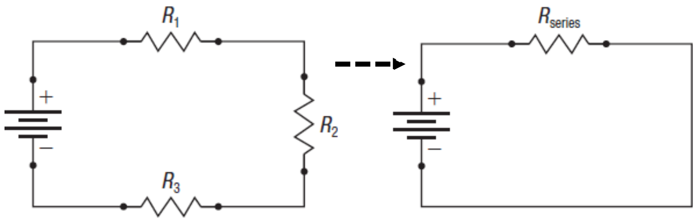
Section 11.8 - Resistors in Circuits

Resistors in Series Circuits - Basic

As previously stated, the current in a Series circuit is CONSTANT and the sum of the voltage drops must equal the source voltage.

No matter how many resistors are placed in the circuit, the current that leaves the power source, will travel through the entire circuit.



Equivalent resistance is the total resistance of a group of resistors in a circuit. You can use Kirchhoff's laws and Ohm's law to derive the equivalent resistance for resistors in a series circuit. Start with KVL for a series circuit:

$$V_{series} = V_1 + V_2 + V_3$$

Substitute Ohm's Law in the form $V = IR$:

$$I_{series}R_{series} = I_1R_1 + I_2R_2 + I_3R_3$$

In a series circuit, the current is constant and the same at all points (KCL). So the currents on the left side will cancel with the currents on the right side:

$$I_{series}R_{series} = I_1R_1 + I_2R_2 + I_3R_3$$
$$\cancel{I_{series}}R_{series} = \cancel{I_{series}}R_1 + \cancel{I_{series}}R_2 + \cancel{I_{series}}R_3$$
$$R_{series} = R_1 + R_2 + R_3$$

Therefore, in a series circuit the equivalent resistance is given by

$$R_{series} = R_1 + R_2 + R_3 + \cdots$$

So the three resistors can be reduced to a single resistor with a value equivalent to the sum of the three resistances. In the following Tutorial, you will apply the equivalent resistance equation to solve for missing values in a series circuit.

Sample Problem 1

Four resistors are connected in series in a circuit. The resistances are as follows: $R_1 = 41\ \Omega$, $R_2 = 51.75\ \Omega$, $R_3 = 11.1\ \Omega$, and $R_4 = 102.008\ \Omega$. Calculate the equivalent resistance.

Given: $R_1 = 41\ \Omega$; $R_2 = 51.75\ \Omega$; $R_3 = 11.1\ \Omega$; $R_4 = 102.008\ \Omega$

Required: R_{series}

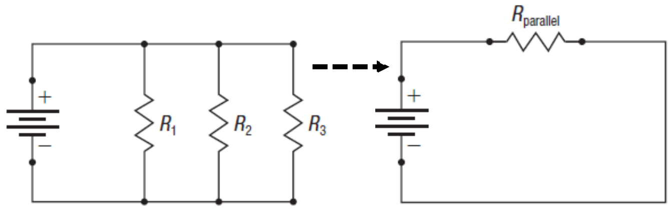
Practice

1. What is the equivalent resistance for a $25.2\ \Omega$ resistor connected in series with a $28.12\ \Omega$ resistor? T/I
2. What is the equivalent resistance for three $53.0\ \Omega$ resistors connected in series? T/I

Resistance in Parallel Circuits - Basic

You can derive an equivalent resistance equation for resistors in a parallel circuit. Start with KCL for the parallel circuit shown in **Figure 2:**

$$I_{parallel} = I_1 + I_2 + I_3$$



Substitute Ohm's law in the form $I = \frac{V}{R}$:

$$\frac{V_{parallel}}{R_{parallel}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

In a parallel circuit, the voltage is constant and the same at all points. Using KVL, the voltages on the left side will cancel with the voltages on the right side:

$$\frac{V_{parallel}}{R_{parallel}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$\frac{\cancel{V_{parallel}}}{R_{parallel}} = \frac{\cancel{V_{parallel}}}{R_1} + \frac{\cancel{V_{parallel}}}{R_2} + \frac{\cancel{V_{parallel}}}{R_3}$$

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Therefore, in a parallel circuit the equivalent resistance is given by

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Sample Problem 1

Three resistors are connected in parallel in a circuit. The resistances are $R_1 = 15\ \Omega$, $R_2 = 12\ \Omega$, and $R_3 = 10\ \Omega$. Calculate the equivalent resistance.

