5.1 Work

Figure 1 Olympic weightlifter Marilou Dozois-Prevost did work on a barbell while lifting it over her head.

It took a lot of work for Canadian Olympic weightlifter Marilou Dozois-Prevost to lift a 76 kg barbell over her head at the 2008 Beijing Olympics (**Figure 1**). In weightlifting, you do work by moving weights from the ground to a position above your head. To lift the weights, you apply an upward force that displaces (moves) the weights in the direction of the force. This form of work is called mechanical work. In science, **mechanical work** is done on an object when a force displaces the object in the direction of the force or a component of the force.

mechanical work (W) applying a force on an object that displaces the object in the direction of the force or a component of the force

Work Done by a Constant Force

Mathematically, the mechanical work, W, done by a force on an object is the product of the magnitude of the force, F, and the magnitude of the displacement of the object, Δd:

$$W = F\Delta d$$

This equation applies only to cases where the magnitude of the force is constant and where the force and the displacement are in the same direction. Since work is a product of the magnitude of the force and the magnitude of the displacement, the symbols for force and displacement in the equation $W = F\Delta d$ are written without the usual vector notations $(\vec{F}, \Delta\vec{d})$. Work is a scalar quantity; there is no direction associated with work.

The SI unit for work is the newton metre (N·m). The newton metre is called the joule (J) in honour of James Prescott Joule, an English physicist who investigated work, energy, and heat (1 N·m = 1 J). Notice that we do not consider the amount of time the force acts on an object or the velocities or accelerations of the object in the calculation of work.

LEARNING TIP

Describing Work
When describing work, you should always mention the object that does the work and the object that the work is done on.

Work Done When Force and Displacement Are in the Same Direction

The equation $W = F\Delta d$ may be used to calculate the amount of mechanical work done on an object when force and displacement are in the same direction. In the following Tutorial, you will solve two Sample Problems involving mechanical work in which a force displaces an object in the same direction as the force.

Tutorial 1 Using the Equation $W = F\Delta d$ to Calculate Work Done

In this Tutorial, you will use the equation $W = F\Delta d$ to calculate the amount of work done by a force when the force and displacement are in the same direction.

Sample Problem 1

How much mechanical work does a store manager do on a grocery cart if she applies a force with a magnitude of 25 N in the forward direction and displaces the cart 3.5 m in the same direction (**Figure 2**)?

We are given the force, F_a, and the cart's displacement, Δd. Since the force and displacement are in the same direction, we may use the equation $W = F_a\Delta d$ to solve the problem.

Given: $F_a = 25$ N; $\Delta d = 3.5$ m

Required: W

Analysis: $W = F_a\Delta d$

Figure 2

Solution: $W = F_a \Delta d$

$= (25\ \text{N})(3.5\ \text{m})$

$= 88\ \text{N·m}$

$W = 88\ \text{J}$

Statement: The store manager does 88 J of mechanical work on the grocery cart.

Sample Problem 2

A curler applies a force of 15.0 N on a curling stone and accelerates the stone from rest to a speed of 8.00 m/s in 3.50 s. Assuming that the ice surface is level and frictionless, how much mechanical work does the curler do on the stone?

In this problem, we are given the force but not the stone's displacement. However, in Chapter 1, you learned that the displacement of an accelerating object may be determined using the equation $\Delta d = \left(\frac{v_1 + v_2}{2}\right)\Delta t$ if you know the initial speed, v_1, the final speed, v_2, and the time interval, Δt, in which the object's change in speed occurs.

Given: $F_a = 15.0\ \text{N}$; $v_1 = 0\ \text{m/s}$; $v_2 = 8.00\ \text{m/s}$; $\Delta t = 3.50\ \text{s}$

Required: W

Analysis: $\Delta d = \left(\frac{v_1 + v_2}{2}\right)\Delta t$; $W = F_a \Delta d$

Solution: First, we calculate Δd using the v_1, v_2, and Δt values given.

$$\Delta d = \left(\frac{v_1 + v_2}{2}\right)\Delta t$$

$$= \left(\frac{0 + 8.00\ \text{m/}\cancel{\text{s}}}{2}\right)(3.50\ \cancel{\text{s}})$$

$$\Delta d = 14.0\ \text{m}$$

We now substitute the values of F_a and Δd into the equation for work and solve.

$$W = F_a \Delta d$$

$$= (15.0\ \text{N})(14.0\ \text{m})$$

$$= 2.10 \times 10^2\ \text{N·m}$$

$$W = 2.10 \times 10^2\ \text{J}$$

Statement: The curler does 2.10×10^2 J of mechanical work on the curling stone.

