Section 8.4 - Wave Speed

Learning Goal: By the end of today, I will be able to calculate the speed of a wave travelling in a medium such as a string or air.

Frequency is the inverse of Period

$$f = \frac{Cycles}{time} \qquad T = \frac{time}{Cycle}$$

Example

$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

$$f = 5Hz$$

$$f = \frac{5cycles}{1\sec}$$

$$T = \frac{1\sec}{5\,cycles}$$

$$T = 0.2\ \sec$$

$$f = \frac{1}{0.2}$$

$$f = 5$$

Universal Wave Equation

Wave Speed (m/s) $$v = \frac{\lambda}{T}$$ wave length (m)

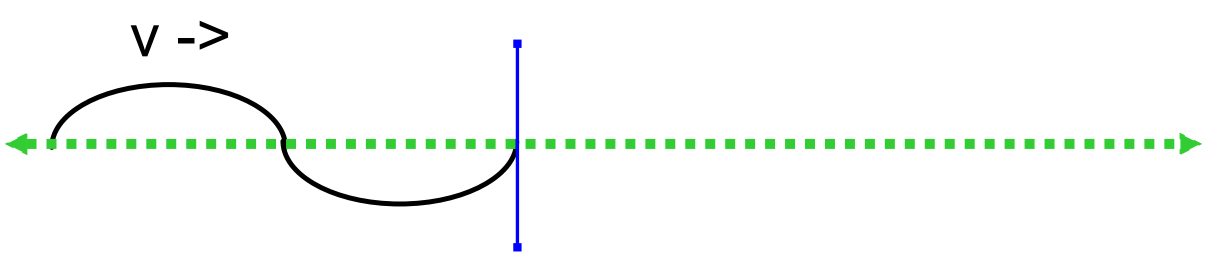
Period (time for one full cycle)

Given: $T = \frac{1}{f}$

then, substituting in for T $$v = \frac{\lambda}{\left(\frac{1}{f}\right)}$$

$$v = f\lambda$$ Universal wave equation

Unit analysis: $$\frac{m}{s} = \frac{cycles}{\sec} \times \frac{m}{cycle}$$



Sample Problem 1: Calculating Wave Speed

A harp string supports a wave with a wavelength of 2.3 m and a frequency of 220.0 Hz. Calculate its wave speed.

Given: $\lambda = 2.3$ m; $f = 220.0$ Hz

Required: v

Sample Problem 2: Calculating Wavelength

A trumpet produces a sound wave that is observed travelling at 350 m/s with a frequency of 1046.50 Hz. Calculate the wavelength of the sound wave.

Given: $v = 350$ m/s; $f = 1046.50$ Hz

Required: λ

Analysis: Rearrange the universal wave equation to solve for wavelength: $v = f\lambda$

Practice

1. If a wave has a frequency of 230 Hz and a wavelength of 2.3 m, what is its speed? T/I
   [redacted]
2. If a wave has a speed of 1500 m/s and a frequency of 11 Hz, what is its wavelength? T/I
   [redacted]
3. If a wave has a speed of 405 m/s and a wavelength of 2.0 m, what is its frequency? T/I
   [redacted]

Factors That Affect Wave Speed

The transfer of energy using waves is more efficient if the particle vibrations do not absorb much energy. For example, a more rigid object such as a soccer ball tends to bounce more effectively if it is fully inflated. If the atoms comprising an object are linked by strong intermolecular forces, the wave energy is transmitted more efficiently and thus the wave speed is faster. If these forces are not as strong, then energy transmission is less efficient and thus slower.

Temperature

In the case of gases, you might think that cooler gases are more effective at transmitting sound because they are denser. However, usually the converse is true because, with an increase in temperature, the molecules move faster and transfer their kinetic energy more efficiently (**Figure 1**).

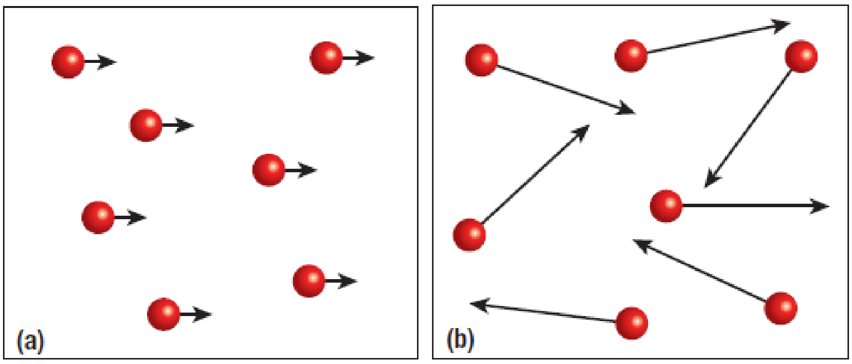


Figure 1 Comparing transmission of sound through (a) a cool gas and (b) a warm gas. The warm molecules jostle neighbouring molecules more rapidly, thus increasing the rate of sound energy transfer.

Linear Density and Tension

The speed of a wave along a string, such as a violin or guitar string, is governed by the properties of the string (**Figure 2**). A string's **linear density**, or mass per unit distance, determines how much force it will take to make the string vibrate. Linear density, μ, is calculated using the equation

$$\mu = \frac{m}{L}$$

where m is the mass of the string, in kilograms, and L is its length, in metres.

