3.5 Using Newton's Laws

Newton's three laws of motion apply to many different situations everywhere around you. They can be used to determine if a train locomotive can make it through a mountain pass while pulling many cars behind it. The laws can also be used to design devices such as prosthetic limbs and bridges (**Figure 1**). To understand how to design these types of devices, you need to know how to apply all three of Newton's laws, and you need to know how they are related to each other. For example, forces act on a prosthetic leg to accelerate it so a person can walk. The tension in suspension bridge cables helps to hold up the roadway against the force of gravity.

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Figure 1 Newton's three laws of motion are used to design devices such as (a) prosthetic limbs and (b) bridges.

In this section, you will practise using all three laws together to solve more complex and interesting problems involving forces and motion. In addition, you will learn more about forces, such as tension, and how to use kinematics to understand motion problems more clearly.

Tension and Newton's Laws

Recall that tension is a pulling force exerted by a device such as a rope or a string. In this course, you may assume that the ropes or strings are light and do not stretch. This means that you do not have to include the mass of the string when calculating the acceleration of two objects tied together. It also means that the tension is uniform throughout the rope or string and pulls with the same force at both ends. Keep in mind that ropes and strings can only pull. This means that tension always acts on the object directed toward the rope or the string.

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For example, two masses are tied together with a string, and a horizontal applied force pulls m_1 to the right (**Figure 2**). If the string exerts a tension of 20 N [left] on m_1, then it also exerts a tension of 20 N [right] on m_2. This is a direct consequence of Newton's third law and the fact that the string does not stretch and has negligible mass.

How can we measure tension directly? One way is with a force sensor or a spring scale. If you tie opposite ends of a string to two separate force sensors and then pull on the string in opposite directions, you can easily see that the readings on both sensors are the same (**Figure 3**). Again, this is a direct consequence of Newton's third law and our assumptions about strings. A single force sensor or spring scale can be used to measure the tension in strings if it is tied between two strings (**Figure 4**).

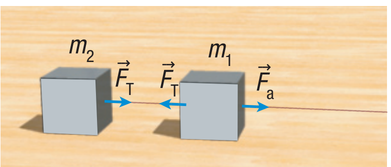


Figure 2 Two objects tied together with a string. The tension force at both ends of the string is the same.

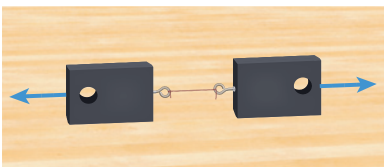


Figure 3 Two force sensors tied together with a string. The readings on the sensors will be equal according to Newton's third law.

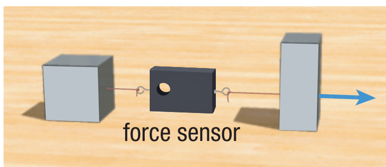


Figure 4 Force sensors can be used to measure tension in strings.

Keep in mind that when two objects are tied together with a single string, if you drag one object forward, the other will also move forward once the string becomes taut. This means that the two objects will move with the same acceleration when they are pulled in a straight line. In this case, tension is an internal force and can be ignored if you are calculating the acceleration of both objects.

Since the tension is the same throughout the string and the tension pulls in opposite directions at the ends, we often just calculate the magnitude of the tension and refer to the FBD for the direction. The following Tutorial will help you to practise solving problems that involve tension and Newton's laws.

Tutorial 1 Solving Tension Problems

To solve the following Sample Problems, we will use Newton's laws and our knowledge of tension. Each problem will help clarify some of the concepts about tension.

Sample Problem 1: Objects Hanging from Strings

Each object in **Figure 5** has a force of gravity of 120 N [down] acting on it. Determine the tension in each string.

