

Determine the resultant displacement

$$d_1 = 15\text{km @ } 50^\circ$$

$$d_2 = 20\text{km @ } 160^\circ$$

$$d_3 = 12\text{km @ due West}$$

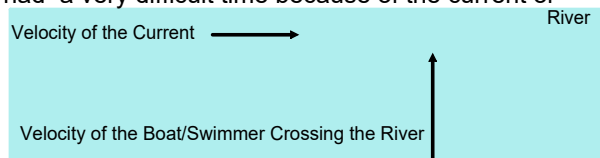
Draw a really detailed picture to start please.

### Adding TWO Velocities - What does that mean?

If you have ever tried to swim or paddle across a river you have experienced the addition of two velocities.

If you tried to swim across a backyard pool to a point exactly across from you, you probably didn't have much difficulty.

If you tried to swim across a rushing river to a point exactly across from you, you probably had a very difficult time because of the current or moving water.



River crossing problems are a type of two-dimensional motion problem that involve perpendicular velocity vectors. The "river crossing problem" is often first introduced in terms of boats crossing rivers, but it may also involve aircraft flying through the air, and so on. These types of problems always involve two perpendicular motions that are independent of each other.

**CASE 1: DETERMINING THE TIME IT TAKES FOR A RIVER CROSSING WITHOUT TAKING CURRENT INTO ACCOUNT**

**Sample Problem 1**

Consider the river shown in Figure 11. A physics student has forgotten her lunch and needs to return home to retrieve it. To do so she hops into her motorboat and steers straight across the river at a constant velocity of 12 km/h [N]. If the river is 0.30 km across and has no current, how long will it take her to cross the river?

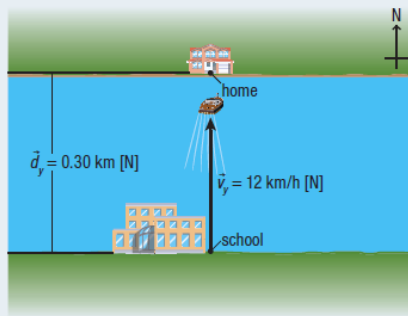


Figure 11 River crossing with no current

Let  $\vec{v}_y$  represent the velocity caused by the boat's motor.

**Given:**  $\Delta \vec{d}_y = 0.30 \text{ km [N]}$ ,  $\vec{v}_y = 12 \text{ km/h [N]}$

**Required:**  $\Delta t$

**Analysis:** Since the boat is travelling at a constant velocity, we can solve this problem using the defining equation for average velocity. Since displacement and average velocity are in the same direction, we can simply divide one magnitude by the other when we rearrange this equation.

$$\vec{v}_y = \frac{\Delta \vec{d}_y}{\Delta t}$$

$$\vec{v}_y(\Delta t) = \Delta \vec{d}_y$$

$$\Delta t = \frac{\Delta d_y}{v_y}$$

$$\begin{aligned} \text{Solution: } \Delta t &= \frac{\Delta d_y}{v_y} \\ &= \frac{0.30 \text{ km}}{12 \frac{\text{km}}{\text{h}}} \end{aligned}$$

$$\Delta t = 0.025 \text{ h}$$

**Statement:** It will take the student 0.025 h or 1.5 min to cross the river.

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**CASE 2: DETERMINING THE DISTANCE TRAVELLED DOWNSTREAM DUE TO A RIVER CURRENT**

**Sample Problem 2**

The river crossing problem in Sample Problem 1 is not very realistic because a river usually has a current. So we introduce a current here, in Sample Problem 2, and see how the current affects the trip across the river. Figure 12 shows the same boat from Sample Problem 1 going at the same velocity. Let us now assume that the gate of a reservoir has been opened upstream, and the river water now flows with a velocity of 24 km/h [E]. This current has a significant effect on the motion of the boat. The boat is now pushed due north by its motor and due east by the river's current. This causes the boat to experience two

velocities at the same time, one due north and another due east. In Figure 12 these two velocity vectors are joined tip to tail to give a resultant velocity represented by  $\vec{v}_r$ . Notice that even though the student is steering the boat due north, the boat does not arrive at her home. Instead it lands some distance farther downstream.

