8.6 Volume and Surface Area of a Sphere

YOU WILL NEED

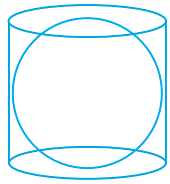
- orange
- scissors and tape
- paper
- sand
- paper plates

GOAL

Develop formulas for the volume and surface area of a sphere.

INVESTIGATE the Math

Exercise balls are **spheres** filled with liquid for weight training. They are sold in cylindrical packages. The manufacturer wants to calculate how much water will fill an exercise ball with a radius of 18 cm, and how much material is needed to make the ball.



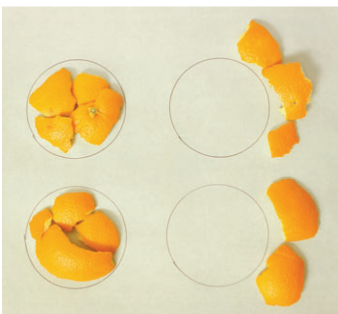
? How can you determine the volume and surface area of a spherical shape like the exercise ball?

A. Use an orange to represent the exercise ball.
Construct a paper tube to represent the cylindrical package.
It should be the same height as the orange and have the same circumference as the equator of the orange.

B. Calculate the volume of the paper tube in millilitres using the formula $V = \pi r^2 h$ (1 mL = 1 cm^3).

C. Place the tube on a paper plate.
Put the orange in the tube.
Pour the sand into the tube, filling the regions above and below the orange.

D. Remove the tube, leaving the sand and orange on the plate.
Pour the same sand back into the tube again, using a second plate.



E. Compare the volume of the sand left in the tube with the volume of the tube.

F. Trace the base of the paper tube several times on paper.

G. Calculate the area of the circles, using the formula $A = \pi r^2$.

H. Peel the orange and place the pieces of peel over the circles that you traced using the base of the paper tube.

I. Estimate the area of the orange.

J. Compare the surface area of the peel (sphere) to the area of the base of the tube.

Reflecting

K. About what fraction of the cylinder did the orange fill?
The cylinder's height was twice its radius.
Use this fact and your result to create a formula to describe the volume of a sphere in terms of its radius.

L. About how many copies of the base of the cylinder did you cover with the orange peel?
How might you use your results to create a formula for the surface area of a sphere?

APPLY the Math

EXAMPLE 1 Using a formula to calculate volume

Dylan must buy 100 spherical balloons for \$0.08 each and enough helium to inflate them. Helium costs \$0.024/L. Each balloon will inflate to a surface area of 900.00 cm^2. How much will it cost to buy and inflate them?

Dylan's Solution

$$SA_{sphere} = 4\pi r^2$$
$$4(3.14 \times r^2) \doteq 900.00$$
$$3.14 \times r^2 = \frac{900.00}{4}$$
$$3.14 \times r^2 = 225.00$$
$$r^2 = \frac{225.00}{3.14}$$
$$r^2 \doteq 71.66$$
$$r \doteq 8.47 \text{ cm}$$

I used the surface area to determine the radius.

I took the square root of 71.66 to calculate r.

$$V = \frac{4}{3}\pi r^3$$
$$\doteq \frac{4}{3}(3.14) \times (8.47)^3$$
$$\doteq 2544 \text{ mL or } 2.544 \text{ L}$$

I calculated the volume of one balloon.

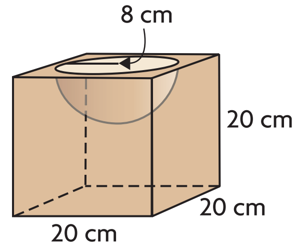
Cost of helium for one balloon
= cost of helium × volume of balloon
= 0.024 × 2.544
= \$0.061

Cost of 100 balloons with helium
100 × (0.08 + 0.061)
= 100 × 0.141
= \$14.10

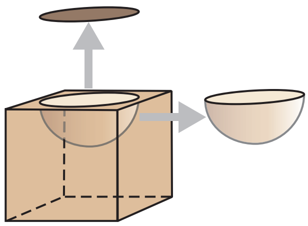
The total cost will be \$14.10.