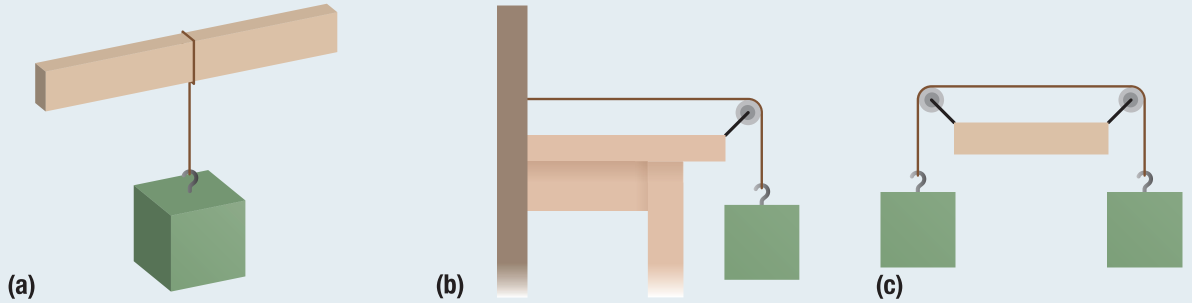
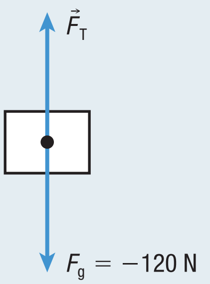


Figure 5

Solution

(a) First draw the FBD of the object. Choose up as positive.



The object is at rest, so it is not accelerating. Newton's first law implies that the net force must be zero.

$$\vec{F}_{net} = \vec{F}_T + \vec{F}_g$$
$$F_{net} = F_T + (-120\text{ N})$$
$$0 = F_T - 120\text{ N}$$
$$F_T = +120\text{ N}$$

The tension in the string is 120 N [up].

(b) In this diagram, the force of gravity has not changed and the object is at rest. This means that the FBD is the same and we will find the same tension. This example reinforces the concept that pulleys only change the direction of force without changing the magnitude of the force.

(c) In this balanced system, both objects are at rest. By drawing an FBD for either object, you will get exactly the same result for the tension. This result is contrary to what most people would expect. Most people would incorrectly say the string is holding up twice as much mass and should have twice the tension. Others incorrectly think the tension is zero since both forces of gravity pull the string at each end and they should cancel. Neither statement is true. The second object is just providing the force necessary to hold up the first object. In other words, the second object is just doing the job of the wall or beam, but otherwise the situation is unchanged.

Sample Problem 2: Objects Connected Horizontally by Strings

Three sleds are tied together and pulled east across an icy surface with an applied force of 120 N [E] (**Figure 6**). The mass of sled 1 is 12.0 kg, the mass of sled 2 is 11.0 kg, and the mass of sled 3 is 7.0 kg. You may assume that no friction acts on the sleds.

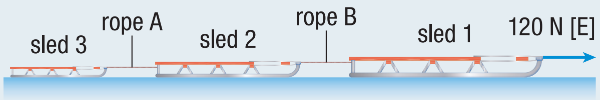


Figure 6

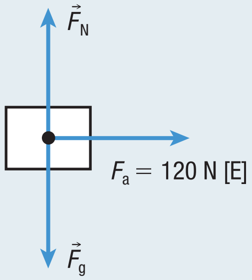
(a) Determine the acceleration of the sleds.

(b) Calculate the magnitude of the tension in rope A.

(c) Calculate the magnitude of the tension in rope B.

Solution

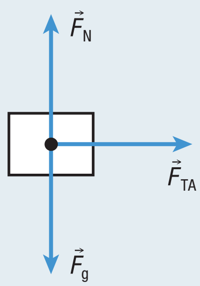
(a) All three sleds will move together with the same acceleration, so we can treat them as one single object. The total mass of all three sleds is $m_T = 12.0\text{ kg} + 11.0\text{ kg} + 7.0\text{ kg} = 30.0\text{ kg}$. There is no need to consider the tension at this point because it is an internal force and will not contribute to the acceleration of the total mass. Choose east as positive. So west is negative.



$$\vec{F}_{net} = m_T\vec{a}$$
$$+120\text{ N} = (30.0\text{ kg})a$$
$$a = \frac{+120\text{ N}}{30.0\text{ kg}}$$
$$a = +4.0\text{ m/s}^2$$

The acceleration of all three sleds is 4.0 m/s^2 [E].

