Topic 1: Measurement and uncertainties

1.2 – Uncertainties and errors

Essential idea:

Scientists aim towards designing experiments that can give a "true value" from their measurements, but due to the limited precision in measuring devices, they often quote their results with some form of uncertainty.

Understandings - Applications:

- Random and systematic errors

Explaining how random and systematic errors can be identified and reduced

- Absolute, fractional and percentage uncertainties

Collecting data that include absolute and/or fractional uncertainties and stating these as an uncertainty range (expressed as: best estimate ± uncertainty range)

→ • Error bars and Calculations

Propagating uncertainties through calculations involving addition, subtraction, multiplication, division and raising to a power

IB Guidance:

- Analysis of uncertainties will not be expected for trigonometric or logarithmic functions in examinations

IB Physics Data Booklet, First Exam 2016, Subtopic 1.2

$$\text{If } y = a \pm b \text{ then } \Delta y = \Delta a + \Delta b$$

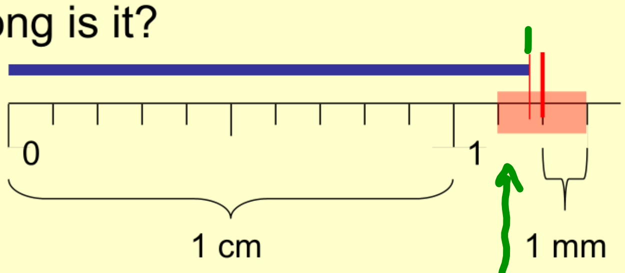
$$\text{If } y = \frac{ab}{c} \text{ then } \frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

$$\text{If } y = a^n \text{ then } \frac{\Delta y}{y} = \left| n \frac{\Delta a}{a} \right|$$

Random and systematic errors

- Error in measurement is expected because of the imperfect nature of us and our measuring devices.
- We say the **precision** or **uncertainty** in our measurement is ± 1 mm.

EXAMPLE: Consider the following line whose length we wish to measure. How long is it?



SOLUTION: It is closer to 1.2 cm than to 1.1 cm, so we say it measures 1.2 cm (or 12 mm or 0.012 m).

12 12mm ± 1mm

FYI • We record L = 12 mm ± 1 mm.

Measure the width of a piece of paper.

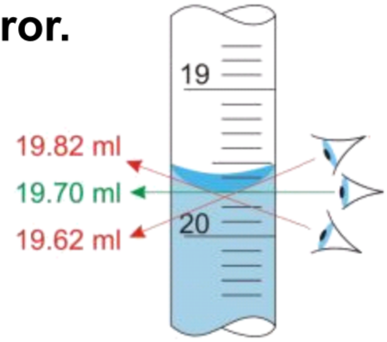
Measure the length of your pen/pencil.

Random and systematic errors

- **Random error** is error due to the recorder, rather than the instrument used for the measurement.
- Different people may measure the same line slightly differently. You may in fact measure the same line differently on two different occasions.
- Perhaps the ruler wasn't perfectly lined up.
- Perhaps your eye was viewing at an angle rather than head-on. This is called a **parallax error.**

FYI

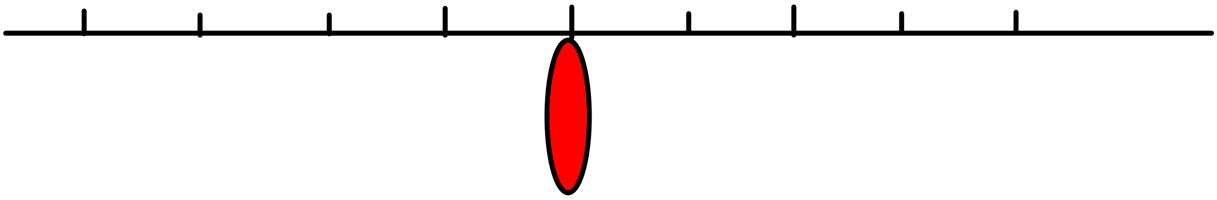
- The only way to minimize random error is to take many readings of the same measurement and to average them all together.