Practice

1. A 0.50 kg laboratory dynamics cart with an initial velocity of 3.0 m/s [right] accelerates for 2.0 s at 1.2 m/s^2 [right] when pulled by a string. Assume there is no friction acting on the cart. T/I
   (a) Calculate the force exerted by the string on the cart. [ans: 0.60 N]
   (b) Calculate the displacement of the cart. [ans: 8.4 m]
   (c) Calculate the mechanical work done by the string on the cart. [ans: 5.0 J]

Work Done When Force and Displacement Are in Different Directions

In some cases, an object may experience a force in one direction while the object moves in a different direction. For example, this occurs when a person pulls on a suitcase with wheels and a handle (**Figure 3(a)**). The free-body diagram (FBD) for this situation is shown in **Figure 3(b)**. In this case, the applied force is directed toward the person's hips, while the suitcase rolls on the floor in the forward direction. The FBD shows all the forces acting on the suitcase, including the force of gravity (F_g), the normal force (F_N), and the applied force resolved into horizontal (F_h) and vertical (F_v) components.

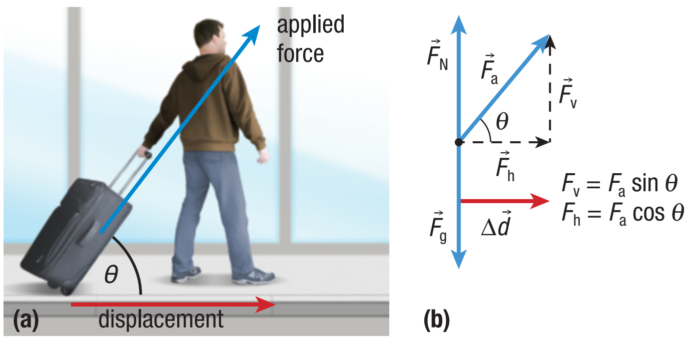

Figure 3 (a) A system diagram of a trolley suitcase displaced to the right (b) FBD of a trolley suitcase displaced to the right

LEARNING **TIP**

Work
Work has a different meaning in science than it does in everyday life. In science, work is done only when a force displaces an object. If a force is applied on an object, but the object does not move, then no work is done. For example, Marilou Dozois-Prevost was applying a force on the barbell while she was lifting it and while she was holding it motionless over her head. However, she did work on the barbell (in the scientific sense) only while she was lifting it, not while holding it over her head as in Figure 1.

(We will assume that the force of friction is negligible.) The FBD also indicates that the applied force vector, $\vec{F}_a$, makes an angle θ with the displacement vector, $\Delta\vec{d}$.

The magnitude of F_h is given by the equation

$$F_h = F_a \cos\theta$$

Since the horizontal component, F_h, is the only force in the direction of the suitcase's displacement, it is the only force that causes the suitcase to move along the floor. Thus, the amount of mechanical work done by F_h on the suitcase may be calculated using the equation $W = F_a(\cos\theta)\Delta d$ or, in general,

$$W = F(\cos\theta)\Delta d$$

where W is the work done, F is the force, and $\cos\theta$ is the angle between the force and the displacement vector, Δd.

What about the vertical component of F_a? F_v does no work on the suitcase because the suitcase is not displaced in the direction of F_v. Since F_v is perpendicular to the suitcase's displacement along the floor ($\theta = 90°$), F_v does no work on the suitcase. This can be calculated using the work equation, $W = F_a(\cos\theta)\Delta d$, where $\theta = 90°$:

$$\begin{aligned} W &= F_a(\cos\theta)\Delta d \\ &= F_a(\cos 90°)\Delta d \\ &= F_a(0)\Delta d, \text{ since } \cos 90° = 0 \\ W &= 0 \text{ J} \end{aligned}$$

This result illustrates an important principle of mechanical work. In general, the work done by a force is zero when the force's direction is perpendicular to the object's displacement; that is, when $\theta = 90°$, $\cos 90° = 0$, and $W = 0$ J.