(b) We could use the FBD for either sled 2 or sled 3 to calculate the tension in rope A. We will use the FBD for sled 3 because it is slightly simpler—it only has rope A pulling on it, whereas sled 2 has both rope A and rope B pulling on it.



$$\vec{F}_{net} = \vec{F}_{TA}$$
$$m_3\vec{a} = \vec{F}_{TA}$$
$$(7.0\text{ kg})(+4.0\text{ m/s}^2) = F_{TA}$$
$$F_{TA} = +28\text{ N}$$

The magnitude of the tension in rope A is 28 N.

(c) To calculate the tension in rope B, we can use the FBD of sled 2, but our answer will depend on the accuracy of the calculated tension in rope A. To avoid this problem, you can either use the FBD of sled 1 or the FBD of sleds 2 and 3 (**Figure 7**). We will choose the latter since it is slightly simpler. In this calculation, we will use a mass of $m_2 + m_3 = 18.0$ kg.

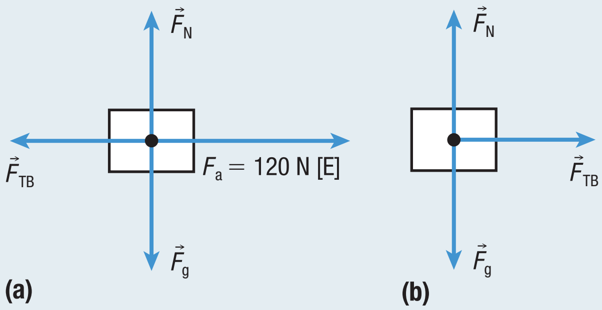


Figure 7 (a) FBD of sled 1 (b) FBD of sleds 2 and 3

$$\vec{F}_{net} = \vec{F}_{TB}$$
$$(m_2 + m_3)\vec{a} = \vec{F}_{TB}$$
$$(18.0\text{ kg})(+4.0\text{ m/s}^2) = F_{TB}$$
$$F_{TB} = +72\text{ N}$$

The magnitude of the tension in rope B is 72 N.

Practice

1. Examine each diagram in **Figure 8**. In each situation, which rope will have the greater tension? Explain your reasoning. K/U
2. A locomotive with a mass of 6.4×10^5 kg is accelerating at 0.12 m/s^2 [W] while pulling a train car with a mass of 5.0×10^5 kg. Assume that negligible friction is acting on the train. T/I
   (a) Calculate the net force on the entire train. [ans: 1.4×10^5 N [W]]
   (b) Determine the magnitude of the tension between the locomotive and the train car. [ans: 6.0×10^4 N]

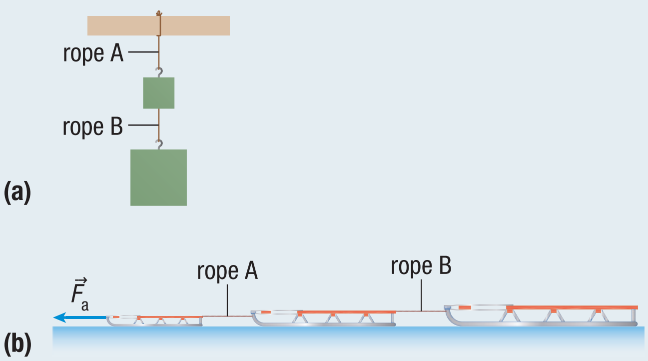


Figure 8

Kinematics and Newton's Laws

When using any of the kinematics equations from Unit 1, it is essential that the acceleration remain constant. Now we can extend this restriction by stating that the net force on an object must also remain constant if you use one of the kinematics equations. This is a direct consequence of Newton's second law, $\vec{F}_{net} = m\vec{a}$, which shows that net force is constant when the acceleration is constant.

Imagine that you start moving east due to a constant net force acting on you. You might start walking, but you gradually speed up and start running. Then your net force drops to zero and you move at a constant velocity. Finally, another constant net force acts in the opposite direction and slows you down until you eventually come to rest.

During the three separate parts of the trip, your acceleration was constant because the net force was constant. This means that you can use a kinematics equation during one part of the trip but not a single equation for the entire trip. The following Tutorial will clarify how to use kinematics concepts with Newton's laws.