- (a) How long does it now take the boat to cross the river?
- (b) How far downstream does the boat land?
- (c) What is the boat's resultant velocity  $\vec{v}_r$ ?

(a) **Given:**  $\Delta \vec{d}_y = 0.30 \text{ km [N]}$ ;  $\vec{v}_y = 12 \text{ km/h [N]}$ ;  $\vec{v}_x = 24 \text{ km/h [E]}$

**Required:**  $\Delta t$

First we will consider the motion of the boat moving across the river.

$$\text{Analysis: } \vec{v}_y = \frac{\Delta \vec{d}_y}{\Delta t}$$

$$\Delta t = \frac{\Delta d_y}{v_y}$$

$$\begin{aligned} \text{Solution: } \Delta t &= \frac{\Delta d_y}{v_y} \\ &= \frac{0.30 \text{ km}}{12 \frac{\text{km}}{\text{h}}} \\ \Delta t &= 0.025 \text{ h} \end{aligned}$$

**Statement:** The time it takes the boat to cross the river is still 0.025 h.

**Given:**  $\vec{v}_x = 24 \text{ km/h [E]}$ ;  $\Delta t = 0.025 \text{ h}$

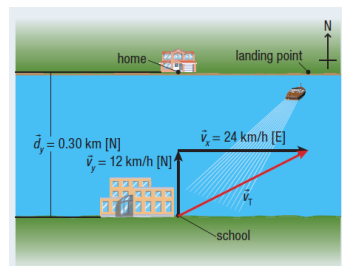
**Required:**  $\Delta \vec{d}_x$

$$\text{Analysis: } \vec{v}_x = \frac{\Delta \vec{d}_x}{\Delta t}$$

$$\Delta \vec{d}_x = \vec{v}_x(\Delta t)$$

$$\begin{aligned} \text{Solution: } \Delta \vec{d}_x &= \vec{v}_x(\Delta t) \\ &= (24 \text{ km/h [E]})(0.025 \text{ h}) \\ \Delta \vec{d}_x &= 0.60 \text{ km [E]} \end{aligned}$$

**Statement:** As a result of the current, the boat will land 0.60 km east, or downstream, of the student's home.



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2. A swimmer swims perpendicular to the bank of a 20.0 m wide river at a velocity of 1.3 m/s. Suppose the river has a current of 2.7 m/s [W]. T/1
- (a) How long does it take the swimmer to reach the other shore?
- (b) How far downstream does the swimmer land from his intended location?

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### Relative Velocity Notation:

A plane is flying at 500 km/hr [E] relative to the air.

The air is travelling at 50 km/hr [E] relative to the ground.

The plane's velocity relative to the ground is:

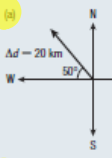
$$\begin{array}{ccccccc}
 \text{plane} & V_{\text{air}} & + & \text{air} & V_{\text{ground}} & = & \text{plane} & V_{\text{ground}} \\
 500 & & + & 50 & & = & 550 \text{ km/hr [E]}
 \end{array}$$

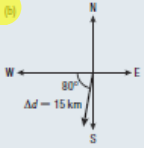

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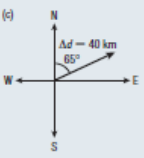
What if the wind was 100 km/hr [W]?