LEARNING TIP

Dot Products
The scalar product of two vectors is commonly indicated by placing a dot between the vector symbols (e.g., $W = \vec{F} \cdot \Delta\vec{d}$) and unit symbols (N·m). It is for this reason that a scalar product is sometimes called a dot product. A dot product may also be represented by using scalar symbols without the dot between them (as we do in this book), $W = F\Delta d$.

Tutorial 2 Using the Equation $W = F(\cos\theta)\Delta d$ to Calculate Work Done

In this Tutorial, you will analyze two cases in which objects experience a force in one direction and a displacement in a different direction.

CASE 1: AN OBJECT IS DISPLACED BY A FORCE THAT HAS A COMPONENT IN THE DIRECTION OF THE DISPLACEMENT

Sample Problem 1

Calculate the mechanical work done by a custodian on a vacuum cleaner if the custodian exerts an applied force of 50.0 N on the vacuum hose and the hose makes a 30.0° angle with the floor. The vacuum cleaner moves 3.00 m to the right on a level, flat surface. The system diagram for this problem is shown in **Figure 4**.

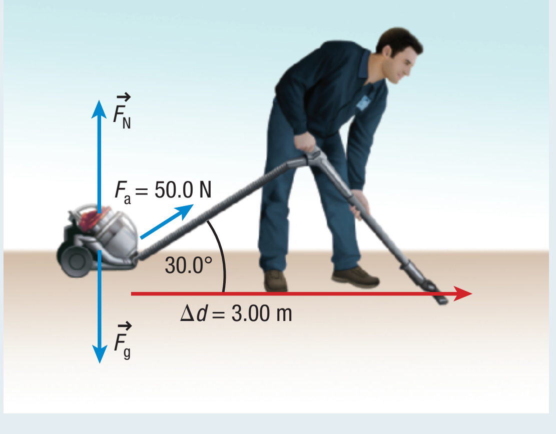

Figure 4

In this problem, the system diagram shows that three forces are acting on the vacuum cleaner: the force of gravity, $\vec{F}_g$; the normal force, $\vec{F}_N$; and the applied force, $\vec{F}_a$. Only the magnitude of the component of $\vec{F}_a$ in the direction of the displacement ($F_a \cos\theta$) does work on the vacuum cleaner. The other two forces ($\vec{F}_N$ and $\vec{F}_g$) do no work on the vacuum cleaner because they are perpendicular to the displacement.

Given: $F_a = 50.0$ N; $\Delta d = 3.00$ m; $\theta = 30.0°$

Required: W

Analysis: $W = F_a(\cos\theta)\Delta d$

Solution:
$$\begin{aligned} W &= F_a(\cos\theta)\Delta d \\ &= (50.0 \text{ N})(\cos 30.0°)(3.00 \text{ m}) \\ &= 1.30 \times 10^2 \text{ N·m} \\ W &= 1.30 \times 10^2 \text{ J} \end{aligned}$$

Statement: The custodian does 1.30×10^2 J of mechanical work on the vacuum cleaner.

CASE 2: AN OBJECT IS DISPLACED, BUT THERE IS NO FORCE OR COMPONENT OF A FORCE IN THE DIRECTION OF THE DISPLACEMENT

Sample Problem 2

Ranbir wears his backpack as he walks forward in a straight hallway. He walks at a constant velocity of 0.8 m/s for a distance of 12 m. How much mechanical work does Ranbir do on his backpack?

Consider the system diagram shown in **Figure 5**. Ranbir walks at constant velocity. Thus, there is no acceleration in the direction of displacement and no applied force on the backpack in that direction. The only applied force on the backpack is the force that Ranbir's shoulders apply on the backpack ($\vec{F}_a$) to oppose the force of gravity on the backpack ($\vec{F}_g$, the backpack's weight). However, neither the applied force nor the force of gravity does work on the backpack because both forces are perpendicular to the displacement. Therefore, Ranbir does no mechanical work at all on the backpack.

