## A–1 Powers

A power is an expression that shows repeated multiplication.

**EXAMPLE**

Evaluate the power $2^4$.

**Solution**

$$2^4 = 2 \times 2 \times 2 \times 2$$
$$= 16$$

2 is the base of the power and 4 is the exponent.

### Practice

**1.** Write the product represented by each power. Then, evaluate the expression.

| | | | | |
|---|---|---|---|---|
| **a)** $2^2$ | **c)** $2^4$ | **e)** $10^3$ | **g)** $4^2$ | **i)** $5^3$ |
| **b)** $2^3$ | **d)** $3^2$ | **f)** $10^4$ | **h)** $4^3$ | **j)** $5^5$ |

## A–2 Order of Operations

You can remember the order of operations by using the memory aid "BEDMAS."

**B**rackets
**E**xponents
**D**ivide and **M**ultiply from left to right
**A**dd and **S**ubtract from left to right

**EXAMPLE**

Evaluate the integer expression $(2 \times 3 - 2^2) + 4(6 - 1)$.

**Solution**

$$(2 \times 3 - 2^2) + 4(6 - 1) = (6 - 4) + 4(5)$$
$$= 2 + 20$$
$$= 22$$

### Practice

**1.** Evaluate using the rules for order of operations.

**a)** $(3 + 6 \div 3)^2$

**b)** $4(2^3 - 3 \times 2)$

**c)** $[(8 + 6 \div 3) - 5]^2$

**d)** $2(3^2 + 1) \div 5$

**e)** $(9 + 1)^3 \div (3^2 + 1)$

**f)** $4[(32 - 5^2) - (2^3 - 1)]$

# A–3 Adding and Subtracting Integers

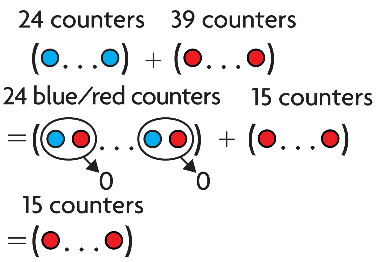
The Zero Principle states that when you add opposite integers, the result is 0.
The + sign is often not used for positive integers.

### EXAMPLE 1

Add $(-24) + (+39)$ using integer counters.

**Solution**

$(-24) + (+39) = 15$

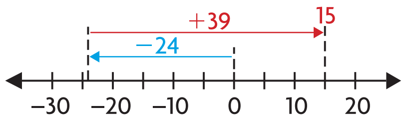



### EXAMPLE 2

Add $-24 + 39$ using a number line.

**Solution**

$-24 + 39 = 15$




### EXAMPLE 3

Add $-24 + 39$ using the Zero Principle.

**Solution**

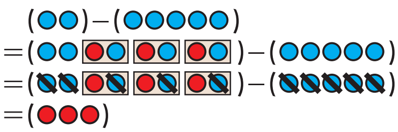
$$\begin{aligned} -24 + 39 &= (-24 + 24) + 15 \\ &= 0 + 15 \\ &= 15 \end{aligned}$$

### EXAMPLE 4

Subtract $-2 - (-5)$ using integer counters.

**Solution**

$-2 - (-5) = 3$

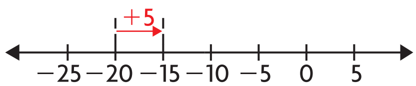


### EXAMPLE 5

Subtract $-15 - (-20)$ using a number line.

**Solution**

$-15 - (-20) = 5$




### EXAMPLE 6

Subtract $-15 - (-20)$ using the Zero Principle.

**Solution**

$$\begin{aligned} -15 - (-20) &= -15 + 20 \\ &\quad + (\cancel{-}20) \\ &\quad - (\cancel{-}20) \\ &= -15 + 20 \\ &= 5 \end{aligned}$$

## Practice

**1.** Represent each operation using integer counters or a number line.

- **a)** $-6 + (-3)$
- **b)** $5 + (-2)$
- **c)** $-23 + 8$
- **d)** $5 - (-4)$
- **e)** $-20 - 16$
- **f)** $-9 - 6$

**2.** Determine each sum.

- **a)** $-3 + (-2)$
- **b)** $2 + (-3)$
- **c)** $-18 + 8$
- **d)** $-6 + 4$
- **e)** $-40 + (-15)$
- **f)** $32 + (-46)$

**3.** Determine each difference.

- **a)** $4 - (-3)$
- **b)** $-5 - (-2)$
- **c)** $5 - (-13)$
- **d)** $-14 - (-7)$
- **e)** $6 - (-6)$
- **f)** $-43 - 4$

**4.** Calculate.

- **a)** $3 - (-4) + 10$
- **b)** $-7 + 2 - (-1)$
- **c)** $-5 - (-3) + 4$
- **d)** $-41 + (-32) + 15$

# A–4 Multiplying and Dividing Integers

The following patterns describe the results of multiplying or dividing two integers:

$(+) \times (+) = +$ $\quad (-) \times (+) = -$ $\quad (+) \times (-) = -$ $\quad (-) \times (-) = +$

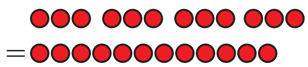
$(+) \div (+) = +$ $\quad (-) \div (+) = -$ $\quad (+) \div (-) = -$ $\quad (-) \div (-) = +$

## EXAMPLE 1

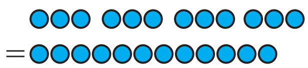
Multiply $4 \times 3$, $4 \times (-3)$, $-4 \times 3$, and $-4 \times (-3)$ using integer counters.

**Solution**

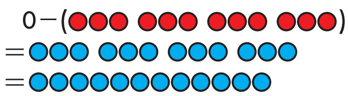
$4 \times 3 = 12$



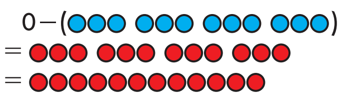
$4 \times (-3) = -12$



$-4 \times 3 = -12$



$-4 \times (-3) = 12$

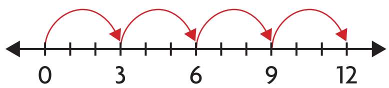


## EXAMPLE 2

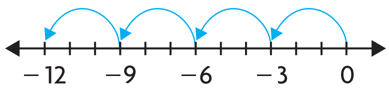
Multiply $4 \times 3$, $4 \times (-3)$, $-4 \times 3$, and $-4 \times (-3)$ using a number line.

**Solution**

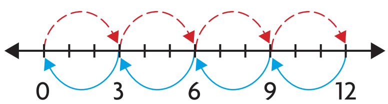
$4 \times 3 = 12$



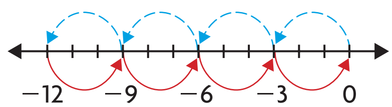

$4 \times (-3) = -12$




$-4 \times 3 = -12$




$-4 \times (-3) = 12$




## EXAMPLE 3

Divide $12 \div 3$, $-12 \div 3$, $12 \div (-3)$, and $-12 \div (-3)$ by solving the related multiplication equations.