Close your left eye, extend your arm out straight, and cover the red shape with your thumb (be about 3 feet from your screen).

Make sure it is hidden from your sight (left eye closed still).

Now without moving your arm, close your right eye and open your left eye. Keep your arm sill and alternate open and closing your eyes. The effect is more pronounced the further from the screen you are.



Random and systematic errors

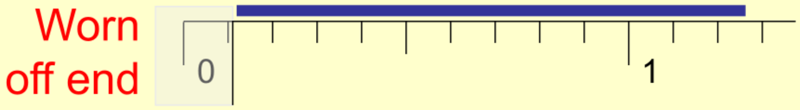
Systematic error is error due to the instrument being "out of adjustment."

•A voltmeter might have a zero offset error.

(**Offset** or **zero** setting **error** in which the instrument does not read **zero** when the quantity to be measured is **zero**.)

•A meter stick might be rounded on one end.

Now Bob measures the same line at 13 mm ± 1 mm.



Furthermore, every measurement Bob makes will be off by that same amount.

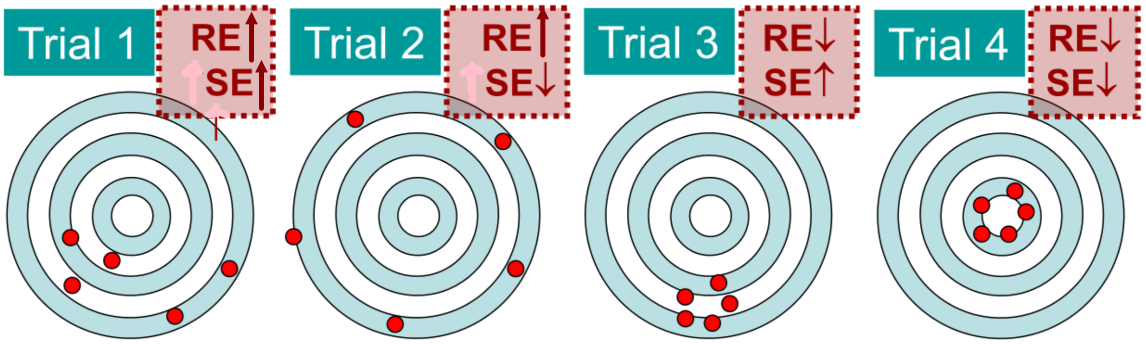
FYI

•Systematic errors are usually difficult to detect.

Random and systematic errors

- The following game where a catapult launches darts with the goal of hitting the bull's eye illustrates the difference between **precision** and **accuracy**.

RE - random error SE - systematic error



Low Precision	Low Precision	High Precision	High Precision
•Hits not grouped	*•Hits not grouped*	*•Hits grouped*	*•Hits grouped*
Low Accuracy	High Accuracy	Low Accuracy	High Accuracy
•Average well below bulls eye	*•Average right at bulls eye*	*•Average well below bulls eye*	*•Average right at bulls eye*

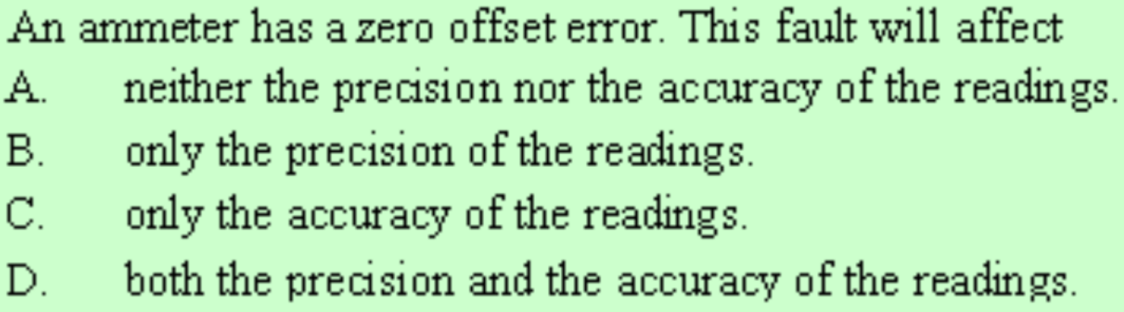
Generaly speaking:

