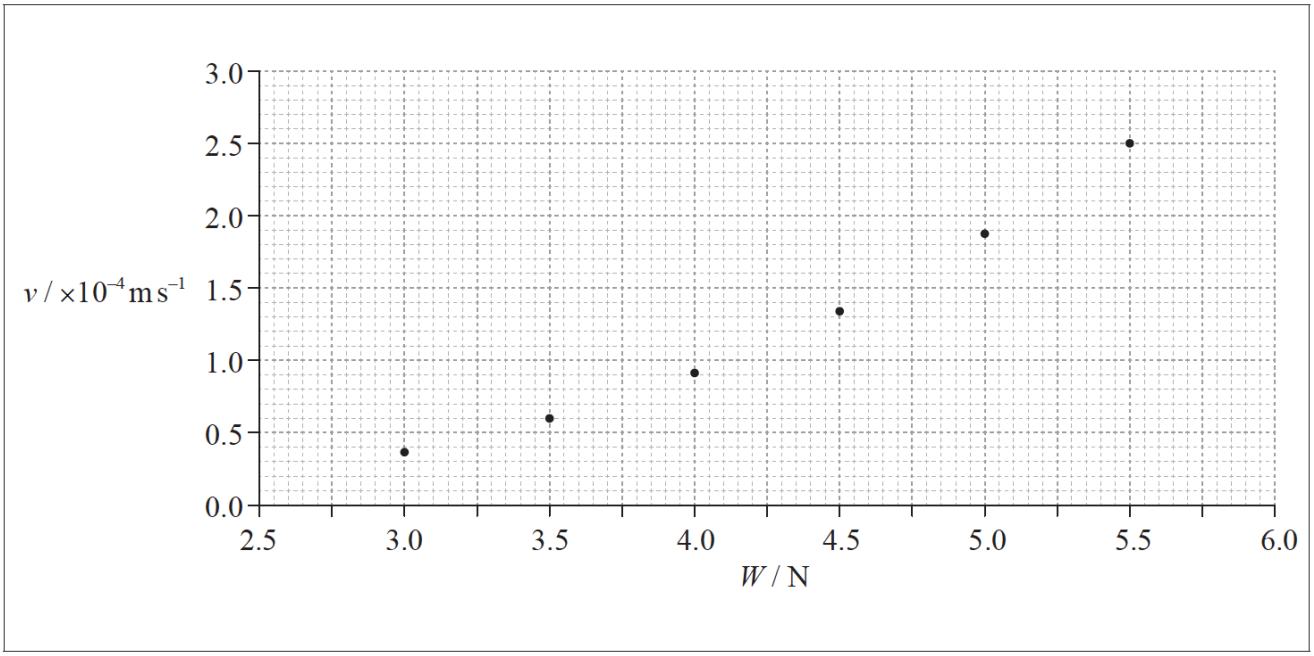
Data analysis question.

The movement of glaciers can be modelled by applying a load to a sample of ice.



After the load has been applied, it is observed to move downwards at a constant speed v as the ice deforms. The constant speed v is measured for different loads. The graph shows the variation of v with load W for a number of identical samples of ice.

The data points are plotted below.



The uncertainty in v is ±20 μm s^{-1} and the uncertainty in W is negligible.

5a. (i) On the graph opposite, draw error bars on the first and last points to show the uncertainty in v. *[2 marks]*