EXAMPLE 2 Using a visualization strategy to understand and solve a problem

Zuri wanted to make a bowl in shop class. She decided to hollow out a half-sphere from a cube. She needed to know the surface area to varnish the bowl. She also wanted to know the final volume of wood used.



Zuri's Solution



I visualized the surface area as a half-sphere plus a cube. But the cube was missing the area of the circle where the half-sphere was cut.

$$SA_{\text{bowl}} = SA_{1\text{ half-sphere}} + SA_{6\text{ squares}} - SA_{1\text{ circle}}$$

I counted the half-sphere and six square sides minus the circle.

$$SA_{\text{sphere}} = 4\pi r^2$$

$$\begin{aligned} SA_{\text{half-sphere}} &= \frac{1}{2} \times 4\pi r^2 \\ &\doteq \frac{1}{2} \times 4 \times 3.14 \times 8^2 \\ &= 2 \times 3.14 \times 64 \\ &= 2 \times 200.96 \\ &\doteq 402 \text{ cm}^2 \end{aligned}$$

The surface area of the half-sphere is about 402 cm^2.

I calculated the surface area of the half-sphere.

$$SA_{\text{square}} = s^2$$

$$SA_{\text{circle}} = \pi r^2$$

$$\begin{aligned} SA_{6\text{ squares}} - SA_{1\text{ circle}} &= 6 \times s^2 - \pi r^2 \\ &\doteq 6 \times 20^2 - 3.14 \times 8^2 \\ &= 6 \times 400 - 3.14 \times 64 \\ &= 2400 - 200.96 \\ &= 2199.04 \\ &\doteq 2199 \text{ cm}^3 \end{aligned}$$

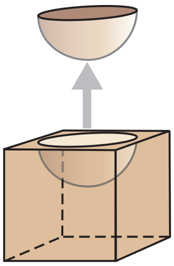
I calculated the surface area of the square sides minus the circle.

The surface area is about 2199 cm^3.

$$SA_{bowl} = 402 + 2199$$
$$= 2601 \text{ cm}^3$$

The surface area of the bowl is 2601 cm^3.

I calculated the total surface area of the bowl.



$$V_{wood} = V_{cube} - V_{half\text{-}sphere}$$

I determined the volume of wood. I visualized the volume as a cube minus a half-sphere.

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$V_{half\text{-}sphere} = \frac{1}{2} \times \frac{4}{3}\pi r^3$$
$$\doteq \frac{1}{2} \times \frac{4}{3} \times 3.14 \times 8^3$$
$$= \frac{2}{3} \times 3.14 \times 512$$
$$= \frac{2}{3} \times 1607.68$$
$$\doteq 1071.79 \text{ cm}^3$$

The volume of the half-sphere is about 1071.79 cm^3.

I calculated the volume of the half-sphere.

$$V_{cube} = s^3$$
$$= 20^3$$
$$= 8000 \text{ cm}^3$$

The volume of the cube is 8000 cm^3.

I calculated the volume of the cube.

$$V_{wood} = 8000 - 1071.79$$
$$= 6928.21 \text{ cm}^3$$

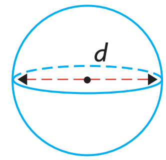
The volume of wood used for the bowl is 6928.21 cm^3.

I calculated the total volume of the wood used.

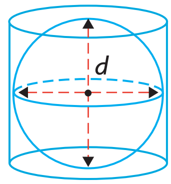
In Summary

Key Ideas

- The surface area of a sphere is four times the area of the circular cross-section that goes through its diameter.



- The volume of a sphere is $\frac{2}{3}$ the volume of a cylinder with the same radius and height.



Need to Know

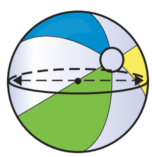
- The formula for the surface area of a sphere with radius r is $SA = 4\pi r^2$.
- The formula for the volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.
- The surface area of a 3-D figure composed of other 3-D figures is the sum of the exposed surface areas of the other figures.
- The volume of a 3-D figure composed of other figures is the combined volume of the other figures.
- When one 3-D figure is removed from another, the volume of the remaining figure is the volume of the original figure minus the volume of the figure that was removed.

CHECK Your Understanding

Give your answers to the same number of decimal places as in the original measurements.