Systematic errors affect accuracy. (tools)

Random errors affect precision. (user)

Random and systematic errors

PRACTICE:



An ammeter has a zero offset error. This fault will affect

A. neither the precision nor the accuracy of the readings.

B. only the precision of the readings.

C. only the accuracy of the readings.

D. both the precision and the accuracy of the readings.

SOLUTION:

- This is like the rounded-end ruler. It will produce a systematic error.
- Thus its error will be in accuracy, not precision.
- Solution is C

Absolute, fractional and percentage uncertainties

•**Absolute error** is the uncertainty or precision of your measurement.

EXAMPLE:

A student measures the length of a line with a wooden meter stick to be 11 mm ± 1 mm. What is the absolute error or uncertainty in her measurement?

SOLUTION:

•The ± number is the **absolute error**. Thus 1 mm is the absolute error.

•1 mm is also the **precision**.

Absolute uncertainty and precision can be the same value, but it doesn't have to be.

A digital stop watch might measure 0.001 of a second (precision), but your reaction time might only be 0.1 of a second (absolute uncertainty).

Absolute, fractional and percentage uncertainties

- **Fractional error** is given by

fractional error

$$\text{Fractional Error} = \frac{\text{Absolute Error}}{\text{Measured Value}}$$

EXAMPLE:

A student measures the length of a line with a wooden meter stick to be 11 mm ± 1 mm. What is the fractional error or uncertainty in her measurement?

SOLUTION:

- *Fractional error* = 1 / 11 = 0.09

FYI

- "Fractional" errors are usually expressed as decimal numbers rather than fractions.

Absolute, fractional and percentage uncertainties

- **Percentage error** is given by

percentage error

$$\text{Percentage Error} = \left(\frac{\text{Absolute Error}}{\text{Measured Value}}\right) \cdot 100\%$$

EXAMPLE:

A student measures the length of a line with a wooden meter stick to be 11 mm ± 1 mm. What is the percentage error or uncertainty in her measurement?

SOLUTION:

- *Percentage error* = (1 / 11) ·100% = 9%

FYI •Don't forget to include the percent sign.

Absolute, fractional and percentage uncertainties

PRACTICE:

A student measures the voltage shown. What are the absolute, fractional and percentage uncertainties of his measurement? Find the precision

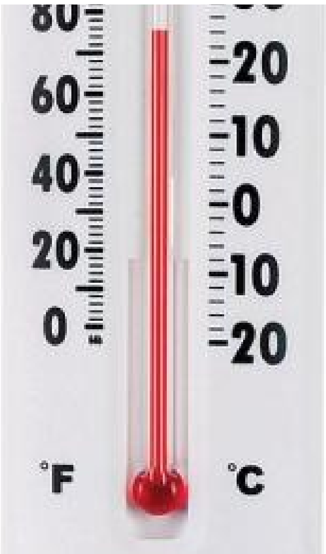
SOLUTION:

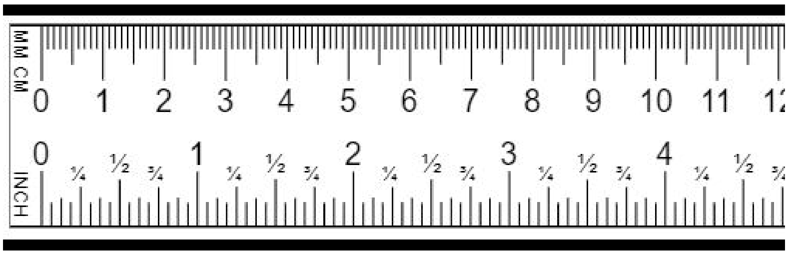
- Absolute uncertainty = 0.001 V.
- Fractional uncertainty = 0.001/0.385 = 0.0026.
- Percentage uncertainty = 0.0026(100%) = 0.26%.
- Precision is 0.001 V.