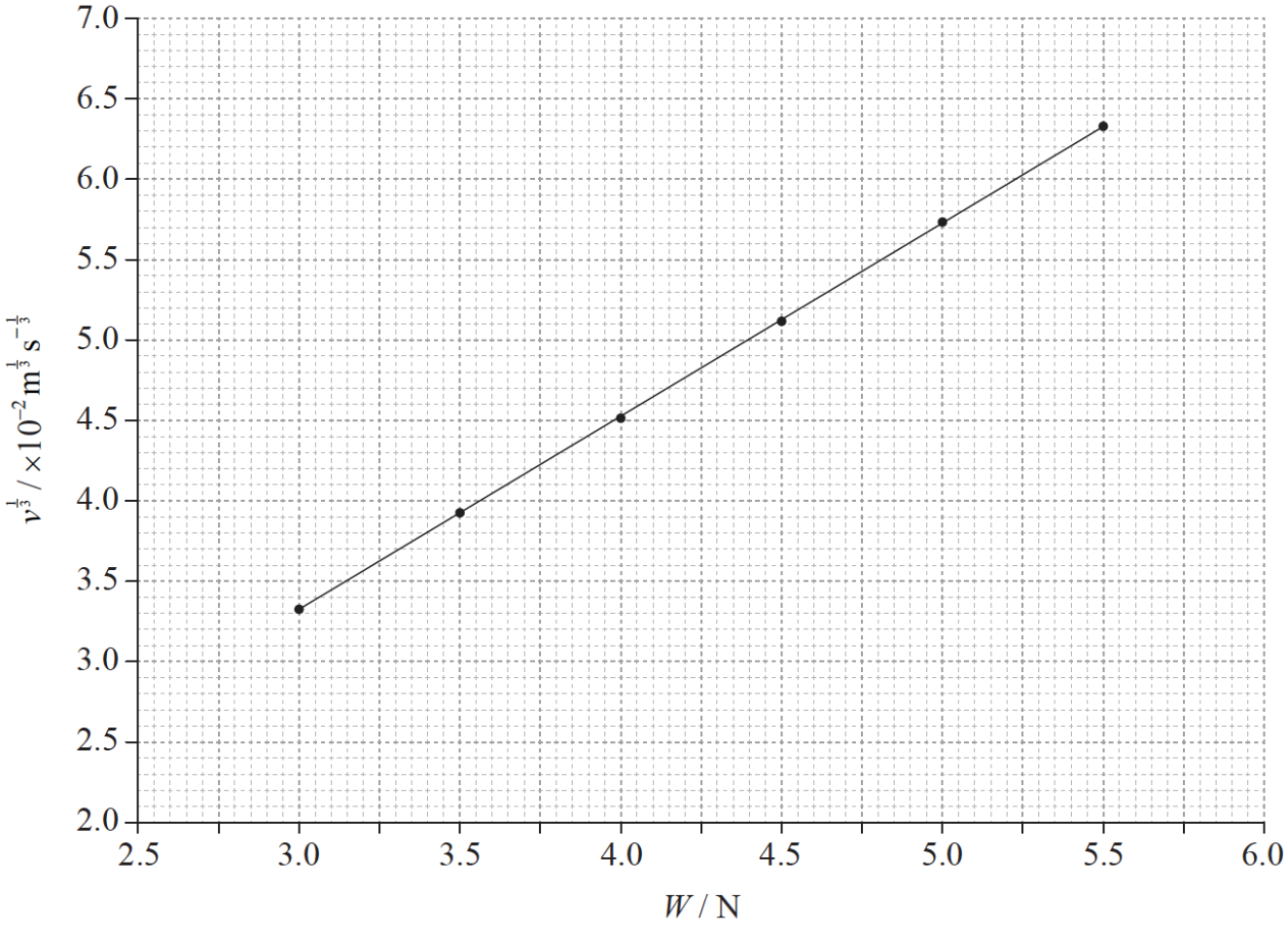
(ii) On the graph opposite, draw the line of best-fit for the data points.

5b. Theory suggests that the relation between v and W is *[3 marks]*

$$v = kW^3$$

where k is a constant.

To test this hypothesis a graph of $v^{\frac{1}{3}}$ against W is plotted.