Tutorial 2 Newton's Laws and Kinematics

When solving the following Sample Problems, keep in mind that the acceleration is constant when the net force is constant, according to Newton's second law.

Sample Problem 1: Skater Pushing on the Boards

Starting from rest, an ice skater (54.0 kg) pushes the boards with a force of 130.0 N [W] and moves 0.704 m. He then moves at a constant velocity for 4.00 s before he digs in his skates and starts to slow down. When he digs in his skates, he causes a net force of 38.0 N [W] to slow him down until he stops.

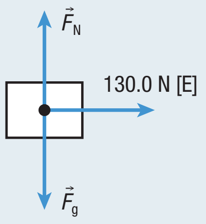
(a) Determine the acceleration of the skater
 (i) when he is pushing on the boards
 (ii) just after he stops pushing on the boards
 (iii) when he starts to slow down

(b) How far does he move?

Solution

Use FBDs to solve this problem.

(a) (i) When the skater pushes the boards with a force of 130.0 N [W], the boards push back on the skater with an equal and opposite force of 130.0 N [E]. Choose east as positive. So west is negative.



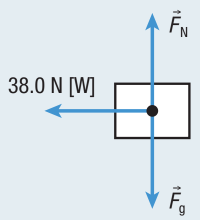
$$\vec{F}_{net} = m\vec{a}$$
$$+130.0 \text{ N} = (54.0 \text{ kg})a$$
$$a = \frac{+130.0 \text{ N}}{54.0 \text{ kg}}$$
$$a = 2.407 \text{ m/s}^2 \text{ (one extra digit carried)}$$

The acceleration of the skater is 2.41 m/s^2 [E] when he is pushing on the boards.

(ii) When he stops pushing on the boards, the net force acting on him is zero. According to Newton's first law, his acceleration will also be zero.

(iii) Now the skater is slowing down and the net force is opposite to the direction of motion.

$$\vec{F}_{net} = m\vec{a}$$
$$-38.0 \text{ N} = (54.0 \text{ kg})a$$
$$a = \frac{-38.0 \text{ N}}{54.0 \text{ kg}}$$
$$a = -0.704 \text{ m/s}^2$$



The acceleration of the skater is 0.704 m/s^2 [W] when he is slowing down.

(b) During part (i), the skater moves 0.704 m [E], but we need to calculate his final velocity in order to calculate his displacement during the other sections of the motion.

Given: $\vec{v}_1 = 0$; $\Delta\vec{d} = 0.704$ m [E]; $\vec{a} = 2.407$ m/s^2 [E]

Required: $\vec{v}_2$

Analysis: $v_2^2 = v_1^2 + 2a\Delta d$

Solution: $v_2^2 = v_1^2 + 2a\Delta d$

$$v_2^2 = (0)^2 + 2(+2.407 \text{ m/s}^2)(+0.704 \text{ m})$$
$$v_2 = 1.841 \text{ m/s (one extra digit carried)}$$

Now we can calculate the displacement for the other sections of the motion. For part (ii),

$$\Delta\vec{d} = \vec{v}\Delta t$$
$$= (+1.841 \text{ m/s})(4.00 \text{ s})$$
$$\Delta d = 7.364 \text{ m (one extra digit carried)}$$

For part (iii),

$$v_2^2 = v_1^2 + 2a\Delta d$$
$$0^2 = (+1.841 \text{ m/s})^2 + 2(-0.704 \text{ m/s}^2)\Delta d$$
$$\Delta d = 1.981 \text{ m (one extra digit carried)}$$

Statement: The total distance travelled by the skater is 0.704 m + 7.364 m + 1.981 m = 10.1 m

Sample Problem 2

A worker pushes two large boxes across the floor from rest with an applied force of 160.0 N [right] on the larger box (**Figure 9**). The boxes are touching. The mass of the larger box is $m_1 = 32.0$ kg and the mass of the smaller box is $m_2 = 8.0$ kg. The force of friction on the large box is 80.0 N [left] and the force of friction on the smaller box is 20.0 N [left].

(a) Calculate the acceleration of the two boxes. Assume that the boxes start to move.