A note about PRECISION (uncertainty):

A digital device will have precision to the smallest number displayed on the screen, there is no ability to estimate past that value.

For an analog device, a ruler, a thermometer, etc. the precision is usually taken to be HALF of the smallest measured interval on the device.





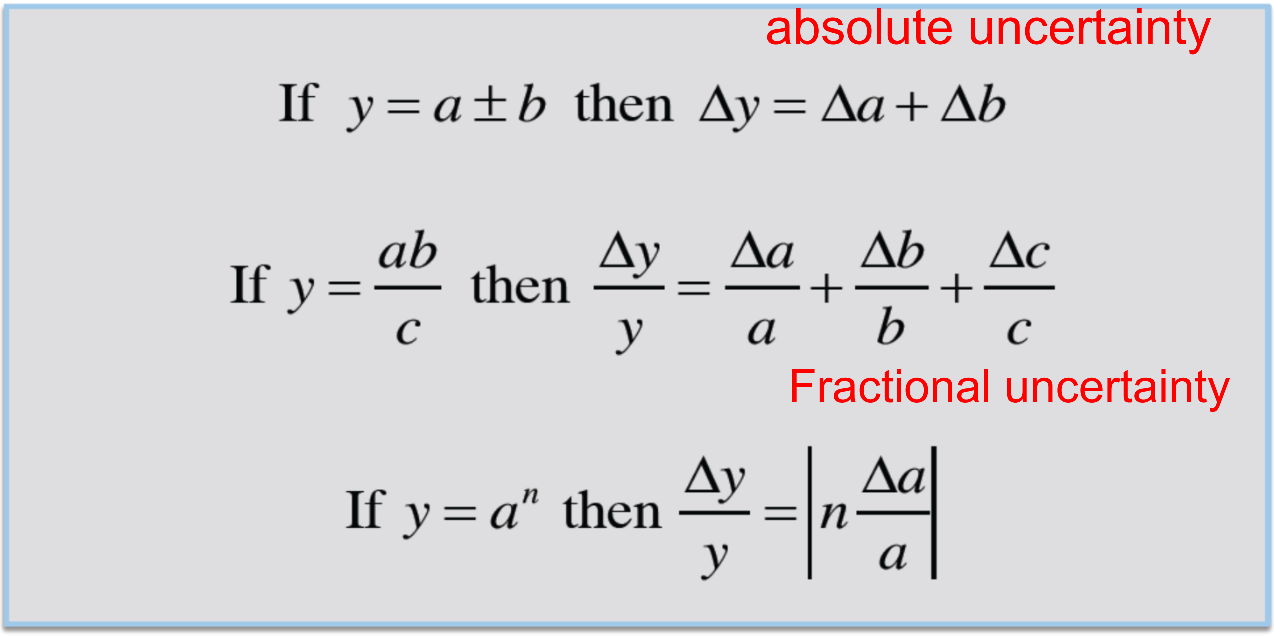
For a ruler using the metric scale, the smallest interval is 1mm, so the uncertainty would be 0.5mm, BUT, that is for only one end of the ruler. Therefore, rulers normally have uncertainty of +-1mm, (2x0.5mm). Both ends are active.

Each right side interval is 2 units, there for the uncertainty is +-1°C. One end is active.

IB Guidance:

- Analysis of uncertainties will not be expected for trigonometric or logarithmic functions in examinations

IB Physics Data Booklet, First Exam 2016, Subtopic 1.2



absolute uncertainty

$$\text{If } y = a \pm b \text{ then } \Delta y = \Delta a + \Delta b$$

$$\text{If } y = \frac{ab}{c} \text{ then } \frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

Fractional uncertainty

$$\text{If } y = a^n \text{ then } \frac{\Delta y}{y} = \left| n \frac{\Delta a}{a} \right|$$

Topic 1: Measurement and uncertainties

1.2 – Uncertainties and errors

Propagating uncertainties through calculations

- To find the **uncertainty in a sum or difference** you just add the uncertainties of all the ingredients.
- In formula form we have

uncertainty in sums and differences

If $y = a \pm b$ then $\Delta y = \Delta a + \Delta b$

FYI

- Note that whether or not the **calculation** has a + or a -, the **uncertainties** are ADDED.
- Uncertainties NEVER REDUCE ONE ANOTHER.