**Solution**

$12 \div 3 = 4$

because

$3 \times 4 = 12$

$-12 \div 3 = -4$

because

$3 \times (-4) = -12$

$12 \div (-3) = -4$

because

$-3 \times (-4) = 12$

$-12 \div (-3) = 4$

because

$-3 \times 4 = -12$

## Practice

**1.** Represent each operation using integer counters or a number line.

**a)** $-2 \times 5$ **b)** $-5 \times (-4)$ **c)** $3 \times (-6)$ **d)** $-6 \times (-9)$

**2.** Calculate each product.

**a)** $(-3)(2)$ **b)** $(-4)(-9)$ **c)** $(4)(-3)$ **d)** $(-7)(-3)$ **e)** $(5)(4)$ **f)** $(-2)(7)$

**3.** Calculate each quotient.

**a)** $-18 \div (-6)$ **b)** $-24 \div 6$ **c)** $51 \div (-17)$ **d)** $-42 \div (-14)$ **e)** $60 \div (-12)$ **f)** $-30 \div (-15)$

**4.** Evaluate.

**a)** $(-5)(-5)$ **b)** $-56 \div 8$ **c)** $(-2)(5)(-4)$ **d)** $(8)(4) \div (-2)$ **e)** $(4)(81) \div (-27)(-2)$ **f)** $64 \div [(-4)(-4)(-4)]$

# A–5 Evaluating Integer Expressions with Several Operations

Expressions involving many integer operations are evaluated using the same order of operations as for whole numbers.

### EXAMPLE 1

Evaluate the expression $-4(-3 - 6) + (-2 + (-1))$.

**Solution**

$$\begin{aligned} -4(-3 - 6) + (-2 + (-1)) &= -4(-9) + (-3) \\ &= 36 + (-3) \\ &= 33 \end{aligned}$$

### EXAMPLE 2

Evaluate the expression $(-2)(4) + (-3)^2$.

**Solution**

$$\begin{aligned} (-2)(4) + (-3)^2 &= (-2)(4) + 9 \\ &= -8 + 9 \\ &= 1 \end{aligned}$$

## Practice

**1.** Evaluate using the order of operations.

**a)** $5 - (3 - 4)$
**b)** $(5 - 7) - (3 - 4)$
**c)** $-3(-4) - (5 - 7)$
**d)** $(3)(2) - (3 + 5)$
**e)** $-(5 - 9) - (-2)(2)$
**f)** $(4 - 3) - 2(3 - 4)$

**2.** Evaluate.

**a)** $2(-3)^2 - 4(-2)$
**b)** $-4(-2)^3 - 3(-4)^2$
**c)** $(-3 - 2)^2 - (2 + 4)^2$
**d)** $3(-2 + 4)^3 - 2(-4 + 1)^2$
**e)** $2(-1 - 3)^2 - (1 + 3)^2$
**f)** $5(-2)^2 - 3(-1 - 2)^3$

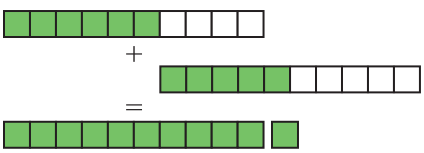
# A–6 Adding and Subtracting Fractions

### EXAMPLE 1

Add $\frac{3}{5} + \frac{1}{2}$ using fraction strips.

**Solution**

$$\begin{aligned} \frac{3}{5} + \frac{1}{2} &= \frac{6}{10} + \frac{5}{10} \\ &= \frac{11}{10} \text{ or } 1\frac{1}{10} \end{aligned}$$

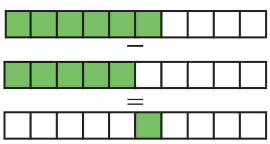



### EXAMPLE 2

Subtract $\frac{3}{5} - \frac{1}{2}$ using fraction strips.

**Solution**

$$\begin{aligned} \frac{3}{5} - \frac{1}{2} &= \frac{6}{10} - \frac{5}{10} \\ &= \frac{1}{10} \end{aligned}$$

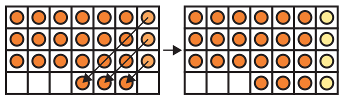


## EXAMPLE 3

Add $\frac{3}{4} + \frac{1}{7}$ using a grid.

**Solution**

$$\frac{3}{4} + \frac{1}{7} = \frac{21}{28} + \frac{4}{28}$$
$$= \frac{25}{28}$$

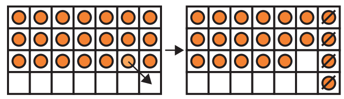


## EXAMPLE 4

Subtract $\frac{3}{4} - \frac{1}{7}$ using a grid.

**Solution**

$$\frac{3}{4} - \frac{1}{7} = \frac{21}{28} - \frac{4}{28}$$
$$= \frac{17}{28}$$

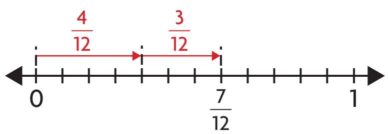


## EXAMPLE 5

Add $\frac{1}{3} + \frac{1}{4}$ using a number line.

**Solution**

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12}$$
$$= \frac{7}{12}$$

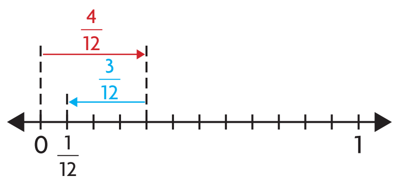



## EXAMPLE 6

Subtract $\frac{1}{3} - \frac{1}{4}$ using a number line.

**Solution**

$$\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12}$$
$$= \frac{1}{12}$$




## EXAMPLE 7

Add $\frac{2}{3} + \frac{1}{6}$ using the least common denominator.

**Solution**

$$\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6}$$
$$= \frac{5}{6}$$

## EXAMPLE 8

Subtract $\frac{2}{3} - \frac{1}{6}$ using the least common denominator.

**Solution**

$$\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6}$$
$$= \frac{3}{6} \text{ or } \frac{1}{2}$$

## Practice

1. Represent each operation using a grid, fraction strips, or a number line.
   a) $\frac{3}{4} + \frac{1}{5}$
   b) $\frac{3}{4} + \frac{5}{6}$
   c) $\frac{4}{5} - \frac{1}{3}$
   d) $\frac{1}{6} - \frac{1}{9}$

2. Add.
   a) $\frac{1}{7} + \frac{3}{7}$
   b) $\frac{2}{9} + \frac{5}{9}$
   c) $\frac{3}{8} + \frac{1}{8}$
   d) $\frac{1}{3} + \frac{1}{9}$
   e) $\frac{1}{3} + \frac{1}{6}$
   f) $\frac{1}{3} + \frac{5}{12}$

3. Subtract.
   a) $\frac{5}{9} - \frac{1}{9}$
   b) $\frac{14}{15} - \frac{7}{15}$
   c) $\frac{7}{15} - \frac{2}{5}$
   d) $\frac{5}{6} - \frac{3}{8}$
   e) $\frac{3}{4} - \frac{1}{6}$
   f) $\frac{1}{3} - \frac{1}{6}$

4. Evaluate.
   a) $\frac{3}{4} + \frac{3}{10}$
   b) $\frac{4}{3} - \frac{2}{11}$
   c) $\frac{2}{5} + \frac{6}{7}$
   d) $\frac{8}{15} - \frac{1}{16}$
   e) $\frac{7}{2} + \frac{3}{5}$
   f) $\frac{14}{5} - \frac{5}{7}$

# A–7 Multiplying and Dividing Fractions

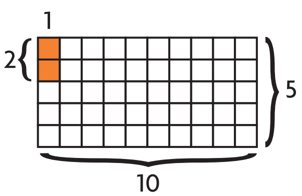
### EXAMPLE 1

Multiply $\frac{2}{5} \times \frac{1}{10}$ using an area model.

**Solution**

The product of the denominators indicates the length and width of the rectangle to create. The product of the numerators indicates the length and width of the rectangle to be shaded.

$\frac{2}{5} \times \frac{1}{10} = \frac{2}{50}$




### EXAMPLE 2

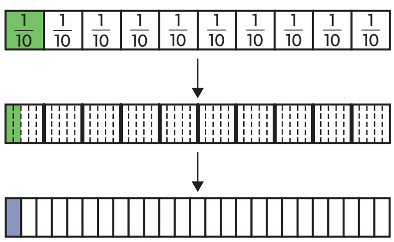
Multiply $\frac{2}{5} \times \frac{1}{10}$ using fraction strips.