(b) Determine the force exerted by the larger box on the smaller box.

(c) Determine the velocity of the boxes after 4.0 s.

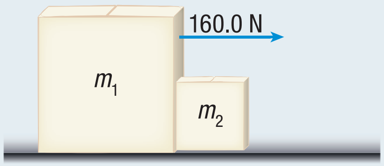
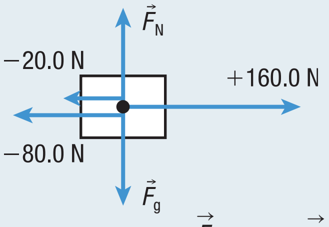


Figure 9

(a) Both boxes must move together with the same acceleration, so for now we will treat them like one single object with a total mass of $m_T = 32.0 \text{ kg} + 8.0 \text{ kg} = 40.0$ kg. From the FBD for both boxes, the normal force and gravity cancel. Choose right as positive. So left is negative.



$$\vec{F}_{\text{net}} = m_T \vec{a}$$

$$+160.0 \text{ N} + (-80.0 \text{ N}) + (-20.0 \text{ N}) = m_T a$$

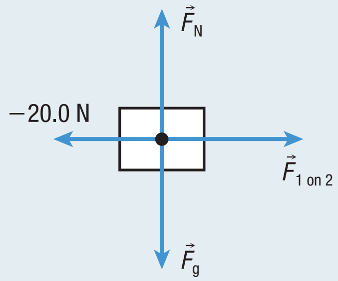
$$+60.0 \text{ N} = (40.0 \text{ kg})a$$

$$a = \frac{+60.0 \text{ N}}{40.0 \text{ kg}}$$

$$= +1.5 \text{ m/s}^2$$

The acceleration of both boxes is 1.5 m/s^2 [right].

(b) The force from the large box on the smaller box is an internal force. To calculate this force, we need to draw the FBD for just one box. Either box will do but we will use the smaller one because it has fewer forces acting on it. Again, the normal force and the force of gravity cancel.



$$F_{\text{net}} = F_{1 \text{ on } 2} + (-20.0 \text{ N})$$

$$m_2 a = F_{1 \text{ on } 2} - 20.0 \text{ N}$$

$$(8.0 \text{ kg})(+1.5 \text{ m/s}^2) = F_{1 \text{ on } 2} - 20.0 \text{ N}$$

$$F_{1 \text{ on } 2} = +32.0 \text{ N}$$

The force exerted by the larger box on the smaller box is 32.0 N [right].

(c) **Given:** $\vec{v}_1 = 0$; $\Delta t = 4.0$ s; $\vec{a} = 1.5$ m/s^2 [right]

Required: $\vec{v}_2$

Analysis: $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

Solution: $\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$$= 0 + (+1.5 \text{ m/s}^2)(4.0 \text{ s})$$

$$v_2 = +6.0 \text{ m/s}$$

Statement: The final velocity of both boxes is 6.0 m/s [right].

Practice

1. Two dynamics carts are placed end to end. Cart 1 (1.2 kg) is stuck to cart 2 (1.8 kg). Cart 1 is pushed with a force of 18.9 N [W], causing cart 1 to push cart 2 forward. Ignore the force of friction. T/I
   (a) Calculate the acceleration of each cart. [ans: 6.3 m/s^2 [W]]
   (b) Calculate the force that cart 1 exerts on cart 2. [ans: 11 N [W]]
   (c) Would your answers change if cart 2 were pushed with an equal but opposite force instead of cart 1? If your answers change, calculate the new results. [ans: 6.3 m/s^2 [E]; 7.6 N [W]]
2. A 1200 kg car is moving at 95 km/h when the driver notices a deer down the road. He immediately moves his foot toward the brake pedal, taking only 0.50 s before the car starts slowing down. The brakes cause a net force of 2400 N [backwards] on the car for 2.0 s. The deer then jumps out of the way and the driver lifts his foot off the brake pedal. How far does the car move in the 2.5 s starting from when the driver sees the deer? T/I [ans: 62 m]

3.5 Summary

- The tensions at both ends of a string or a rope are equal in magnitude.
- Tension can be measured with a spring scale or a force sensor.
- The key equations of motion from the Kinematics unit and Newton's laws can be used together to solve motion problems.

