Optimal Word Problems

- There are times where we need to optimize a solution. In these cases we are looking for a maximum or minimum value, in other words, the vertex.

- Depending on how we are given, or choose to set up, an equation we can locate the vertex through one of two methods:
  1) Find and average the zeros.
  2) Use the equation for the axis of symmetry $x = \frac{-b}{2a}$.

Ex/ Determine the min or max value and when it occurs for:

a) $y = x^2 + 12x + 10$

positive -> opens up -> minimum

$x = \frac{-12}{2(1)} = \frac{-12}{2} = -6$

$y = (-6)^2 + 12(-6) + 10$
$= -26$

∴ Min of −26 when x = −6

b) $y = -4(x+1)(x-3)$

negative -> opens down -> max

$x = -1,\ x = 3$

axis: $x = \frac{-1+3}{2} = \frac{2}{2} = 1$

$y = -4(1+1)(1-3)$
$= -4(2)(-2)$
$= 16$

∴ Max of 16 when x = 1

Ex/ Jasmine makes necklaces for her market stall. Her daily profit from making x necklaces is given by $P = -x^2 + 20x$ dollars.

a) How many necklaces should Jasmine make per day to maximize her profits?
b) Find the maximum daily profit that Jasmine can make?
c) What happens if Jasmine sells no necklaces? What point on the parabola is this?

a) $x = \frac{-20}{2(-1)} = \frac{-20}{-2} = 10$

∴ She should make 10 necklaces

b) $P = -(10)^2 + 20(10)$
$= 100$

∴ Her max profit is \$100

c) $x = 0,\ P = 0$

the y-int

Ex/ A computer software company models the profit on its latest video game using the relation $P = -4x^2 + 20x - 9$, where x is the number of games produced in hundred thousands and P is the profit in millions?

a) What happens if the company doesn't sell any games?
b) What happens if the company sells 400,000 games?
c) How many games must the company produce to earn the maximum profit?
d) What is the maximum profit that the company can earn?

a) $x = 0,\ P = -9$

∴ They will lose \$9 million

b) $x = 4$

$P = -4(4)^2 + 20(4) - 9$
$= 7$

∴ They will earn \$7 million

c) $x = \frac{-20}{2(-4)} = 2.5$

∴ They need to sell 250,000 games to maximize

d) $x = 2.5$

$P = -4(2.5)^2 + 20(2.5) - 9$
$= 16$

∴ Max profit is \$16 million

Ex/ The population of a shrinking town, P is modeled by the function $P = 8x^2 - 112x + 570$, where $t = 0$ represents the year 2000.

a) What will the population be in 2030?

b) When does the town reach its minimum population?

a) $t = 30$ (years since 2030)

$P = 8(30)^2 - 112(30) + 570$

$= 4410$

b) $x = \frac{112}{2(8)}$

$= 7$

$\therefore$ In 2007 it reached the minimum.

Ex/ A rapid transit company has 5000 passengers daily, each currently paying a \$2.25 fare. For each \$0.25 increase, the company estimates that it will lose 100 passengers daily.

a) Determine the optimal pricing strategy?

b) How much revenue will the company make?

$x = \#$ of changes

a) $R = (2.25 + 0.25x)(5000 - 100x)$

$2.25 + 0.25x = 0$

$0.25x = -2.25$

$x = -9$

$5000 - 100x = 0$

$5000 = 100x$

$50 = x$

AoS : $x = \frac{-9 + 50}{2}$

$= \frac{41}{2}$

$= 20.5$ changes

$R = (2.25 + 0.25(20.5))(5000 - 100(20.5))$

$= 7.375\ (2950)$

* have to round

If you charge \$7.38, 2950 people will ride

b) Will make a max of

$7.38(2950) =$ \$21,771

Homework: Pg. 302 #'s: 11-14,17