**Solution**

$\frac{2}{5} \times \frac{1}{10}$ is the same as $\frac{2}{5}$ of $\frac{1}{10}$.

Each box representing $\frac{1}{10}$ must be divided into 5 equal parts. Two of these parts must be shaded.

This is $\frac{2}{50}$ or $\frac{1}{25}$ in lowest terms.




### EXAMPLE 3

Multiply $\frac{2}{5} \times \frac{1}{10}$ by multiplying numerators and denominators.

**Solution**

$$\frac{2}{5} \times \frac{1}{10} = \frac{2 \times 1}{5 \times 10}$$

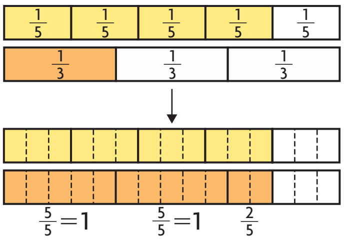
$$= \frac{2}{50} \text{ or } \frac{1}{25} \text{ in lowest terms}$$

## EXAMPLE 4

Divide $\frac{4}{5} \div \frac{1}{3}$ using fraction strips.

**Solution**

$$\frac{4}{5} \div \frac{1}{3} = \frac{12}{5} \text{ or } 2\frac{2}{5}$$




## EXAMPLE 5

Divide $\frac{4}{5} \div \frac{1}{3}$ using the least common denominator.

**Solution**

$$\begin{aligned}\frac{4}{5} \div \frac{1}{3} &= \frac{12}{15} \div \frac{5}{15} \\ &= 12 \div 5 \\ &= \frac{12}{5} \text{ or } 2\frac{2}{5}\end{aligned}$$

## EXAMPLE 6

Divide $\frac{4}{5} \div \frac{1}{3}$ using a reciprocal.

**Solution**

$$\begin{aligned}\frac{4}{5} \div \frac{1}{3} &= \frac{4}{5} \times \frac{3}{1} \\ &= \frac{12}{5} \text{ or } 2\frac{2}{5}\end{aligned}$$

## Practice

**1.** Represent each operation using an area model or fraction strips.

**a)** $\frac{1}{3} \times \frac{1}{5}$ **c)** $\frac{2}{5} \div \frac{1}{2}$

**b)** $\frac{3}{4} \times \frac{2}{5}$ **d)** $\frac{3}{4} \div \frac{2}{3}$

**2.** Multiply.

**a)** $\frac{1}{2} \times \frac{3}{5}$ **c)** $\frac{3}{4} \times \frac{8}{15}$

**b)** $\frac{3}{4} \times \frac{7}{10}$ **d)** $\frac{2}{3} \times \frac{9}{11}$

**3.** Divide.

**a)** $\frac{3}{7} \div \frac{4}{5}$ **c)** $\frac{3}{4} \div \frac{7}{8}$

**b)** $\frac{2}{11} \div \frac{3}{5}$ **d)** $\frac{5}{8} \div \frac{13}{16}$

**4.** Evaluate.

**a)** $\frac{2}{3} \times \frac{8}{13}$ **c)** $\frac{5}{8} \div \frac{1}{4}$

**b)** $\frac{3}{5} \times \frac{3}{5}$ **d)** $\frac{8}{9} \times \frac{3}{8}$

# A–8 Evaluating Fraction Expressions with Several Operations

Expressions involving many fraction operations are evaluated using the same order of operations as for whole numbers.

**EXAMPLE**

Evaluate the fraction expression $\frac{3}{2} - \frac{2}{5} \div \frac{1}{5} \times \left(\frac{3}{8} + \frac{1}{8}\right)^2 + \frac{2}{3}$.

**Solution**

$$\begin{aligned}
\frac{3}{2} - \frac{2}{5} \div \frac{1}{5} \times \left(\frac{3}{8} + \frac{1}{8}\right)^2 + \frac{2}{3} &= \frac{3}{2} - \frac{2}{5} \div \frac{1}{5} \times \left(\frac{1}{2}\right)^2 + \frac{2}{3} \\
&= \frac{3}{2} - \frac{2}{5} \div \frac{1}{5} \times \frac{1}{4} + \frac{2}{3} \\
&= \frac{3}{2} - 2 \times \frac{1}{4} + \frac{2}{3} \\
&= \frac{3}{2} - \frac{1}{2} + \frac{2}{3} \\
&= 1\frac{2}{3}
\end{aligned}$$

## Practice

**1.** Calculate using the order of operations.

**a)** $\frac{1}{2} - \frac{1}{3} \times \frac{1}{4} + \frac{1}{5} \div \frac{1}{6}$

**b)** $\left(\frac{1}{2} - \frac{1}{3}\right) \times \left(\frac{1}{4} + \frac{1}{5} \div \frac{1}{6}\right)$

**c)** $\left(\frac{1}{2} - \frac{1}{3} \times \frac{1}{4} + \frac{1}{5}\right) \div \frac{1}{6}$

**d)** $\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{6}\right)^3 + \frac{2}{3} \times \frac{4}{5}$

**e)** $\frac{5}{4} \times \frac{1}{2} - \frac{2}{3} \div 2 + \frac{1}{2}$

**f)** $\left(\frac{2}{3} + \frac{1}{6}\right)^2$

# A–9 Multiplying and Dividing Decimals

Estimate the answer before solving a decimal problem. Then, compare your result to your prediction.

**EXAMPLE 1**

Estimate the product $0.6 \times 0.5$.

**Solution**

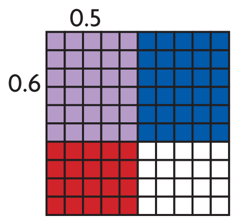
0.6 is a little more than $\frac{1}{2}$ and 0.5 is $\frac{1}{2}$, so the product will be a little more than $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ or 0.25

**EXAMPLE 2**

Multiply $0.6 \times 0.5$ using a grid model.

**Solution**

$$\begin{aligned}
0.6 \times 0.5 &= \frac{30}{100} \\
&= 0.30
\end{aligned}$$



## EXAMPLE 3

Multiply 0.6 × 0.5 using equivalent whole numbers.

**Solution**

$$\begin{aligned} 0.6 &= 6 \div 10 \text{ and } 6 \times 0.5 = 3.0. \\ \text{So, } 0.6 \times 0.5 &= 3.0 \div 10 \\ &= 0.3 \end{aligned}$$

## EXAMPLE 4

Multiply 0.6 × 0.5 using an algorithm.

**Solution**

$$\begin{array}{r} 0.6 \\ \times\ 0.5 \\ \hline 3\ 0 \\ 0\ 0\phantom{\ 0} \\ \hline 0.3\ 0 \end{array}$$

## EXAMPLE 5

Estimate the quotient 2.5 ÷ 0.45.

**Solution**

0.45 is close to 0.5, which is $\frac{1}{2}$. Dividing by $\frac{1}{2}$ is the same as multiplying by 2. So, 2.5 ÷ 0.45 is about 2.5 × 2 = 5.0.

## EXAMPLE 6

Divide 2.5 ÷ 0.45 using equivalent whole numbers.

**Solution**

2.5 ÷ 0.45 is the same as $\frac{2.5}{0.45}$.

$$\text{So, } \frac{2.5}{0.45} = \frac{2.5 \times 100}{0.45 \times 100} = \frac{250}{45}$$

$$\begin{array}{r} 5.5 \\ 45\overline{)250} \\ \underline{225} \\ 25.0 \\ \underline{22.5} \\ 2.5 \\ \dots \end{array}$$

The remainder repeats, so the answer is $5.\overline{5}$.