Another variable affecting wave speed is tension. A loose string, for example, will quickly absorb all of the energy. A taut (tight) string, however, will transmit energy very effectively. Linear density and tension are the only variables that control the speed that waves can travel along a string. The equation for the speed of a wave along a string is

$$v = \sqrt{\frac{F_T}{\mu}}$$

Proof

where F_T is the tension in the string (in newtons) and μ is the linear density (in kilograms per metre)

Linear Density and Tension in a String Instrument

Figure 2 The diameters of the guitar strings shown here are getting progressively larger from left to right. The linear density is therefore increasing from left to right. The speed of sound is progressively slower in these strings.

Sample Problem 1: Determining String Tension

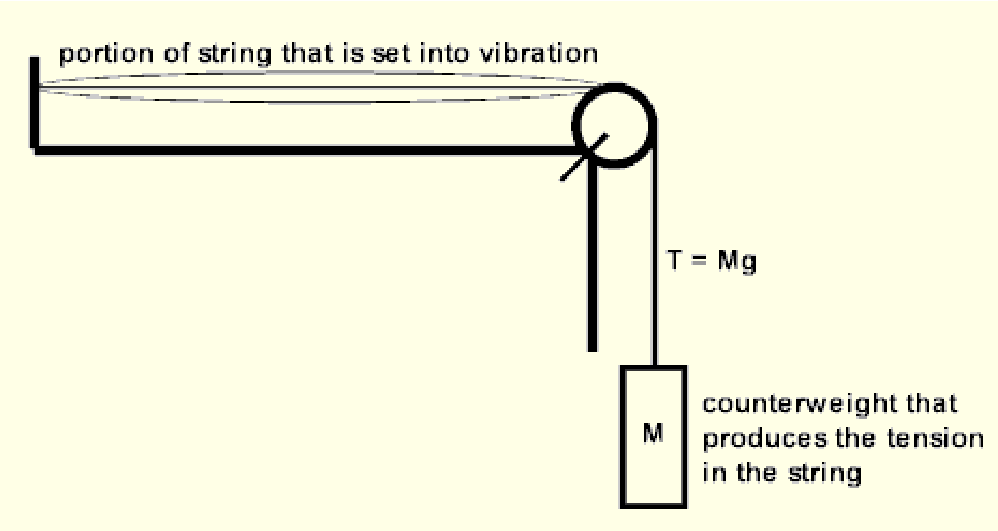
On your class wave machine, you have a string of mass 350 g and length 2.3 m. You would like to send a wave along this string at a speed of 50.0 m/s. What must the tension of the string be?

Given: $m = 350$ g or 0.350 kg; $L = 2.3$ m; $v = 50.0$ m/s

Required: F_T

$$\mu = \frac{m}{L}$$

$$v = \sqrt{\frac{F_T}{\mu}}$$


portion of string that is set into vibration
T = Mg
M
counterweight that produces the tension in the string

Demo

Practice

1. If a 2.5 m long string on the same wave machine has a tension of 240 N, and the wave speed is 300 m/s, what is the mass of the string? T/I [redacted]
2. If a wave machine string has a linear density of 0.2 kg/m and a wave speed of 200 m/s, what tension is required? T/I [redacted]
3. If a string on a wave machine has a linear density of 0.011 kg/m and a tension of 250 N, what is the wave speed? T/I [redacted]

8.4 Questions

1. A wave has a speed of 123 m/s and a frequency of 230 Hz. What is its wavelength? T/I
2. A guitar string has a tension of 37 N. The linear density is 0.03 g/m. What is the speed of sound along this string? T/I
3. The period of a sound wave from a piano is 1.20×10^{-3} s. If the speed of the wave in the air is 3.40×10^{2} m/s, what is its wavelength? T/I
4. Earthquakes produce seismic waves, which travel through Earth. Primary waves, or P-waves, are longitudinal. They can travel through both solids and liquids. Secondary waves, or S-waves, are transverse. They can travel through solids only. P-waves travel at approximately 8.0 km/s, and S-waves travel at approximately 4.5 km/s. Following an earthquake, vibrations are recorded at seismological stations around the world. K/U T/I A
   (a) Calculate how long P-waves and S-waves take to travel from an earthquake to a seismological station that is 2.4×10^{3} km away. Express your answers in minutes.
   (b) Why do you think that transverse waves are called secondary waves?
   (c) By referring to **Figure 3**, explain how observing P-waves and S-waves helps geophysicists analyze the structure of Earth's interior.
5. Predict what happens to the wavelength of a wave on a string when the frequency is doubled. Assume that the tension in the string remains the same. Confirm your prediction mathematically. K/U T/I

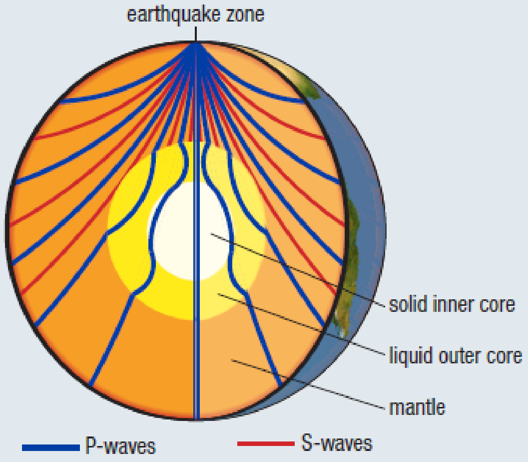


Figure 3

6. Predict what happens to the speed of a wave on a string when the frequency is doubled. Assume that the tension in the string remains the same. Confirm your prediction mathematically. K/U T/I
7. By what factor would you have to multiply the tension in a taut spring in order to double the wave speed? Confirm your answer mathematically. T/I C
8. Develop the equation for wave speed on a string. Use research if you wish. T/I C

Section 8.4 #1-6