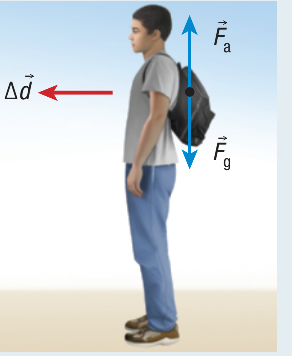

Figure 5

Practice

1. A person cutting a flat lawn pushes a lawnmower with a force of 125 N at an angle of 40.0° below the horizontal for 12.0 m. Determine the mechanical work done by the person on the lawnmower. T/I [ans: 1.15 kJ]
2. A father pulls a child on a toboggan along a flat surface with a rope angled at 35.0° above the horizontal. The total mechanical work done by the father over a horizontal displacement of 50.0 m is 2410 J. Determine the work done on the toboggan by the normal force and the force of gravity, and explain your reasoning. T/I [ans: 0]

Work Done When a Force Fails to Displace an Object

In some cases, a force is applied on an object, but the object does not move: no displacement occurs. For example, when you stand on a solid floor, your body applies a force on the floor equal to your weight ($\vec{F}_a = \vec{F}_g$), but the floor does not move. Your body does no work on the floor because the floor is not displaced. In the following Tutorial, you will examine a situation in which a force is applied but no displacement occurs.

Tutorial 3 Applied Force Causing No Displacement

In the following Sample Problem, you will determine how much work is done when an applied force does not cause displacement.

Sample Problem 1

How much mechanical work is done on a stationary car if a student pushing with a 300 N force fails to displace the car?

Given: $F_a = 300$ N; $\Delta d = 0$ m

Required: W

Analysis: $W = F\Delta d$

Solution:
$$W = F\Delta d$$
$$= (300\text{ N})(0\text{ m})$$
$$W = 0\text{ J}$$

Statement: No mechanical work is done by the student on the car.

In general, if a force fails to displace an object, then the force does no work on the object. Another case in which no work is done occurs when an object moves on a frictionless surface at constant velocity with no horizontal forces acting on it. For example, an air hockey puck that travels 2 m at constant velocity on an air hockey table experiences a displacement but no force (assuming no friction). In this case,

$$F_a = 0 \text{ N and } \Delta d = 2 \text{ m}$$
$$W = F_a \Delta d$$
$$= (0 \text{ N})(2 \text{ m})$$
$$W = 0 \text{ J}$$

No work is done on the puck as it glides at constant velocity on the air hockey table.

Practice

1. There is no work done for each of the following examples. Explain why. K/U C
   (a) A student leans against a brick wall of a large building.
   (b) A space probe coasts at constant velocity toward a planet.
   (c) A textbook is sitting on a shelf.

Figure 6 Marilou Dozois-Prevost applied a force on the barbell in the upward direction, while Earth applied a force of gravity in the downward direction.

Positive and Negative Work

In many cases, an object experiences several forces at the same time. For example, when Marilou Dozois-Prevost lifted weights at the Beijing Olympics, she applied a force, $\vec{F}_a$, on the barbell in the upward direction, while Earth applied a force of gravity, $\vec{F}_g$, on the barbell in the opposite direction (**Figure 6**).

In cases such as this, the total work done on the object is equal to the algebraic sum of the work done by all of the forces acting on the object. We assume that the forces act in the same direction as the object's displacement or in a direction opposite the object's displacement ($\theta = 0°$ or $\theta = 180°$). In the following Tutorial, you will calculate the total work done on an object when the object experiences forces in opposite directions.

Tutorial 4 Positive and Negative Work

In the following Sample Problem, we will consider the total work done on an object when the object experiences two forces: an applied force in the same direction as its displacement and another force (a force of friction) in the opposite direction.

Sample Problem 1

A shopper pushes a shopping cart on a horizontal surface with a horizontal applied force of 41.0 N for 11.0 m. The cart experiences a force of friction of 35.0 N. Calculate the total mechanical work done on the shopping cart.