## Practice

1. Determine each product. Round to the nearest hundredth if necessary.
   - **a)** 1.4 × 2.5
   - **b)** 0.75 × 2.0
   - **c)** 3.25 × 1.4
   - **d)** 4.5 × 2.5
   - **e)** 3.73 × 2.17
   - **f)** 5.81 × 1.01

2. Determine each quotient. Round to the nearest hundredth if necessary.
   - **a)** 8.37 ÷ 3.1
   - **b)** 15.84 ÷ 3.2
   - **c)** 10.25 ÷ 4.1
   - **d)** 7.14 ÷ 4.76
   - **e)** 24.375 ÷ 8.125
   - **f)** 20.265 ÷ 2.1

# A–10 Expanded Form and Scientific Notation

Expanded form is a way of writing a number that shows the value of each digit using a power of 10.
Scientific notation is a way of writing a number as a decimal between 1 and 10, multiplied by a power of 10.

### EXAMPLE 1

Write the number 70 120 in expanded form.

**Solution**

$$70\,120 = 7 \times 10\,000 + 1 \times 100 + 2 \times 10$$
$$= 7 \times 10^4 + 1 \times 10^2 + 2 \times 10$$

### EXAMPLE 2

Write the number 70 120 using scientific notation.

**Solution**

The answer is $7.012 \times 10^4 = 70\,120$. This is because 7.012 is between 1 and 10 and $10^4$ is a power of 10.

## Practice

**1.** Express each number in expanded form.
- **a)** 1234
- **b)** 11 125
- **c)** 10 005
- **d)** 1 045 301

**2.** Express each number using scientific notation.
- **a)** 1234
- **b)** 11 125
- **c)** 10 005
- **d)** 1 045 301

**3.** Copy and complete the table.

| Standard Form | Expanded Form | Scientific Notation |
|---|---|---|
| 451 | $4 \times 10^2 + 5 \times 10 + 1 \times 10^0$ | $4.51 \times 10^2$ |
| 1026 | | |
| | $2 \times 10^3 + 5 \times 10$ | |
| | | $4.72 \times 10^5$ |

# A–11 Patterns and Relationships

A geometric pattern is a sequence of figures made up of several pieces. There is often a relationship between the number of a figure in a pattern and the number of pieces required to build it. The pattern can be described in various ways.

**A geometric pattern:**

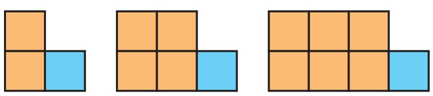


figure 1 figure 2 figure 3

## EXAMPLE 1

Draw figure 4 in the pattern.

**Solution**

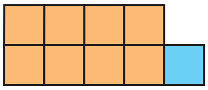


figure 4

## EXAMPLE 2

Determine the number of squares in figure 4 in the pattern using a table of values.

**Solution**

The table of values shows the number of squares in the first four figures of the pattern. Figure 4 has 9 squares.

| Figure Number | Number of Squares |
|---|---|
| 1 | 3 |
| 2 | $3 + 2 = 5$ |
| 3 | $5 + 2 = 7$ |
| 4 | $7 + 2 = 9$ |

## EXAMPLE 3

Determine the number of squares in figure 4 in the pattern using words or an algebraic expression.

**Solution**

The number of squares seems to be 1 more than twice the figure number. The pattern is $s = 2n + 1$ where $s$ is the number of squares and $n$ is the figure number. So, figure 4 has $s = 2(4) + 1 = 9$ squares.

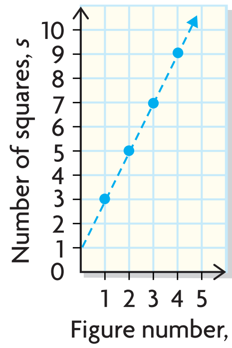
## EXAMPLE 4

Determine the number of squares in figure 4 in the pattern using a scatter plot.

**Solution**

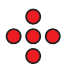
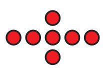
The scatter plot shows the relationship between the figure number and the number of squares in the figure. Figure 4 has 9 squares.

**Number of Squares vs. Figure Number**




# Practice

**1. a)** Copy and complete the table of values.

| Figure Number | Figure | Number of Counters |
|---|---|---|
| 1 |  | |
| 2 |  | |
| 3 |  | |
| 4 | | |
| 5 | | |

**b)** Construct a scatter plot that represents the pattern.

**2. a)** Describe the pattern rule using an algebraic expression. Represent the figure number as $n$. Represent the number of toothpicks in the figure as $t$.

figure 1 figure 2 figure 3

**b)** Construct a table of values that shows the toothpicks required to build the first five figures in the pattern.

**c)** Construct a scatter plot that represents the pattern.

# A–12 The Cartesian Coordinate System

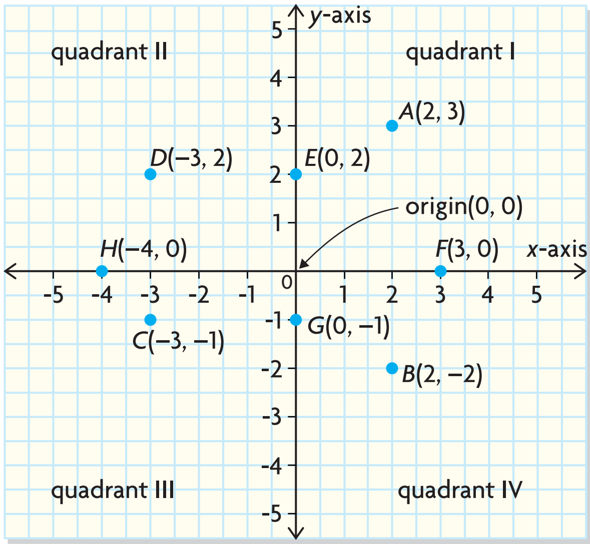
A Cartesian coordinate system uses a horizontal number line (the *x*-axis) and a vertical number line (the *y*-axis) to determine the coordinates of points.

**EXAMPLE**

Determine the location and the signs of the coordinates of each point.

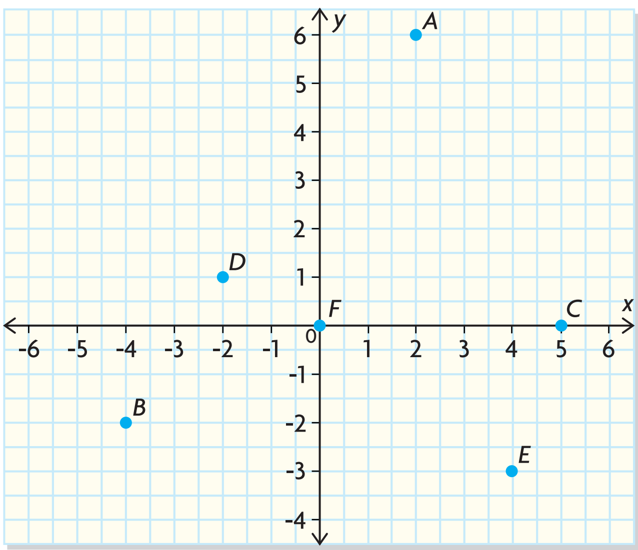
**Solution**

| Point | Location | Signs of Coordinates $(x, y)$ |
|---|---|---|
| *A* | quadrant I | $(+, +)$ |
| *B* | quadrant IV | $(+, -)$ |
| *C* | quadrant III | $(-, -)$ |
| *D* | quadrant II | $(-, +)$ |
| *E* and *G* | *y*-axis | $(0, y)$ |
| *F* and *H* | *x*-axis | $(x, 0)$ |




## Practice

1. Write the coordinates of each point shown in the graph.




2. Plot each point on a Cartesian coordinate system.
   - a) $(2, 3)$
   - b) $(0, 0)$
   - c) $(0, -3)$
   - d) $(5, 0)$
   - e) $(-4, -6)$
   - f) $(-7, 2)$

3. Answer the following questions about the points in the previous question.
   - a) Which points are on an axis? Which axis?
   - b) In which quadrant is each point not on an axis located?

