Section 1.5 - Developing Equations for Motion

Learning Goal: By the end of today, I will know how to use the 5 kinematic equations to solve a problem.

Sep 11-8:28 PM

Key Concepts
Part 1 - Motion in a Straight Line (One Dimension)

After completing this chapter you will be able to:

- explain how distance, position, and displacement are different
- explain how speed, velocity, and acceleration are different
- explain how vectors and scalars are different
- add and subtract vectors using scale diagrams and algebraic methods
- obtain motion information from position–time, velocity–time, and acceleration–time graphs
- solve uniform velocity and uniform acceleration problems using algebraic methods
- describe how the acceleration due to gravity affects the motion of objects close to the surface of Earth
- assess the impact on society and the environment of a technology that applies concepts related to kinematics

Sep 4-9:02 PM

A Displacement Equation for Uniformly Accelerated Motion

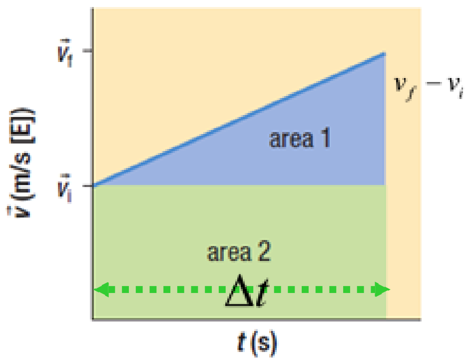

Figure 1 A velocity–time graph for an object undergoing uniform acceleration

$$\Delta \vec{d} = A_{triangle} + A_{rectangle}$$
$$= \frac{1}{2}bh + lw$$
$$= \frac{1}{2}\Delta t(\vec{v}_f - \vec{v}_i) + \Delta t\vec{v}_i$$
$$= \frac{1}{2}\vec{v}_f\Delta t - \frac{1}{2}\vec{v}_i\Delta t + \vec{v}_i\Delta t$$
$$= \frac{1}{2}\vec{v}_f\Delta t + \frac{1}{2}\vec{v}_i\Delta t$$

$$\Delta \vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t \text{ (Equation 1)}$$

Sep 11-8:21 PM

Additional Motion Equations

Consider the defining equation for acceleration: $\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

If we rearrange this equation to solve for final velocity ($\vec{v}_f$), we get Equation 2:

$$\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t \text{ (Equation 2)}$$

You may use Equation 2 in problems that do not directly involve displacement.

If we substitute the expression $v_i + \vec{a}_{av}\Delta t$ from Equation 2 into Equation 1, we get

$$\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t \text{ (Equation 2)}$$

$$\Delta \vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t \text{ (Equation 1)}$$
$$= \frac{1}{2}(\vec{v}_i + \vec{a}_{av}\Delta t + \vec{v}_i)\Delta t$$
$$= \frac{1}{2}(2\vec{v}_i + \vec{a}_{av}\Delta t)\Delta t$$

$$\Delta \vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}_{av}\Delta t^2 \text{ (Equation 3)}$$

Sep 11-8:24 PM

The Five Key Equations of Accelerated Motion

Table 1 shows the five key equations of accelerated motion. You should be able to solve any kinematics question by correctly choosing one of these five equations. You have seen how the first three are developed. We will leave the others to be developed as an exercise.

Table 1 The Five Key Equations of Accelerated Motion

	Equation	Variables found in equation	Variables not in equation
Equation 1	$\Delta\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$	$\Delta\vec{d}$, Δt, $\vec{v}_f$, $\vec{v}_i$	$\vec{a}_{av}$
Equation 2	$\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t$	$\vec{a}_{av}$, Δt, $\vec{v}_f$, $\vec{v}_i$	$\Delta\vec{d}$
Equation 3	$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}_{av}\Delta t^2$	$\Delta\vec{d}$, $\vec{a}_{av}$, Δt, $\vec{v}_i$	$\vec{v}_f$
Equation 4	$v_f^2 = v_i^2 + 2a_{av}\Delta d$	Δd, a_{av}, v_f, v_i	Δt
Equation 5	$\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}_{av}\Delta t^2$	$\Delta\vec{d}$, $\vec{a}_{av}$, Δt, $\vec{v}_f$	$\vec{v}_i$

Sep 11-8:24 PM

Example

A sports car approaches a highway on-ramp at a velocity of 20.0 m/s [E]. If the car accelerates at a rate of 3.2 m/s^2 [E] for 5.0 s, what is the displacement of the car?

What is given?

What equation might be used?

Sep 11-8:31 PM

Example

A sailboat accelerates uniformly from 6.0 m/s [N] to 8.0 m/s [N] at a rate of 0.50 m/s^2 [N]. What distance does the boat travel?

What is given?

What equation might be used?

Sep 11-8:32 PM

Example

A dart is thrown at a target that is supported by a wooden backstop. It strikes the backstop with an initial velocity of 35 m/s [E]. The dart comes to rest in 0.0050 s.

(a) What is the acceleration of the dart?

(b) How far does the dart penetrate into the backstop?

What is given?

What equation might be used?

Sep 11-8:34 PM

Practice

A football player initially at rest accelerates uniformly as she runs down the field, travelling 17 m [E] in 3.8 s. What is her final velocity?

A child on a toboggan sits at rest on the top of a tobogganing hill. If the child travels 70.0 m [downhill] in 5.3 s while accelerating uniformly, what acceleration does the child experience?

Sep 11-8:35 PM

1.5 Questions

1. A car accelerates from rest at a rate of 2.0 m/s^2 [N]. What is the displacement of the car at t = 15 s?
2. An astronaut is piloting her spacecraft toward the International Space Station. To stop the spacecraft, she fires the retro-rockets, which cause the spacecraft to slow down from 20.0 m/s [E] to 0.0 m/s in 12 s.
   (a) What is the acceleration of the spacecraft?
   (b) What is the displacement of the spacecraft when it comes to rest?
3. A helicopter travelling at a velocity of 15 m/s [W] accelerates uniformly at a rate of 7.0 m/s^2 [E] for 4.0 s. What is the helicopter's final velocity?
4. Two go-carts, A and B, race each other around a 1.0 km track. Go-cart A travels at a constant speed of 20.0 m/s. Go-cart B accelerates uniformly from rest at a rate of 0.333 m/s^2. Which go-cart wins the race and by how much time?
5. A boat increases its speed from 5.0 m/s to 7.5 m/s over a distance of 50.0 m. What is the boat's acceleration?
6. Within 4.0 s of liftoff, a spacecraft that is uniformly accelerating straight upward from rest reaches an altitude of 4.50×10^2 m [up].
   (a) What is the spacecraft's acceleration?
   (b) At what velocity is the spacecraft travelling when it reaches this altitude?
7. Derive Equation 4 and Equation 5 in Table 1 on page 37 by substituting other expressions.

Sep 11-8:36 PM