1. Calculate the surface area of a tennis ball with a radius of 3.0 cm.

2. Calculate the volume of the beach ball.



PRACTISING

3. Calculate the surface area of a soccer ball with a radius of 12 cm. Explain what you did.

4. **K** Calculate how much water you would need to fill a round water balloon with a radius of 5 cm.

5. Jim runs a company that makes ball bearings. The bearings are shipped in boxes that are then loaded onto trucks. Each bearing has a diameter of 0.96 cm.

a) Each box can hold 8000 cm^3 of ball bearings. How many ball bearings can a box hold?

b) Each ball bearing has a mass of 0.95 g. Determine the mass of each box.

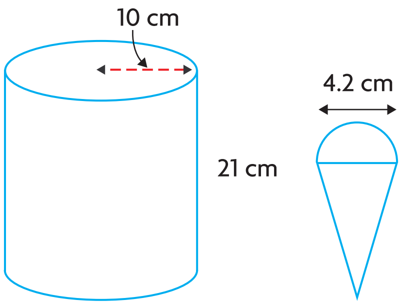
c) The maximum mass a truck can carry is 11 000 kg. What is the maximum number of boxes that can be loaded into a truck?

d) Besides the ball bearings' mass, what else must Jim consider when loading a truck?

6. Ice cream is sold to stores in cylindrical containers as shown. Each scoop of ice cream in a cone is a sphere with a diameter of 4.2 cm.

a) How many scoops of ice cream are in each container?

b) An ice cream cone with one scoop sells for 86¢. How much money will the ice cream store charge for each full cylinder of ice cream that it sells in cones?



7. a) Earth has a circumference of about 40 000 km. Estimate its radius to the nearest tenth of a kilometre and use the radius to calculate the surface area to the nearest hundred square kilometres.

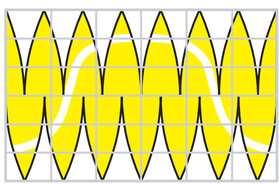
b) Mars has a surface area of about 144 800 000 km^2. Determine the circumference of Mars to the nearest hundred kilometres.

8. a) Frederic has a sphere of clay with a radius of 10 cm. What additional volume of clay does he need to enlarge his sphere to one with a radius of 20 cm?

b) How much foil would be needed to wrap the larger sphere?

9. T **a)** A tennis ball has a radius of 3.4 cm. What volume of this cylinder is empty?

b) This pattern is used to create the surface of one tennis ball. How much material will be left over?



10. A baseball has an inner core covered with string. The ball's circumference is between 23 cm and 23.5 cm. Between what values must the surface area fall?

11. A cylinder just fits inside a 10 cm by 10 cm by 10 cm cubic box. Which shape has the smaller surface area? Verify your answer by determining the surface area of each shape.

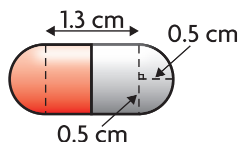
12. **a)** Complete the table.

Shape	Surface Area (cm^2)	Dimensions (cm)	Volume (cm^3)
square-based prism	1000	$s = 10$, $h = ■$	
cylinder	1000	$r = 10$, $h = ■$	
sphere	1000	$r \doteq ■$	

b) Which shape has the greatest volume?

13. Determine the surface area of a ball bearing with a volume of 6.75 cm^3.

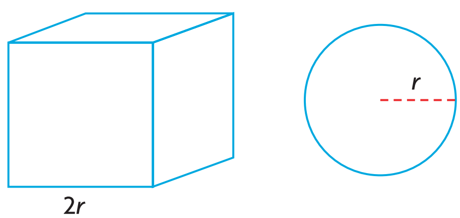
14. **A** A pharmaceutical company creates a capsule for medication in the shape of a cylinder with hemispherical ends as shown. How much medication will the capsule hold?



15. **C** How can you calculate the volume and surface area of a sphere if you know its radius? Create a diagram and dimensions for a sphere from your experience to support your explanation.

Extending

16. Which has a larger volume: a sphere with a radius of r or a cube with a side length of $2r$? Which has a larger surface area?



17. A balloon is inflated to a radius of 10 cm. By how much will the radius increase if you add 1 L of air to the balloon?