4. State the coordinates of a point that satisfies each set of conditions.
   - a) The point is in quadrant I. The *x*-coordinate is greater than the *y*-coordinate.
   - b) The point is on the *x*-axis between quadrants II and III.
   - c) The point is in quadrant III. The *x*- and *y*-coordinates have the same value.
   - d) The point is in quadrant II. The *y*-coordinate is the square of the *x*-coordinate.

# A–13 Equations

An equation is a statement that two mathematical quantities or expressions have the same value.
A solution to an equation is a value for which the equation is true.
Solutions to equations can be found in different ways.

## EXAMPLE 1

Solve the equation $3n + 2 = 17$ using inspection and logical reasoning.

### Solution

$3n + 2 = 17$
That means $3n = 15$.
So, $n$ must be 5.

## EXAMPLE 2

Solve the equation $3n + 2 = 17$ using systematic trial.

### Solution

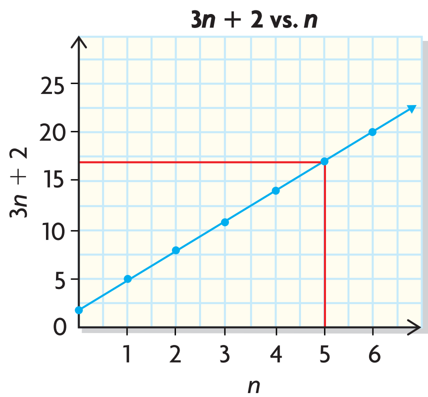
| $n$ | $3n + 2$ | Comparison to 17 |
|---|---|---|
| 10 | $3(10) + 2 = 32$ | too big |
| 4 | $3(4) + 2 = 14$ | too small |
| 5 | $3(5) + 2 = 17$ | correct |

The answer is $n = 5$.

## EXAMPLE 3

Estimate the solution to the equation $3n + 2 = 17$ by using a graph.

### Solution




From the graph, $n$ is about 5.

## EXAMPLE 4

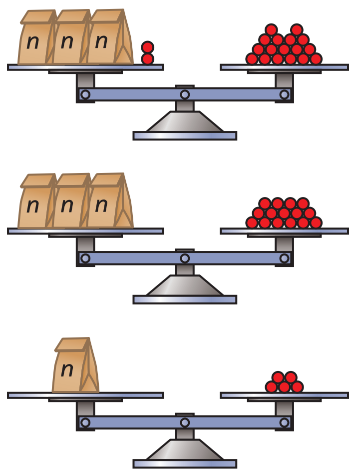
Solve the equation $3n + 2 = 17$ using a balancing strategy.

### Solution

$$3n + 2 = 17$$
$$3n + 2 - 2 = 17 - 2$$

$$3n = 15$$
$$3n \div 3 = 15 \div 3$$

The answer is $n = 5$.



### Practice

**1.** Solve.

**a)** $n + 3 = 7$
**b)** $f + 5 = 2$
**c)** $9 = 3 + x$
**d)** $9 + g = 3$
**e)** $n - 4 = 7$
**f)** $z - 2 = 13$

**2.** Solve.

**a)** $2x = 6$
**b)** $3n = 18$
**c)** $4c = -16$
**d)** $-4m = 20$
**e)** $-30 = 6h$
**f)** $-25 = -5a$

**3.** Solve.

**a)** $2k + 1 = 7$
**b)** $6 + 3k = 27$
**c)** $18 = 4a - 2$
**d)** $11 = 2 - 3y$
**e)** $4p - 3 = 9$
**f)** $-6 = 8 - 7v$
**g)** $3h - 4 = -4$
**h)** $4y - 2 = 2$

**4.** Solve.

**a)** $\frac{1}{3}m = 4$
**b)** $\frac{3}{4}e = 15$
**c)** $-6 = \frac{3}{4}h$
**d)** $20 = \frac{-5}{8}a$
**e)** $\frac{-5}{8}y = -30$
**f)** $\frac{-3}{11}c = 0$

## A–14 Ratios and Rates

Ratios compare quantities measured with the same units or no units at all. Rates show how one quantity changes with respect to the other.
Both ratios and rates can be written using ":" to separate the terms, or as fractions.
A proportion is an equation that states that two ratios or two rates are equivalent.

### EXAMPLE 1

Provide examples of a ratio and a rate.

**Solution**

**A ratio example:** Suppose a bag contains 40 green and 60 red jelly beans. The ratio of green to red is 40 : 60. This simplifies to 4 : 6 or 2 : 3 in lowest terms.

**A rate example:** Suppose a car travels 40 km in 60 min. The rate at which the distance is covered in terms of time is the speed and it is $\frac{40}{60}$ km/min. This simplifies to a unit rate of $\frac{2}{3}$ km/min.

### EXAMPLE 2

Determine the missing value in the proportion $\frac{2}{23} = \frac{\blacksquare}{92}$.

**Solution**

$$\frac{2}{23} = \frac{\blacksquare}{92}$$

$$\frac{2}{23} = \frac{2 \times 4}{23 \times 4}$$

$$= \frac{8}{92}$$

The missing value is 8.

## Practice

1. Write each ratio in lowest terms.
   a) 4 : 8
   b) 6 : 18
   c) 8 : 20
   d) 12 : 42
   e) $\frac{15}{25}$
   f) $\frac{30}{42}$

2. Write each comparison as a ratio.
   a) 7 mm to 3 cm
   b) 17 s to 1 min
   c) 25 m to 5 cm
   d) 15 s to 1 min

3. Calculate each missing term.
   a) $2 : 5 = \blacksquare : 10$
   b) $3 : 7 = \blacksquare : 21$
   c) $\frac{4}{7} = \frac{8}{\blacksquare}$
   d) $\frac{5}{8} = \frac{15}{\blacksquare}$

4. Express each comparison as a rate.
   a) 4 tins for $2
   b) $75 for 8 h work
   c) $4 for 3 novels
   d) 79 km in 4 h
   e) 3 goals for 4 shots
   f) 17 min to deliver 23 papers

# A–15 Percent

A percent is a ratio of the form $\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole amount}}$.

### EXAMPLE 1

Write a ratio or a fraction as a percent.

**Solution**

For example,

$$\frac{3}{5} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100} = 60\%$$

### EXAMPLE 2

Calculate 6% of 120 using a proportion.

**Solution**

$6 : 100 = \blacksquare : 120$

Multiply 6 by 1.2 since $100 \times 1.2 = 120$.

$$\frac{6}{100} = \frac{6 \times 1.2}{120} = \frac{7.2}{120}$$

So, 6% of 120 is 7.2.