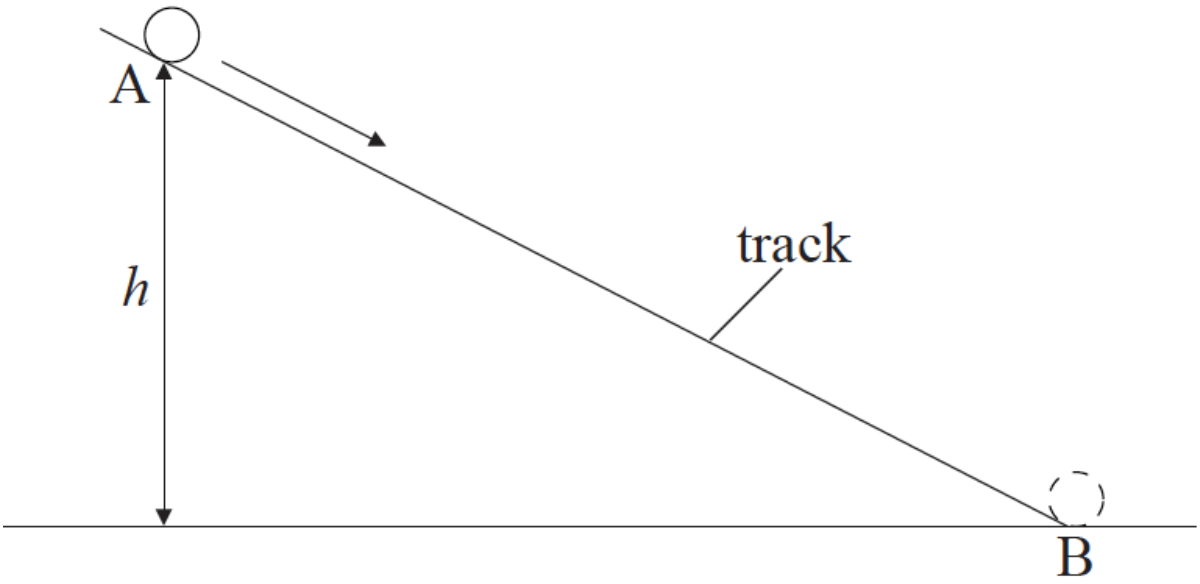
At W=5.5 N the speed is 250±20 μm s^{-1}.

Calculate the uncertainty in $v^{\frac{1}{3}}$ for a load of 5.5 N.

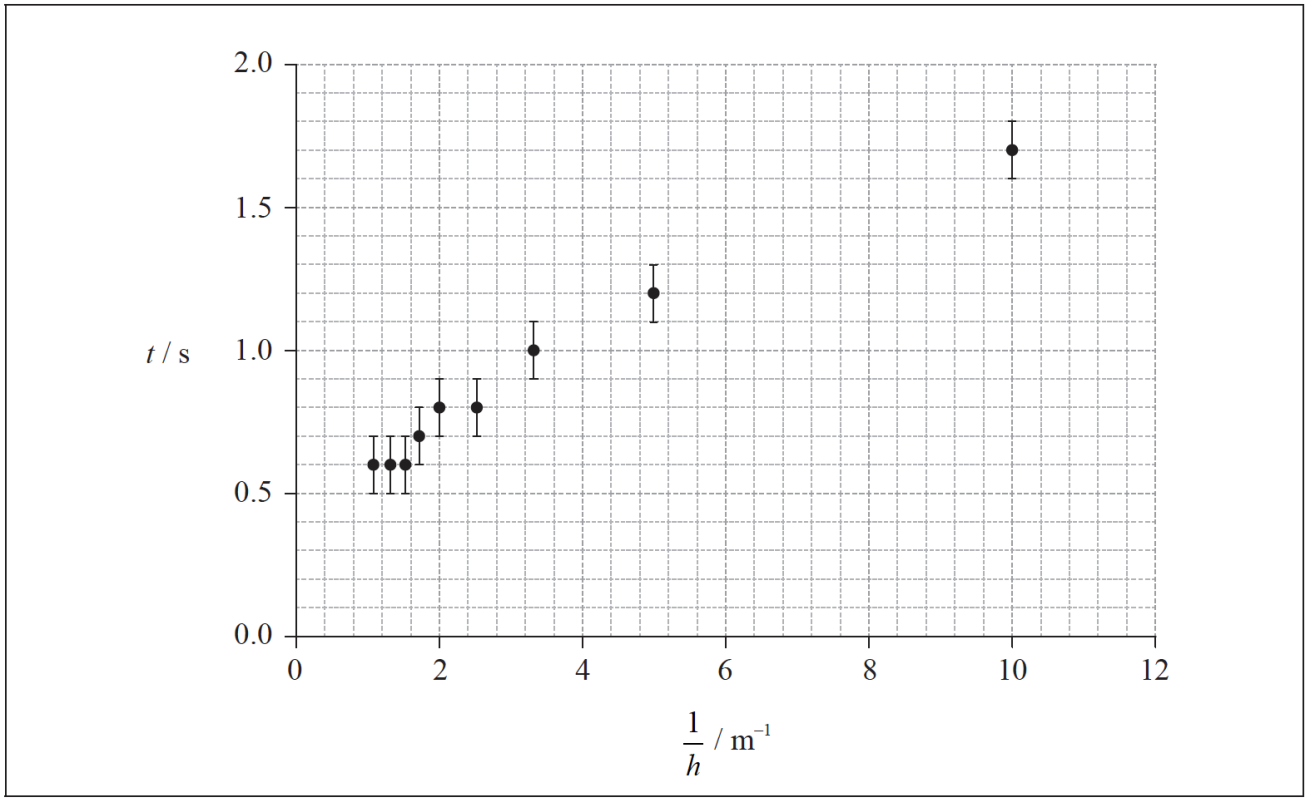
5c. (i) Using the graph in (c), determine k without its uncertainty. *[5 marks]*