In this problem, the applied force, $\vec{F}_a$, does work, W_a, on the cart, and the force of friction, $\vec{F}_f$, does work, W_f, on the cart. While $\vec{F}_a$ acts in the same direction as the cart's displacement, $\vec{F}_f$ acts in the opposite direction. Thus, we will use the equation $W = F(\cos\theta)\Delta d$ to calculate work. We will solve this problem in three parts.

(a) Calculate the work done by the applied force using the equation $W_a = F_a(\cos\theta)\Delta d$.

(b) Calculate the work done by the force of friction using the equation $W_f = F_f(\cos\theta)\Delta d$.

(c) Determine the total, or net, work done on the cart, W_{net}, by calculating the sum of W_a and W_f.

Given: $F_a = 41.0$ N; $\Delta d = 11.0$ m; $F_f = 35.0$ N

Required: W_a; W_f; W_{net}

(a) mechanical work done by the applied force on the cart

Analysis: $W_a = F_a(\cos\theta)\Delta d$

Solution: Since the force of friction acts in the same direction as the displacement, $\theta = 0°$ here.

$$W_a = F_a(\cos\theta)\Delta d$$
$$= (41.0 \text{ N})(\cos 0°)(11.0 \text{ m})$$
$$= 451 \text{ N·m}$$
$$W_a = 451 \text{ J}$$

Statement: The applied force does 451 J of mechanical work on the cart.

(b) mechanical work done by the force of friction on the cart

Analysis: $W_f = F_f(\cos\theta)\Delta d$

Solution: Since the force of friction acts in the opposite direction of the displacement, $\theta = 180°$ here.

$$\begin{aligned} W_f &= F_f(\cos\theta)\Delta d \\ &= (35.0\ \text{N})(\cos 180°)(11.0\ \text{m}) \\ W_f &= -385\ \text{J} \end{aligned}$$

Statement: The force of friction does -385 J of mechanical work on the cart.

(c) net work done by the applied force and the force of friction on the cart

Solution:
$$\begin{aligned} W_{net} &= W_a + W_f \\ &= 451\ \text{J} + (-385\ \text{J}) \\ W_{net} &= 66\ \text{J} \end{aligned}$$

Statement: The net work done by the applied force and the force of friction on the cart is 66 J.

Practice

1. Curtis pushes a bowl of cereal along a level counter a distance of 1.3 m. What is the net work done on the bowl if Curtis pushes the bowl with a force of 4.5 N and the force of friction on the bowl is 2.8 N? T/I [ans: 2.2 J]
2. A crane lifts a 450 kg beam 12 m straight up at a constant velocity. Calculate the mechanical work done by the crane. T/I [ans: 53 kJ]

Graphing Work Done

The work done by a force on an object may be represented graphically by plotting the magnitude of the force, F, on the y-axis and the magnitude of the object's position, d, on the x-axis. In such cases, we assume that the force acts in the same direction as the object's displacement or in a direction opposite the object's displacement ($\theta = 0°$ or $\theta = 180°$). This is known as a force–position, or F–d, graph. **Figure 7** shows an F–d graph for a constant force acting through a displacement Δd. The work done, W, is equal to the area under the F–d graph (the shaded rectangle) and is equal in value to the product $F\Delta d$.

Positive and negative work may also be represented using an F–d graph. In **Figure 8**, positive work (work done when force and displacement are in the same direction) is represented by the blue rectangle above the d-axis. Negative work (work done when force and displacement are in opposite directions) is represented by the red rectangle below the d-axis.

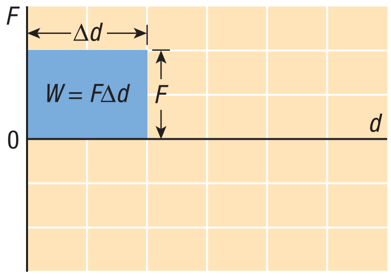

Figure 7 F–d graph for a constant force acting through a displacement

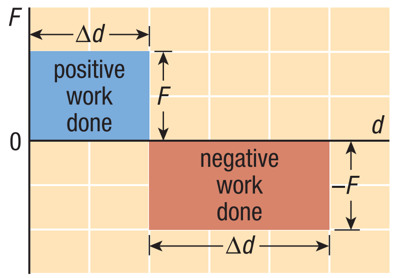

Figure 8 F–d graph representing positive and negative work done

Work Done by a Changing Force

The equation $W = F\Delta d$ or $W = F(\cos\theta)\Delta d$ can be used only to calculate the mechanical work done on an object when the force on the object is constant. However, in many cases, a force varies in magnitude during a displacement. For example, when a bus driver steadily depresses the accelerator pedal of a bus, the force the engine applied on the wheels increases uniformly. The force is not constant.

