Midpoint and Distance

- The coordinates of the middle of a line segment is referred to as the midpoint. The midpoint is the average of the two end points.

$$mp = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Ex/ Determine the coordinates of the midpoint of the line segment that joins:

a) A(0,0) and B(4,0)

$mp_{AB} = \left(\frac{0+4}{2}, \frac{0+0}{2}\right)$

$= \left(\frac{4}{2}, \frac{0}{2}\right)$

$= (2,0)$

b) F(-4,2) and G(-10,-7)

$m_{FG} = \left(\frac{-4+(-10)}{2}, \frac{2+(-7)}{2}\right)$

$= \left(\frac{-14}{2}, \frac{-5}{2}\right)$

$= (-7, -2.5)$

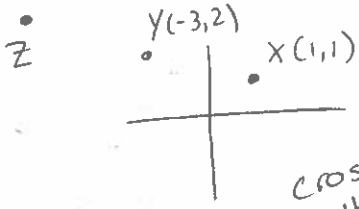
c) M(-3,7) and N(8,-1)

$mp_{MN} = \left(\frac{-3+8}{2}, \frac{7+(-1)}{2}\right)$

$= \left(\frac{5}{2}, \frac{6}{2}\right)$

$= (2.5, 3)$

Ex/ One of the endpoints of a line segment is X(1,1). The midpoint of the line segment is Y(-3,2). Find the other endpoint.



$\left(\frac{x+1}{2}, \frac{y+1}{2}\right) = (-3, 2)$

* make an equation for x-values and another for y-values

cross multiply ↳

$\frac{x+1}{2} = -3$

$x + 1 = -6$

$x = -6 - 1$

$x = -7$

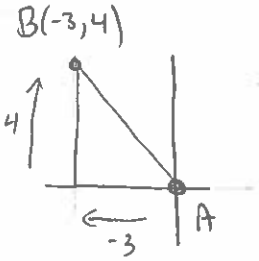
$\frac{y+1}{2} = 2$

$y + 1 = 4$

$y = 4 - 1$

$y = 3$

Ex/ Find the distance from A(0,0) to B(-3,4).



Pythagoras!!

$a^2 + b^2 = c^2$

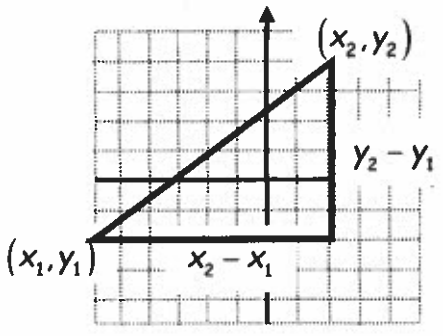
$(-3)^2 + 4^2 = c^2$

$25 = c^2$

$\sqrt{25} = c$

$c = 5$

- It is possible to find the distance between any two points using Pythagoras, all that is needed is the difference between the coordinates.



$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ex/ Determine the length of the line joining the following points:

a) A(-2,4) and B(7,3)

$$d_{AB} = \sqrt{(7-(-2))^2 + (3-4)^2}$$
$$= \sqrt{9^2 + (-1)^2}$$ ← Brackets
$$= \sqrt{81+1}$$ ← square both
$$= \sqrt{82}$$ ← Add
$$= 9.06$$ ← square root

b) C(-4,-2) and D(-12,8)

$$d_{CD} = \sqrt{(-12-(-4))^2 + (8-(-2))^2}$$
$$= \sqrt{(-8)^2 + 10^2}$$
$$= \sqrt{64+100}$$
$$= \sqrt{164}$$
$$= 12.81$$

Ex/ Given P(1,5), Q(5,7) and R(3,1) show that triangle PQR is isosceles.

$$d_{PQ} = \sqrt{(5-1)^2 + (7-5)^2}$$
$$= \sqrt{4^2 + 2^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$

$$d_{QR} = \sqrt{(3-5)^2 + (1-7)^2}$$
$$= \sqrt{(-2)^2 + (-6)^2}$$
$$= \sqrt{4+36}$$
$$= \sqrt{40}$$

$$d_{PR} = \sqrt{(3-1)^2 + (1-5)^2}$$
$$= \sqrt{2^2 + (-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$

∴ Since two sides are equal (and not the third), this is isosceles

Ex/ In a video game, three animated characters are programmed to run out of a building at F(1,-1) and hard in three different directions. After 2 seconds, Animal is at A(22,18), Beast is at B(-3,35) and Creature is at C(7,-29). Which character is the fastest?

$$d_{AF} = \sqrt{(22-1)^2 + (18-(-1))^2}$$
$$= \sqrt{21^2 + 19^2}$$
$$= \sqrt{802}$$

$$d_{BF} = \sqrt{(-3-1)^2 + (35-(-1))^2}$$
$$= \sqrt{(-4)^2 + 36^2}$$
$$= \sqrt{1312}$$

↑ Went furthest = Fastest!

$$d_{CF} = \sqrt{(7-1)^2 + (-29-(-1))^2}$$
$$= \sqrt{6^2 + (-28)^2}$$
$$= \sqrt{820}$$

Homework: Pg. 79 #'s: 3,5,6,9
Pg. 86 #'s: 3,5ace,7