## Practice

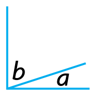
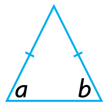
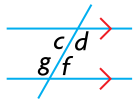
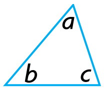
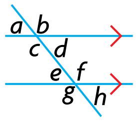
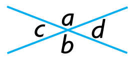
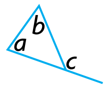
1. Write each percent as a fraction or ratio in lowest terms.
   a) 49%
   b) 75%
   c) 1%
   d) $\frac{1}{2}\%$
   e) $33\frac{1}{3}\%$
   f) $7\frac{1}{2}\%$

2. Write each fraction as a percent.
   a) $\frac{73}{100}$
   b) $\frac{3}{10}$
   c) $\frac{7}{50}$
   d) $\frac{1}{4}$
   e) $\frac{5}{8}$
   f) 1

3. Calculate each percent to one decimal place.
   a) 15% of 75
   b) 75% of 68
   c) 150% of 60
   d) $\frac{1}{2}\%$ of 244
   e) $2\frac{3}{4}\%$ of 748

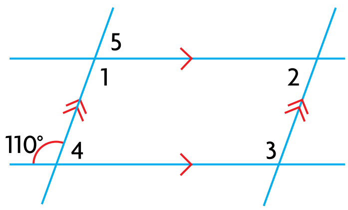
# A–16 Angle Properties

Here is a review of special angle relationships.

| | | | |
|---|---|---|---|
| Complementary angles<br>$a + b = 90°$<br><br> | Isosceles triangle<br>$a = b$<br><br> | Alternate interior angles<br>$c = f, d = g$<br><br> | Supplementary angles<br>$a + b = 180°$<br><br> |
| Sum of the angles of a triangle<br>$a + b + c = 180°$<br><br> | Corresponding angles<br>$a = e, b = f, c = g, d = h$<br><br> | Vertically opposite angles<br>$a = b, c = d$<br><br> | Exterior angle of a triangle<br>$a + b = c$<br><br> |

## EXAMPLE 1

Determine the angles formed by the parallel lines.




### Solution

$\angle 4 = 180° - 110°$
$= 70°$

$\angle 5 = \angle 4$
$= 70°$

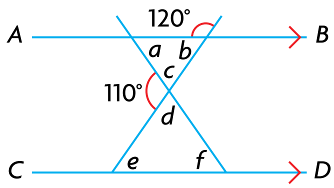
$\angle 1 = 180° - \angle 5$
$= 180° - 70°$
$= 110°$

$\angle 2 = \angle 5$
$= 70°$

$\angle 3 = 180° - \angle 4$
$= 180° - 70°$
$= 110°$

## EXAMPLE 2

Determine the angles in the triangles.




### Solution

$\angle b = 180° - 120°$
$= 60°$

$\angle e = \angle b$
$= 60°$

$\angle a = \angle f$
$= 50°$

$\angle c = 180° - 110°$
$= 70°$

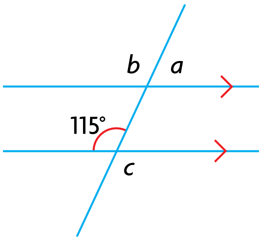
$\angle d = \angle c$
$= 70°$

$\angle f = 180° - \angle d - \angle e$
$= 180° - 70° - 60°$
$= 50°$

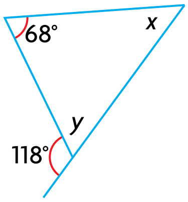
## Practice

**1.** Find the measure of each unknown angle.

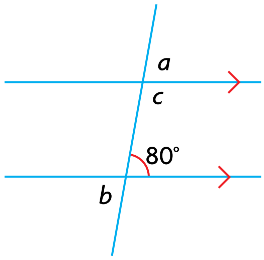
**a)**



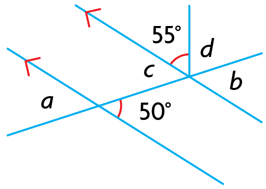

**c)**



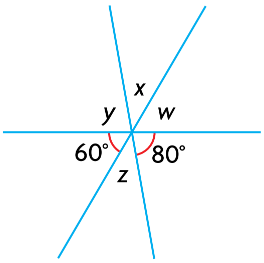

**e)**



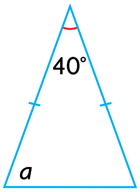

**g)**



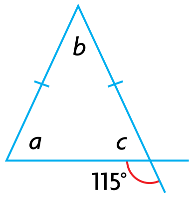

**b)**



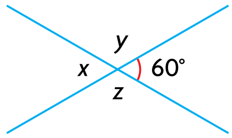

**d)**




**f)**

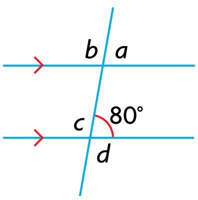



**h)**

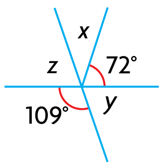



**2.** Find each missing measure.

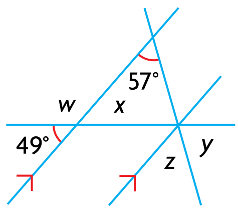
**a)**



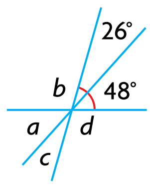

**b)**




**c)**

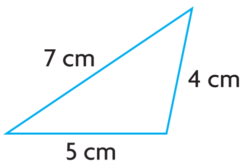



**d)**



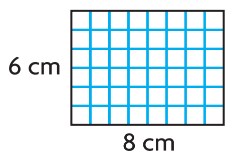

## A–17 Area and Perimeter of Polygons

Perimeter measures the distance around the outside of a closed figure.

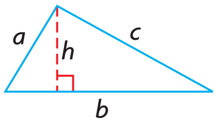
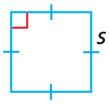
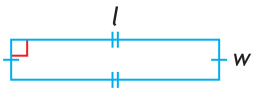
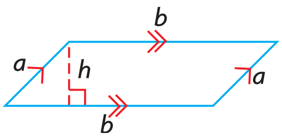
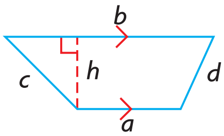



The perimeter of the triangle is
$P = 7 + 4 + 5$
$= 16$ cm

Area measures the number of square units needed to cover a surface.

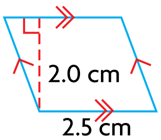



The area of the rectangle is
$A = 6 \times 8$
$= 48\text{ cm}^2$

| Shape | Perimeter | Area |
|---|---|---|
| triangle  | $P = a + b + c$ | $A = \frac{1}{2}(b \times h)$ |
| square  | $P = 4s$ | $A = s^2$ |
| rectangle  | $P = 2(l + w)$ | $A = lw$ |
| parallelogram  | $P = 2(a + b)$ | $A = bh$ |
| trapezoid  | $P = a + b + c + d$ | $A = \frac{1}{2}(a + b)h$ |

### EXAMPLE 1

Calculate the area of the parallelogram.

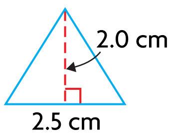



**Solution**

$$A = bh$$
$$= 2.5 \times 2.0$$
$$= 5.0 \text{ cm}^2$$

### EXAMPLE 2

Calculate the area of the triangle.

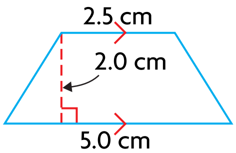



**Solution**

$$A = \frac{1}{2}(b \times h)$$
$$= \frac{1}{2}(2.5 \times 2.0)$$
$$= 2.5 \text{ cm}^2$$

## EXAMPLE 3

Calculate the area of the trapezoid.




**Solution**

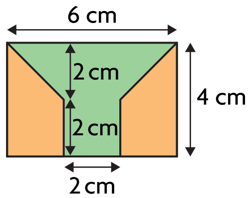
$$A = \frac{1}{2}(a + b)h$$

$$= \frac{1}{2}(5.0 + 2.5) \times 2.0$$

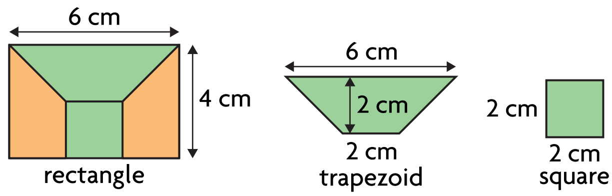
$$= 7.5 \text{ cm}^2$$

## EXAMPLE 4

Determine the area of the orange region.