The work done on an object by a changing force that is in the same direction as the object's displacement may be represented using an F–d graph. **Figure 9** shows an F–d graph for a uniformly increasing force. As in the case of a constant force, the work done is equal to the area under the F–d graph. In this case, the work done, W, is equal in value to the product $F_{av}\Delta d$. F_{av} represents the average force applied to the object as it is displaced.

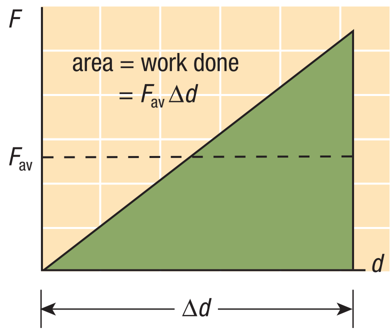

Figure 9 F–d graph representing a uniformly increasing force

Mini Investigation

Human Work

Skills: Predicting, Performing, Observing, Analyzing — SKILLS HANDBOOK A2.1

In this activity, you will determine the work done by a person lifting a book and a shoe.

Equipment and Materials: scale; textbook; tape measure or metre stick; desk; shoe

1. Use a scale to measure the mass of a textbook. Calculate the textbook's weight in newtons.
2. Use a tape measure or metre stick to measure the vertical distance from the floor to the top of a desk.
3. Place the textbook on the floor beside the desk and lift it straight up to the top of the desk at a constant speed.
4. Calculate the work you did on the book.
5. Obtain a clean shoe. Hold the shoe in your hand for a few seconds to get a feel for its weight.
6. Predict the amount of work you would have to do on the shoe to lift the shoe straight up from the floor to the top of the desk at constant speed.
7. Test your prediction by determining the weight of the shoe and then lifting the shoe from the floor to the top of the desk and calculating the work done.

A. Compare the amount of work you did on the textbook to the amount of work you did on the shoe. T/I

B. Evaluate the prediction you made in Step 6 on the basis of the results you obtained in Step 7. T/I

C. Calculate the height above the floor to which you would have to lift the shoe to do the same amount of work as you did on the textbook. T/I

5.1 Summary

- Mechanical work is done when a force displaces an object in the direction of the force or a component of the force.
- The equation $W = F(\cos\theta)\Delta d$ may be used to calculate the amount of mechanical work done on an object.
  - If the force on an object and the object's displacement are in the same direction, then $\theta = 0°$, $\cos\theta = 1$, the equation $W = F(\cos\theta)\Delta d$ becomes $W = F\Delta d$, and the amount of work is a positive value.
  - If the force on an object and the object's displacement are perpendicular, then $\theta = 90°$, $\cos\theta = 0$, and $W = 0$ J (no work is done).
  - If a force acts on an object in a direction opposite the object's displacement, then $\theta = 180°$, $\cos\theta = -1$, the equation $W = F(\cos\theta)\Delta d$ becomes $W = (F\Delta d)(-1)$, and the amount of work is a negative value.
- When the force on an object and the displacement of the object are parallel (in the same direction or opposite in direction), the work done on the object may be determined from an F–d graph by finding the area between the graph and the position axis. The work is positive if the area is above the position axis and negative if it is below the position axis.
- When a force varies in magnitude during a displacement, the work done is equal to the product of the average force, F_{av}, and the displacement, Δd.