3.5 Questions

1. You hold one end of a rope and pull horizontally with a force of 65 N. Calculate the tension in the rope if the other end is
   (a) tied to a wall
   (b) held by a friend who pulls with 65 N in the opposite direction
   (c) tied to a 12 kg object on smooth ice T/I
2. A 72 kg sled is pulled forward from rest by a snowmobile and accelerates at 2.0 m/s^2 [forward] for 5.0 s. The force of friction acting on the sled is 120 N [backwards]. The total mass of the snowmobile and driver is 450 kg. The drag force acting on the snowmobile is 540 N [backwards]. T/I
   (a) Determine the tension in the rope.
   (b) Calculate the force exerted by the snowmobile that pushes the sled forward.
3. Two people, each with a mass of 70 kg, are wearing inline skates and are holding opposite ends of a 15 m rope. One person pulls forward on the rope by moving hand over hand and gradually reeling in more of the rope. In doing so, he exerts a force of 35 N [backwards] on the rope. This causes him to accelerate toward the other person. Assuming that the friction acting on the skaters is negligible, how long will it take for them to meet? Explain your reasoning. T/I
4. A 1200 kg car pulls an 820 kg trailer over a rough road. The force of friction acting on the trailer is 650 N [backwards]. Calculate the force that the car exerts on the trailer if
   (a) the trailer is moving at a constant velocity of 30 km/h [forward]
   (b) the trailer is moving at a constant velocity of 60 km/h [forward]
   (c) the trailer is moving forward at 60 km/h and starts accelerating at 1.5 m/s^2 [forward]
   (d) the trailer is moving forward at 60 km/h and starts accelerating at 1.2 m/s^2 [backwards] T/I
5. An old rope can now only safely suspend 120 kg. When the rope is tied to a beam, it hangs down with a vertical length of 12.0 m. Calculate the minimum time required for an 85 kg person starting from rest to climb the entire length of the rope without breaking it. T/I
6. Three dynamics carts have force sensors placed on top of them. Each force sensor is tied to a string that connects all three carts together (**Figure 10**). You use a sixth force sensor to pull the three dynamics carts forward. The reading on force sensor 2 is 3.3 N. Assume that the force sensors are light and that there is negligible friction acting on the carts. T/I

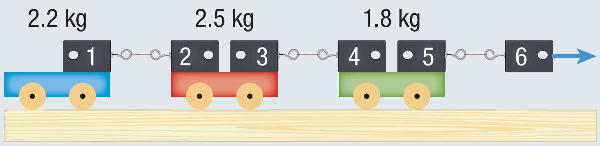


Figure 10

   (a) What is the acceleration of all the carts?
   (b) What is the reading on each force sensor?
   (c) What force are you applying to force sensor 6?
7. A locomotive (6.4×10^5 kg) is used to pull two railway cars (**Figure 11**). Railway car 1 (5.0×10^5 kg) is attached to railway car 2 (3.6×10^5 kg) by a locking mechanism. A railway engineer tests the mechanism and estimates that it can only withstand 2.0×10^5 N of force. Determine the maximum acceleration of the train that does not break the locking mechanism. Explain your reasoning. Assume that friction is negligible. T/I C

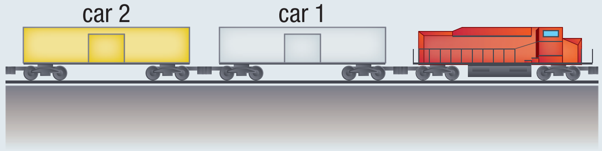


Figure 11

8. A skier (68 kg) starts from rest but then begins to move downhill with a net force of 92 N for 8.2 s. The hill levels out for 3.5 s. On this part of the hill, the net force on the skier is 22 N [backwards]. T/I
   (a) Calculate the speed of the skier after 8.2 s.
   (b) Calculate the speed of the skier at the end of the section where the hill levels out.
   (c) Calculate the total distance travelled by the skier before coming to rest.