(ii) State an appropriate unit for your answer to (d)(i).

Data analysis question.
A small sphere rolls down a track of constant length AB. The sphere is released from rest at A.
The time t that the sphere takes to roll from A to B is measured for different values of height h.



A student suggests that t is proportional to $\frac{1}{h}$. To test this hypothesis a graph of t against $\frac{1}{h}$ is plotted as shown on the axes below. The uncertainty in t is shown and the uncertainty in $\frac{1}{h}$ is negligible.



6a. (i) Draw the straight line that best fits the data. *[2 marks]*

(ii) State why the data do not support the hypothesis.

6b. Another student suggests that the relationship between t and h is of the *[9 marks]*

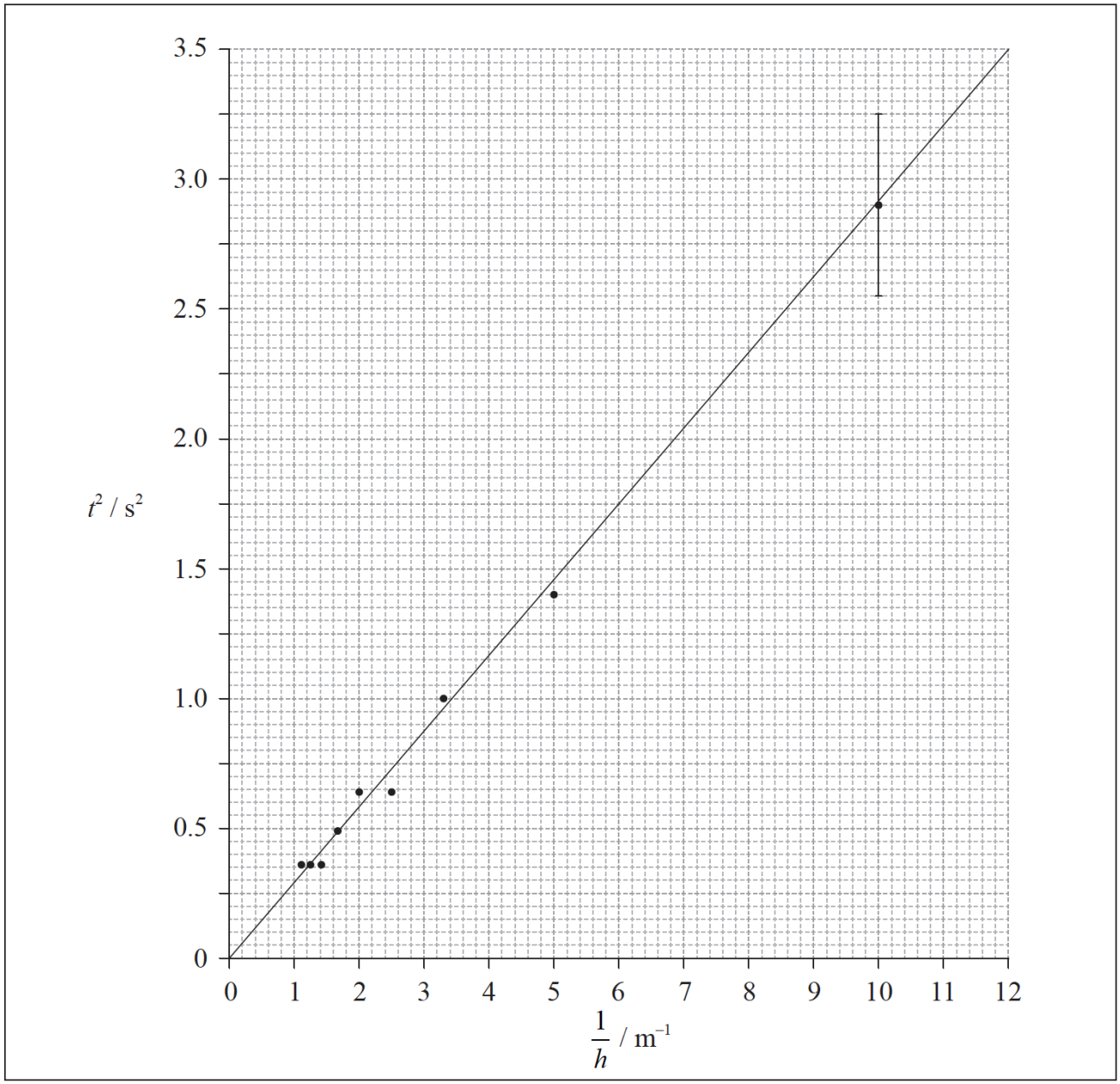