**Solution**




$$A_{\text{orange region}} = A_{\text{rectangle}} - A_{\text{trapezoid}} - A_{\text{square}}$$

$$= 6 \times 4 - \frac{1}{2}(6 + 2) \times 2 - 2 \times 2$$

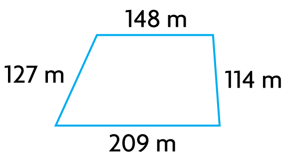
$$= 24 - 8 - 4$$

$$= 12 \text{ cm}^2$$

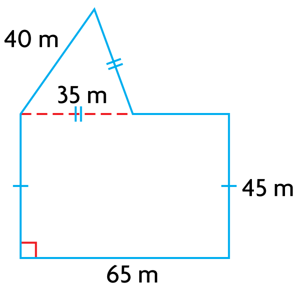
# Practice

**1.** Calculate the perimeter of each figure.

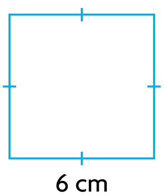
a)



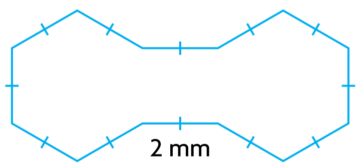

b)




c)

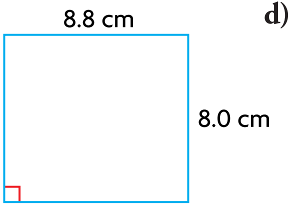



d)

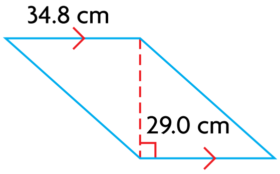


**2.** Calculate the area of each figure.

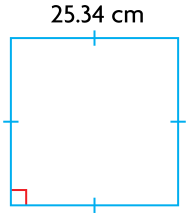
**a)**



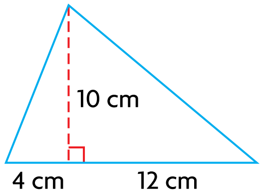

**d)**



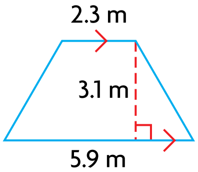

**b)**



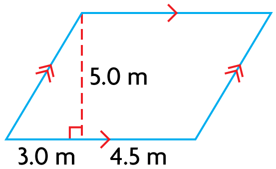

**e)**




**c)**

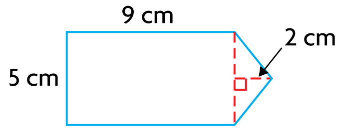



**f)**

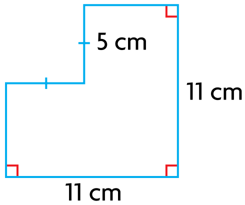



**3.** Calculate the area of each figure.

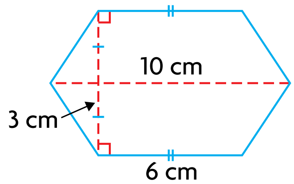
**a)**



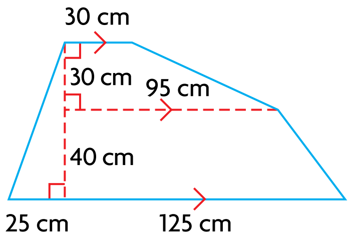

**b)**



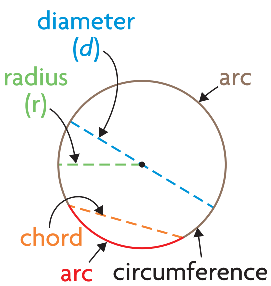

**c)**




**d)**

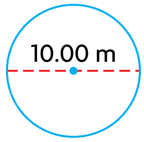



## A–18 Circumference and Area of a Circle

| Circle | Perimeter | Area |
|---|---|---|
|   | In a circle, the distance around the outside is called the circumference. $C = 2\pi r$ or $C = \pi d$ | $A = \pi r^2$ |

## EXAMPLE

Calculate the circumference and area of the circle.




**Solution**

$$C = \pi d$$
$$= \pi \times 10.00$$
$$\doteq 31.42 \text{ m}$$

$$A = \pi r^2$$
$$= \pi \times 5.00^2$$
$$= \pi \times 25.00$$
$$\doteq 78.54 \text{ m}^2$$

## Practice

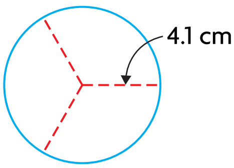
**Record your answers to two decimal places.**

**1.** Calculate the area and circumference of a circle with each measurement.

**a)** 2 cm radius
**b)** 2 cm diameter
**c)** 20 cm radius
**d)** 20 cm diameter

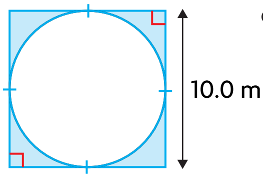
**2.** The circle is divided into three identical sections.

**a)** Determine the area of each section.
**b)** Determine the length of the arc for each section.

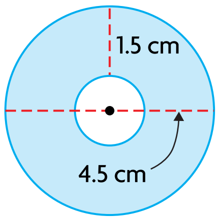



**3.** Determine the area of each shaded region.

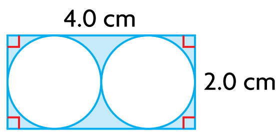
**a)**



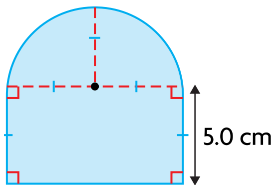

**b)**




**c)**

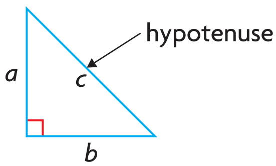


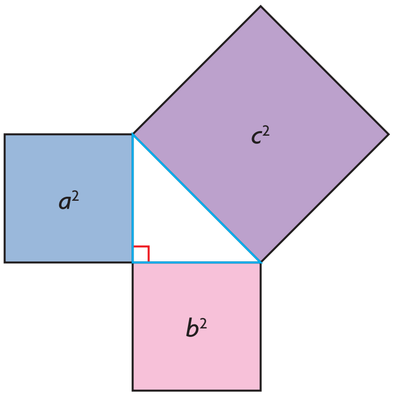

**d)**

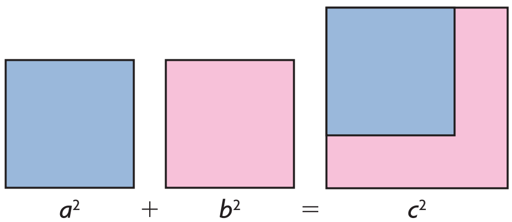



# A–19 The Pythagorean Theorem

The Pythagorean theorem states that $c^2 = a^2 + b^2$ when $c$ is the hypotenuse of a right triangle and $a$ and $b$ are its other sides.

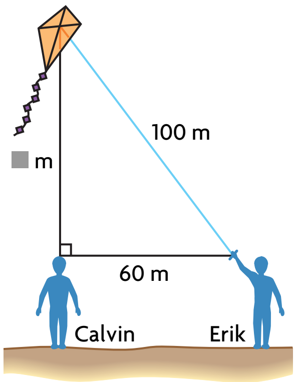


## EXAMPLE

Erik and Calvin are flying a kite as shown in the diagram. How high is the kite above Calvin?




### Solution

$H$ stands for the height of the kite in metres above Calvin.