Topic 1: Measurement and uncertainties

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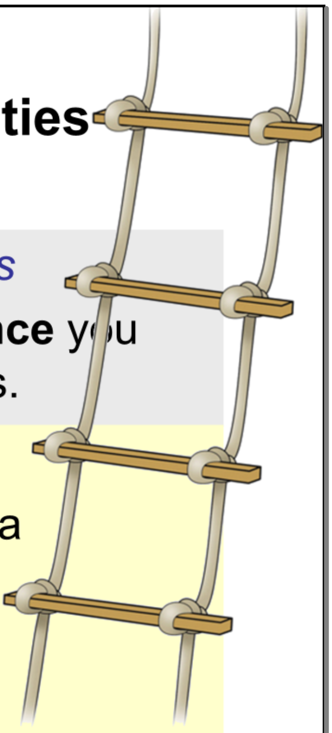

Propagating uncertainties through calculations

- To find the **uncertainty in a sum or difference** you just add the uncertainties of all the ingredients.

EXAMPLE:

A 9.51 ± 0.15 meter rope ladder is hung from a roof that is 12.56 ± 0.07 meters above the ground. How far is the bottom of the ladder from the ground?

SOLUTION:

- $y = a - b$ = 12.56 - 9.51 = 3.05 m
- $\Delta y = \Delta a + \Delta b$ = 0.15 + 0.07 = 0.22 m
- Thus the bottom is 3.05 ± 0.22 m from the ground.

The decimal placement of the main data should be similar to the error value. (the uncertainty should be at most two decimals places, usually one)

Topic 1: Measurement and uncertainties

1.2 – Uncertainties and errors

Propagating uncertainties through calculations

- To find the **uncertainty in a product or quotient** you just add the percentage or fractional uncertainties of all the ingredients.
- In formula form we have

uncertainty in products and quotients

If $y = a \cdot b / c$ then $\Delta y / y = \Delta a / a + \Delta b / b + \Delta c / c$

FYI

- Whether or not the **calculation** has a × or a ÷, the **uncertainties** are ADDED.
- You can't add numbers having different units, so we use fractional uncertainties for products and quotients.

$$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

error quantity

measured quantity

Topic 1: Measurement and uncertainties

1.2 – Uncertainties and errors

Propagating uncertainties through calculations

- To find the **uncertainty in a product or quotient** you just add the percentage or fractional uncertainties of all the ingredients.

EXAMPLE: A car travels 64.7 ± 0.5 meters in 8.65 ± 0.05 seconds. What is its speed?

SOLUTION: Use *rate* = *distance* divided by *time*.

$$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

- $r = d / t = 64.7 / 8.65 = 7.48 \text{ m s}^{-1}$
- $\Delta r / r = \Delta d / d + \Delta t / t = .5 / 64.7 + .05 / 8.65 = 0.0135$
- $\Delta r / 7.48 = 0.0135$ so that
- $\Delta r = 7.48(0.0135) = 0.10 \text{ m s}^{-1}$.
- Thus, the car is traveling at 7.48 ± 0.10 m s^{-1}.

Summary

The measurement tools dictate the potential precision of the data.

The absolute uncertainty of the data can be the same as the precision value, or can be adjusted based on the collection method (by hand, by eye, by hearing, etc.).

The data values will carry a certain number of significant figures. (operations with significant figures have to follow the respective adding-subtracting or multiplying-dividing rules).

The uncertainty values of the data also have operational rules to follow during calculations.

Task - Uncertainty