form

$$t = k\sqrt{\frac{1}{h}}$$

where k is a constant.

To test whether or not the data support this relationship, a graph of t^2 against $\frac{1}{h}$ is plotted as shown below.

The best-fit line takes into account the uncertainties for all data points.



The uncertainty in t^2 for the data point where $\frac{1}{h} = 10.0\text{m}^{-1}$ is shown as an error bar on the graph.

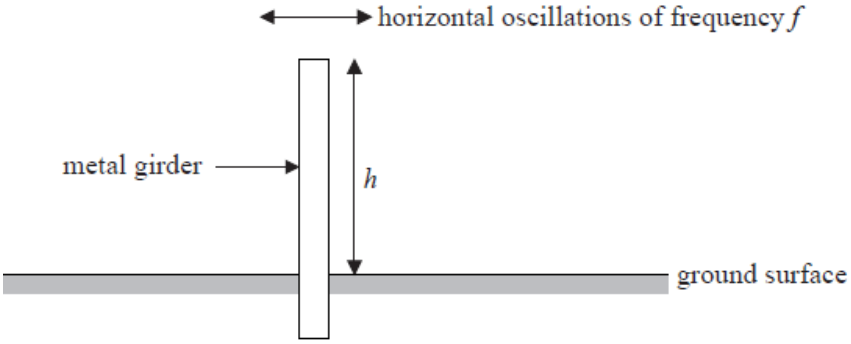
(i) State the value of the uncertainty in t^2 for $\frac{1}{h} = 10.0\text{m}^{-1}$.

(ii) Calculate the uncertainty in t^2 when $t = 0.8 \pm 0.1$s. Give your answer to an appropriate number of significant digits.

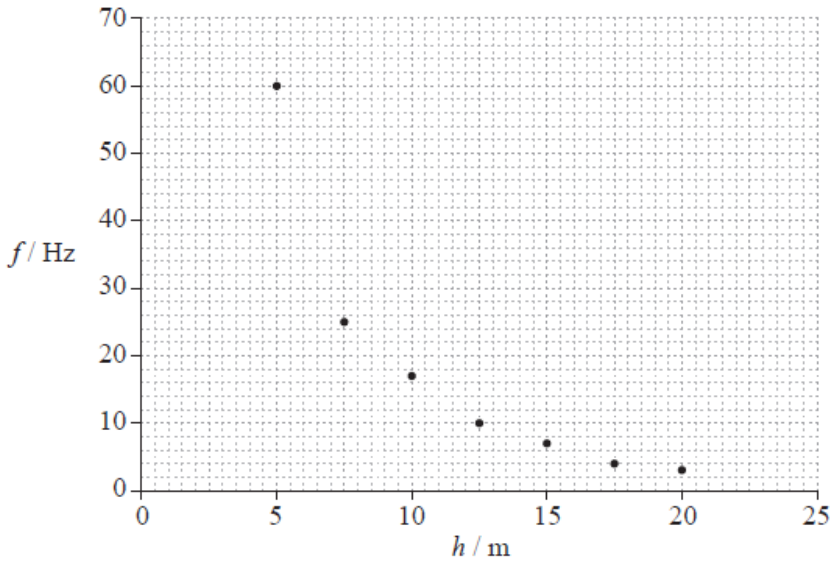
(iii) Use the graph to determine the value of k. Do not calculate its uncertainty.