Given: $R_1 = 15\ \Omega$; $R_2 = 12\ \Omega$; $R_3 = 10\ \Omega$

Required: R_{parallel}

Analysis: $\dfrac{1}{R_{\text{parallel}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

Practice

1. What is the equivalent resistance of a 120 Ω resistor connected in parallel with a 60 Ω resistor? T/I [redacted]
2. What is the equivalent resistance of four 20 Ω resistors connected in parallel? T/I [redacted]

Mixed Circuits - Using Equivalence to Reduce and Simplify

Sample Problem 1

Calculate the equivalent resistance for the circuit shown in **Figure 3**.

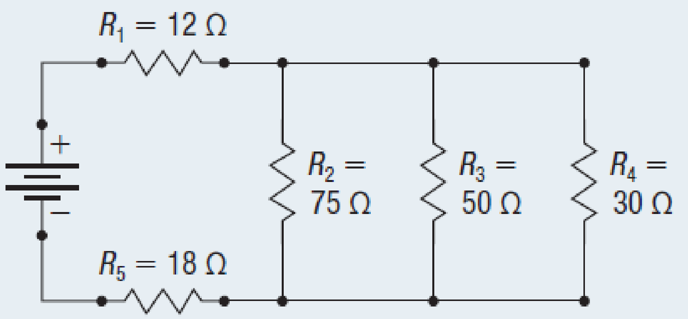


Figure 3

Step 1 - remove parallel

Step 2 - simplify series

Practice

1. What is the total resistance of the mixed circuits shown in **Figure 6**? Note that each resistor has resistance 5.0 Ω. T/I [ans[redacted]

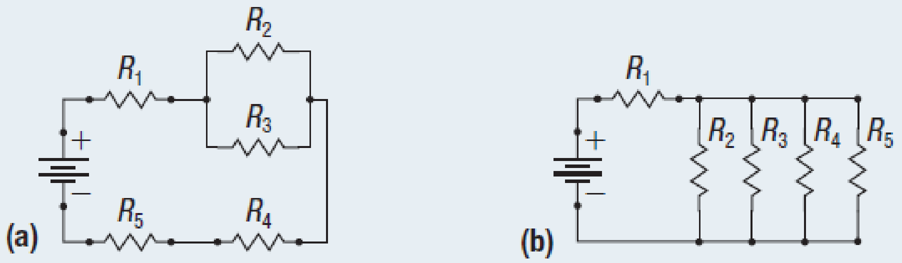


Figure 6

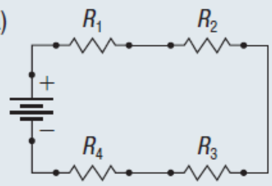
11.8 Summary

- Connecting resistors in series causes an increase in the total resistance and a decrease in the current.
- A group of resistors connected in series can be reduced to a single equivalent resistor with resistance $R_{series} = R_1 + R_2 + R_3 + \cdots$.
- Connecting resistors in parallel causes a decrease in the total resistance and an increase in current.
- A group of resistors connected in parallel can be reduced to a single equivalent resistor with resistance $\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$.

11.8 Questions

1. Using $R_{series} = R_1 + R_2 + R_3 + \cdots$ and Ohm's law, derive Kirchhoff's voltage law for a series circuit. T/I
2. Using $\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$ and Ohm's law, derive Kirchhoff's current law for a parallel circuit. T/I
3. Prove that the equivalent resistance of two identical resistors in parallel is equal to half the resistance of one of the resistors. T/I
4. Recall that all loads have electrical resistance. Suppose that you connect a number of loads in your home in parallel. What will happen to the amount of electric current with each load that you add? Would you be concerned about this? Explain your answer. K/U C
5. What is the equivalent resistance of the following circuits? Each resistor has resistance 12.0 Ω. K/U T/I

(a)



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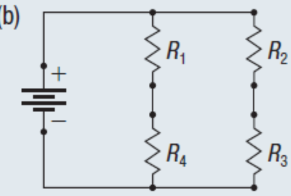

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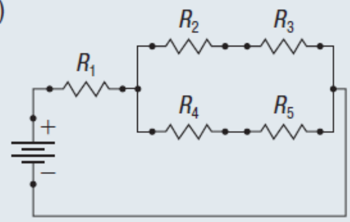

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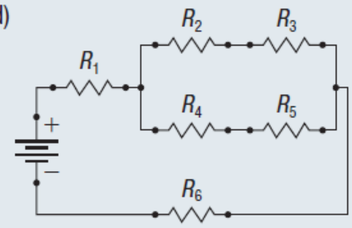

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