

The Quadratic Formula

- Where possible factoring can be used to find the zeros, but unfortunately there are many situations where a quadratic relation has zeros, but we cannot factor.
- In these situations, we must use a more powerful tool, the Quadratic Formula.
- The Quadratic Formula can be used to find the zeros in any situation (as long as they exist).
- To use the formula, a quadratic relation must be in standard form and equal to zero.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex/ Solve.

a) $2x^2 + 5x - 6 = 0$
a b c

opposite sign ✓

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-6)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{73}}{4} \leftarrow \begin{array}{l} \text{positive} \\ \text{so} \\ \text{has} \\ \text{solutions} \end{array}$$

↓ separate

$$x = \frac{-5 + \sqrt{73}}{4} \quad x = \frac{-5 - \sqrt{73}}{4}$$

figure out top before dividing

$$= \frac{3.54}{4} \quad = \frac{-13.54}{4}$$

$$x = 0.886 \quad x = -3.39$$

d) $5x^2 + 12x = 0$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(5)(0)}}{2(5)}$$

$$= \frac{-12 \pm \sqrt{144}}{10}$$

$$= \frac{-12 \pm 12}{10}$$

$$x = 0, \quad x = -2.4$$

← also could have factored

$$5x^2 + 12x = 0$$

$$x(5x + 12) = 0$$

↓ ↘

$$x = 0 \quad 5x + 12 = 0$$

$$5x = -12$$

$$x = -2.4$$

b) $x^2 - 4x + 2 = 0$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{8}}{2}$$

$$x = 3.41, \quad x = 0.586$$

c) $x^2 + 20 = 9x$ equal to zero
↓ First

$$x^2 - 9x + 20 = 0$$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(20)}}{2(1)}$$

$$= \frac{9 \pm \sqrt{1}}{2}$$

$$x = 4, \quad x = 5$$

★ Could have factored to solve

$$x^2 - 9x + 20 = 0 \quad M=20$$

$$A=-9$$

$$(x-4)(x-5) = 0 \quad N=-4$$

$$-5$$

$$x = 4, \quad x = 5$$

Ex/ Marc has a summer tree-trimming business. Based on experience Marc knows that $P = -2x^2 + 200x - 600$ models his profit, P , in dollars, where x is the amount he charges per hour. How much does he need to charge if he wants to have a profit of \$1000?

make equal zero

$$1000 = -2x^2 + 200x - 600$$

$$0 = -2x^2 + 200x - 600 - 1000$$

$$0 = -2x^2 + 200x - 1600$$

solve with formula

$$x = \frac{-200 \pm \sqrt{200^2 - 4(-2)(-1600)}}{2(-2)}$$

$$= \frac{-200 \pm \sqrt{27200}}{-4}$$

$$x = 8.768 \quad x = 91.23$$

∴ If he charges \$8.77 or \$91.23 he will have a profit of \$1000.

Ex/ The sum of the squares of two consecutive integers is 113. Find the integers.

one after the other

$$x = 1^{\text{st}} \#$$

$$x+1 = 2^{\text{nd}} \#$$

$$x^2 + (x+1)^2 = 113$$

squares

sum

FOIL to standard form and make equal zero

$$x^2 + (x+1)(x+1) - 113 = 0$$

$$x^2 + x^2 + 1x + 1x + 1 - 113 = 0$$

$$2x^2 + 2x - 112 = 0$$

Factor OR formula

$$2(x^2 + 1x - 56) = 0$$

$$2(x+8)(x-7) = 0$$

$$x = -8 \quad x = 7$$

$$x+1 = -7 \quad x+1 = 8$$

$$M = -56$$

$$A = 1$$

$$N = 8, -7$$

∴ The numbers are -8 and -7 OR 8 and 7