(iv) State the unit of k.

Data analysis question.
Metal girders are often used in buildings that have been constructed to withstand earthquakes. To aid the design of these buildings, experiments are undertaken to measure how the natural frequency ƒ of horizontal oscillations of metal girders varies with their dimensions. In an experiment, ƒ was measured for vertically supported girders of the same cross-sectional area but with different heights h.



The graph shows the plotted data for this experiment. Uncertainties in the data are not shown.



7a. Draw a best-fit line for the data. *[1 mark]*

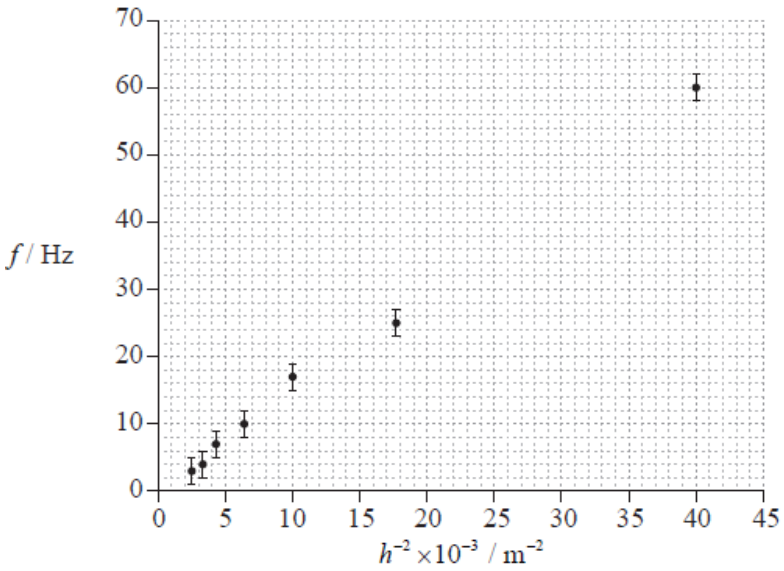
7b. It is hypothesized that the frequency ƒ is inversely proportional to the height h. *[4 marks]*

By choosing **two** well separated points on the best-fit line that you have drawn in (a), show that this hypothesis is incorrect.

7c. Another suggestion is that the relationship between ƒ and h is of the form shown below, where k is a constant. *[5 marks]*

$$f = \frac{k}{h^2}$$

The graph shows a plot of ƒ against h^{-2}.



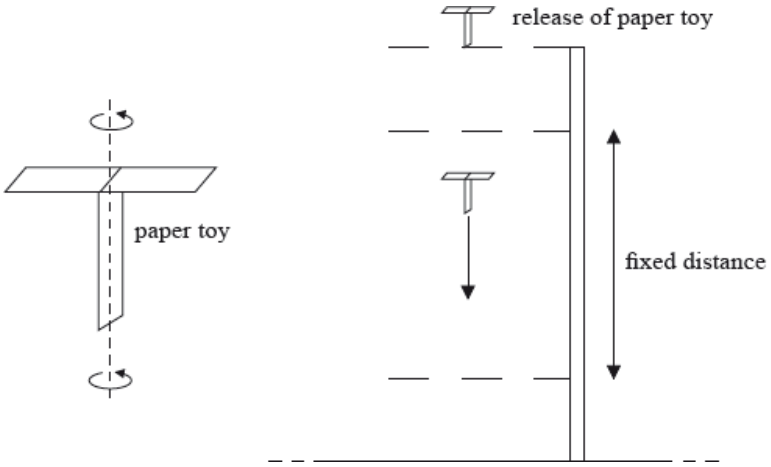
The uncertainties in h^{-2} are too small to be shown.

(i) Draw a best-fit line for the data that supports the relationship $f = \frac{k}{h^2}$.

(ii) Determine, using the graph, the constant k.