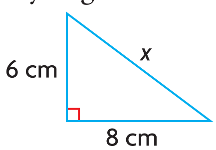
$$H^2 + 60^2 = 100^2$$
$$H^2 + 3600 = 10\,000$$
$$H^2 = 10\,000 - 3600$$
$$= 6400$$
$$H = \sqrt{6400}$$
$$= 80 \text{ m}$$

The kite is 80 m above Calvin.

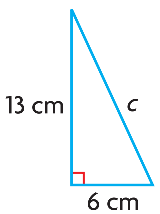
## Practice

**1.** For each right triangle, write the equation for the Pythagorean theorem.

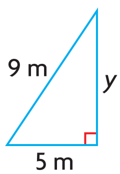
**a)**



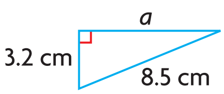

**b)**




**c)**




**d)**

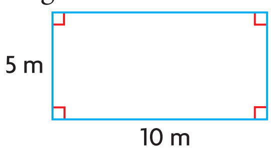



**2.** Calculate the length of the unknown side of each triangle in the previous question. Record each answer to one decimal place.

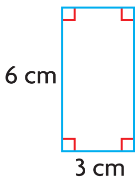
**3.** Calculate the distance saved if Jim hops the fence and takes a shortcut by walking along the diagonal of a rectangular lot that measures 150 m by 200 m.

**4.** Determine the length of the diagonals of each rectangle to the nearest tenth.

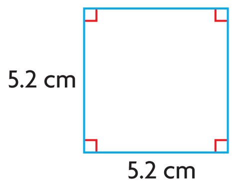
**a)**



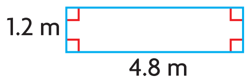

**b)**




**c)**




**d)**




**5.** An apartment building casts a shadow. From the tip of the shadow to the top of the building is 100 m. The tip of the shadow is 72 m from the base of the building. How tall is the building?

**6.** A communications tower is supported by four guy wires. The tower is 155 m tall, and each guy wire is staked into the ground at a distance of 30 m from the base of the tower. What is the total length of wire used to support the tower?

# A–20 Surface Area and Volume of Prisms and Cylinders

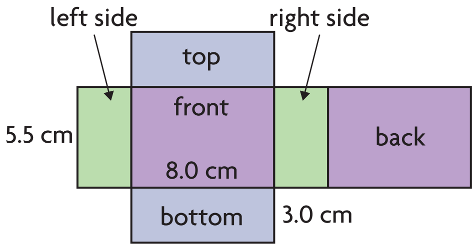
The surface area of a prism or cylinder is the sum of the areas of its faces. This is the area of the object's net.

The volume of a prism or cylinder is the product of the area of the base and the height.

**EXAMPLE 1**

Determine the volume of the gift and the area of paper that covers it.


**Solution**

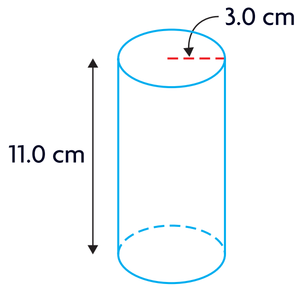



$$SA = 2 \times A_{top} + 2 \times A_{side} + 2 \times A_{front}$$
$$= 2 \times (8.0 \times 3.0) + 2 \times (3.0 \times 5.5) + 2 \times (8.0 \times 5.5)$$
$$= 48.0 + 33.0 + 88.0$$
$$= 169.0 \text{ cm}^2$$

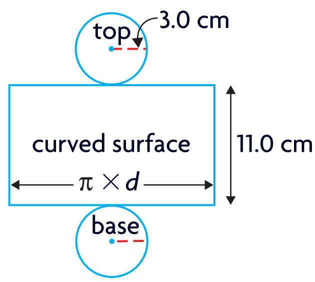
$$V = A_{base} \times h$$
$$= (3.0 \times 8.0) \times 5.5$$
$$= 132.0 \text{ cm}^3$$

**EXAMPLE 2**

Determine the surface area and volume of the cylinder.




**Solution**




$$SA = 2 \times A_{base} + A_{curved\ surface}$$
$$= 2 \times \pi r^2 + \pi dh$$
$$= 2\pi \times 3.0^2 + \pi \times 6.0 \times 11.0$$
$$\doteq 263.9 \text{ cm}^2$$

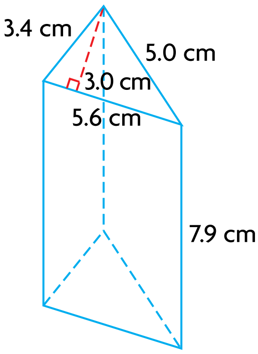
$$V = A_{base} \times h$$
$$= \pi r^2 h$$
$$= \pi \times 3.0^2 \times 11.0$$
$$\doteq 311.0 \text{ cm}^3$$

## Practice

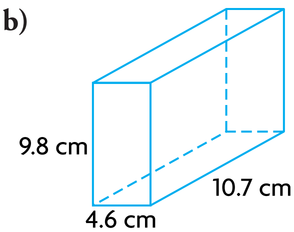
**Round all answers to the nearest tenth of a unit.**

**1.** Calculate the surface area and volume for each shape.

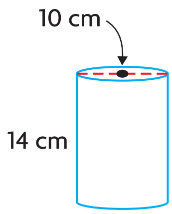
**a)**



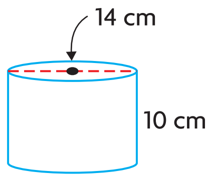

**b)**




**c)**

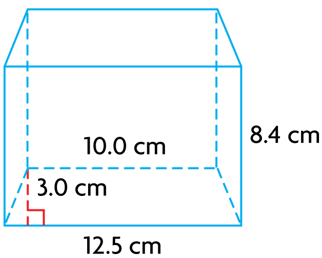



**d)**

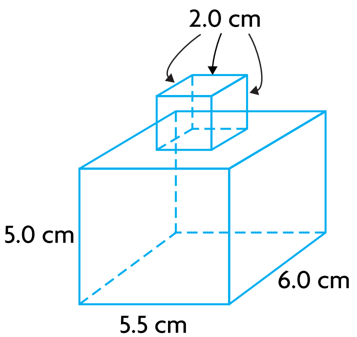



**2.** Determine the total volume of each figure.

**a)**

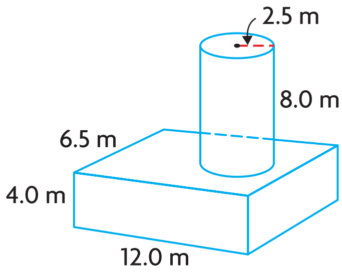



**b)**

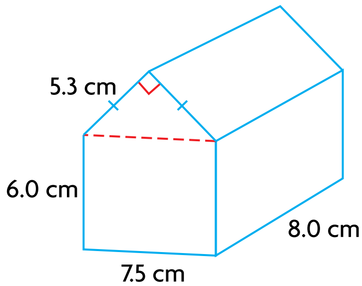



**3.** Determine the total volume of these figures.

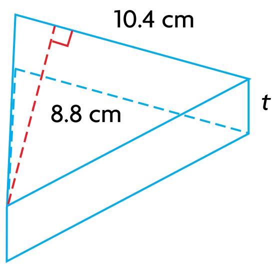
**a)**



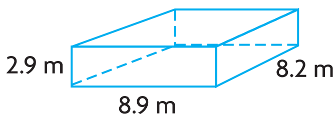

**b)**




**4.** A triangular piece of cheese has a volume of 146.4 $cm^3$. Find the thickness, *t*, of the cheese.




**5.** The dimensions of a room are shown.




**a)** Calculate the volume of the room.

**b)** Calculate the total surface area of the room.

**c)** The walls and ceiling are to be painted. A 4 L can of paint covers an area of 52.2 $m^2$. How many cans of paint are needed?