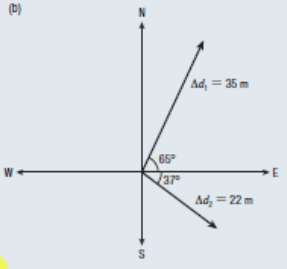
**2.2 Questions**

1. Break each vector down into an x-component and a y-component.

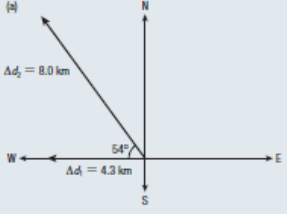
(a)   $\Delta d = 20 \text{ km}$ ,  $50^\circ$

(b)   $\Delta d = 15 \text{ km}$ ,  $80^\circ$

(c)   $\Delta d = 40 \text{ km}$ ,  $65^\circ$

(d)   $\Delta d_1 = 35 \text{ m}$ ,  $65^\circ$ ,  $\Delta d_2 = 22 \text{ m}$ ,  $37^\circ$

- A motorcyclist drives 5.1 km [E] and then turns and drives 14 km [N]. What is her total displacement?
- A football player runs 11 m [N 20° E]. He then changes direction and runs 9.0 m [E]. What is his total displacement?
- What is the total displacement for a boat that sails 200.0 m [S 25° W] and then tacks (changes course) and sails 150.0 m [N 30° E]?
- Determine the total displacement of an object that travels 25 m [N 20° W] and then 35 m [S 15° E].
- Use the component method to determine the total displacement given by the two vectors shown in each diagram.
 

(a)   $\Delta d_1 = 8.0 \text{ km}$ ,  $54^\circ$ ,  $\Delta d_2 = 4.3 \text{ km}$

7. Use the component method to add the following displacement vectors.  
 $\Delta \vec{d}_1 = 25 \text{ m [N } 30^\circ \text{ W]}$ ,  $\Delta \vec{d}_2 = 30.0 \text{ m [N } 40^\circ \text{ E]}$ ,  
 $\Delta \vec{d}_3 = 35 \text{ m [S } 25^\circ \text{ W]}$

8. A swimmer jumps into a 5.1 km wide river and swims straight for the other side at 0.87 km/h [N]. There is a current in the river of 2.0 km/h [W].  
 (a) How long does it take the swimmer to reach the other side?  
 (b) How far downstream has the current moved her by the time she reaches the other side?

9. A conductor in a train travelling at 4.0 m/s [N] walks across the train car at 1.2 m/s [E] to validate a ticket. If the car is 4.0 m wide, how long does it take the conductor to reach the other side? What is his velocity relative to the ground?

10. Vectors can be added algebraically and by scale diagram.  
 (a) Write a letter to your teacher explaining which method you prefer and why.  
 (b) Describe a situation for which the method that you do not prefer might be more suitable.

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**Given:**  $\vec{v}_y = 12 \text{ km/h [N]}$ ;  $\vec{v}_x = 24 \text{ km/h [E]}$

**Required:**  $\vec{v}_T$

**Analysis:**  $\vec{v}_T = \vec{v}_y + \vec{v}_x$

**Solution:**  $\vec{v}_T = \vec{v}_y + \vec{v}_x$   
 $\vec{v}_T = 12 \text{ km/h [N]} + 24 \text{ km/h [E]}$

Figure 14 shows the vector addition to determine the resultant velocity. This is the same technique we used in Tutorial 1.

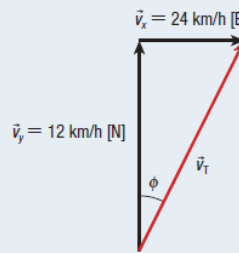


Figure 14 Determining the resultant velocity

$$v_T^2 = v_y^2 + v_x^2$$

$$v_T = \sqrt{v_y^2 + v_x^2}$$

$$v_T = \sqrt{(12 \text{ km/h})^2 + (24 \text{ km/h})^2}$$

$$v_T = 27 \text{ km/h}$$

$$\tan \phi = \frac{v_x}{v_y}$$

$$\tan \phi = \frac{24 \text{ km/h}}{12 \text{ km/h}}$$

$$\phi = 63^\circ$$

**Statement:** The boat's resultant velocity is 27 km/h [N 63° E].

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