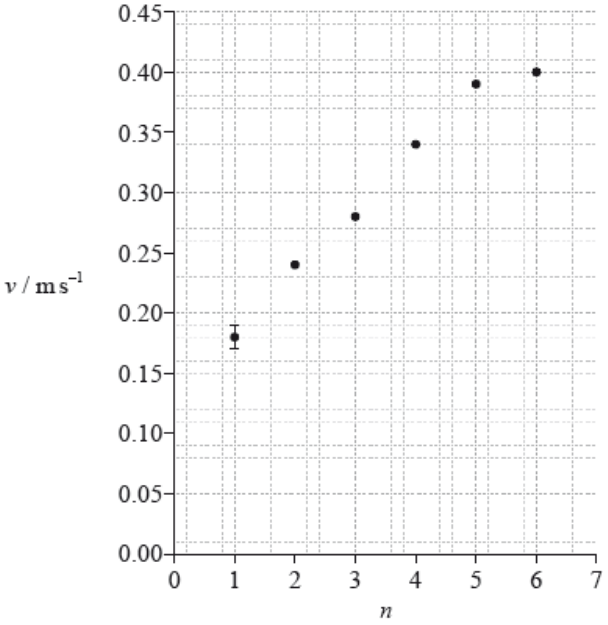
7d. State **one** reason why the results of the experiment could not be used to predict the natural frequency of oscillation for girders of height 50 m. *[1 mark]*

A student performs an experiment with a paper toy that rotates as it falls slowly through the air. After release, the paper toy quickly attains a constant vertical speed as measured over a fixed vertical distance.



The aim of the experiment was to find how the terminal speed of the paper toy varies with its weight. The weight of the paper toy was changed by using different numbers of paper sheets in its construction.

The graph shows a plot of the terminal speed v of the paper toy (calculated from the raw data) and the number of paper sheets n used to construct the toy. The uncertainty in v for $n = 1$ is shown by the error bar.



The fixed distance is 0.75 m and has an absolute uncertainty of 0.01 m. The percentage uncertainty in the time taken to fall through the fixed distance is 5%.

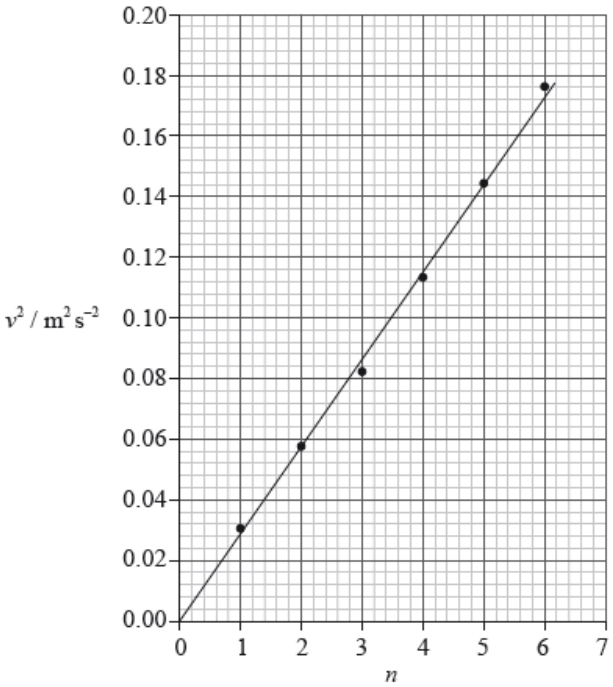
8a. Calculate the absolute uncertainty in the terminal speed of the paper toy for $n = 6$. *[3 marks]*

8b. On the graph, draw an error bar on the point corresponding to $n = 6$. *[1 mark]*

8c. On the graph, draw a line of best-fit for the data points. *[1 mark]*

8d. The student hypothesizes that *v* is proportional to *n*. Use the data points for $n = 2$ and $n = 4$ from the graph opposite to show that this hypothesis is incorrect. *[3 marks]*

8e. Another student hypothesized that v might be proportional to n. To verify this hypothesis he plotted a graph of v^2 against $\sqrt{n}$ as shown below. *[3 marks]*



Explain how the graph verifies the hypothesis that v is proportional to $\sqrt{n}$.