5.1 Questions

1. A 25.0 N applied force acts on a cart in the direction of the motion. The cart moves 13.0 m. How much work is done by the applied force? T/I
2. A tow truck pulls a car from rest onto a level road. The tow truck exerts a horizontal force of 1500 N on the car. The frictional force on the car is 810 N. Calculate the work done by each of the following forces on the car as the car moves forward 12 m: T/I
   (a) the force of the tow truck on the car
   (b) the force of friction
   (c) the normal force
   (d) the force of gravity
3. A child pulls a wagon by the handle along a flat sidewalk. She exerts a force of 80.0 N at an angle of 30.0° above the horizontal while she moves the wagon 12 m forward. The force of friction on the wagon is 34 N. T/I
   (a) Calculate the mechanical work done by the child on the wagon.
   (b) Calculate the total work done on the wagon.
4. A horizontal rope is used to pull a box forward across a rough floor doing 250 J of work over a horizontal displacement of 12 m at a constant velocity. T/I C
   (a) Draw an FBD of the box.
   (b) Calculate the tension in the rope.
   (c) Calculate the force of friction and the work done by the force of friction. Explain your reasoning.
5. A 62 kg person in an elevator is moving up at a constant speed of 4.0 m/s for 5.0 s. T/I C
   (a) Draw an FBD of the person in the elevator.
   (b) Calculate the work done by the normal force on the person.
   (c) Calculate the work done by the force of gravity on the person.
   (d) How would your answers change if the elevator were moving down at 4.0 m/s for 5.0 s?
6. A force sensor pulls a cart horizontally from rest. The position of the cart is recorded by a motion sensor. The data were plotted on a graph as shown in **Figure 10**. The applied force and the displacement are parallel. What is the work done on the cart by the force sensor after a displacement of 0.5 m? T/I

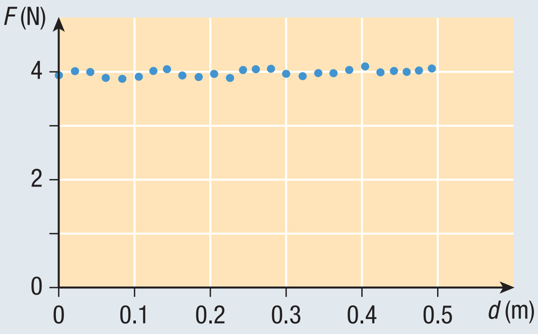

Figure 10

7. A rope pulls a 2.0 kg bucket straight up, accelerating it from rest at 2.2 m/s^2 for 3.0 s. T/I
   (a) Calculate the displacement of the bucket.
   (b) Calculate the work done by each force acting on the bucket.
   (c) Calculate the total mechanical work done on the bucket.
   (d) Calculate the net force acting on the bucket and the work done by the net force. Compare your answer to the total mechanical work done on the bucket as calculated in (c).
8. In your own words, explain if mechanical work is done in each of the following cases: K/U C
   (a) A heavy box sits on a rough horizontal counter in a factory.
   (b) An employee pulls on the box with a horizontal force and nothing happens.
   (c) The same employee goes behind the box and pushes even harder, and the box begins to move. After a few seconds, the box slides onto frictionless rollers and the employee lets go, allowing the box to move with a constant velocity.
9. The graph in **Figure 11** shows the force acting on a cart from a spring. The force from the spring is either in the same direction as the cart's displacement or in the opposite direction. T/I C

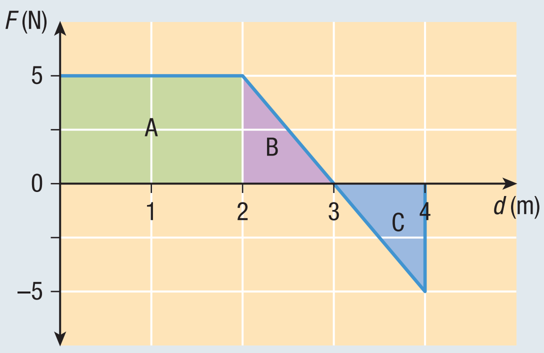

Figure 11

   (a) Calculate the work done in sections A, B, and C.
   (b) Calculate the total work done.
   (c) Explain why the work done in section C must be negative.
10. Describe two ways to determine the total work done by one object on another object. K/U C
11. Consider the equation $W = F(\cos\theta)\Delta d$. K/U C
    (a) Using the equation, explain why a force perpendicular to the displacement does zero work.
    (b) Using the equation, explain why a force opposite to the displacement does negative work.