Volumes of Pyramids and Cones

GOAL

Investigate formulas for the volume of pyramids and cones.

YOU WILL NEED

- 3-D containers: cylinders, pyramids, cones, prisms
- sand or other fine filler

INVESTIGATE the Math

The student art club recycles used candles. The students store yellow wax in cylinders, and red wax in prisms, as shown. They will pour the red wax into pyramids and the yellow wax into cones. Then, they will sell them with wicks as candle kits to raise money for field trips. The pyramids and prisms have the same height and base area. Likewise, the cylinders and cones have the same height and base area.

? How many pyramidal candles can be made from one prism, and how many conical candles can be made from one cylinder?

A. Estimate the pyramid's volume as a fraction of the prism's volume.

B. Fill the prism with sand.

C. How many times can you fill the pyramid with sand from the prism?

D. Estimate the cone's volume as a fraction of the cylinder's volume.

E. Fill the cylinder with sand.

**F.** How many times can you fill the cone with sand from the cylinder?

Reflecting

G. How is calculating the volume of the pyramid, from the volume of its corresponding prism, like calculating the volume of the cone from the volume of its corresponding cylinder?

H. What formula describes how to calculate the volume of a square-based pyramid with base length l and height h?

I. What formula describes how to calculate the volume of a cone with radius r and height h?

APPLY the Math

EXAMPLE 1 | Calculating the volume of a pyramid

John and Lisa bought an oil lamp with a reservoir in the shape of a pyramid with a regular pentagonal base. A diagram of the base is shown. The reservoir has a height of 8.1 cm. The oil comes in 750 mL bottles. How many times can they fill the lamp completely with one bottle of oil?
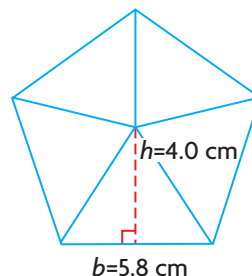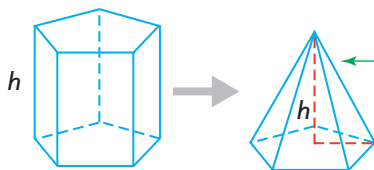
h=4.0 cm

b=5.8 cm

Henri's Solution

$$A_{\text{pentagon}} = 5 \times \frac{bh}{2}$$

I divided the base into five triangles.

$$= 5 \times \frac{5.8 \times 4.0}{2}$$

I multiplied by 5 to determine the total area.

$$= 5 \times \frac{23.2}{2}$$

$$= 5 \times 11.6$$

$$= 58.0 \text{ cm}^2$$

The area of the base was 58.0 cm^2.

h

h

I drew the prism that corresponds to the pyramid. Its volume is Ah. The volume of the pyramid is $\frac{1}{3}$ that of the prism.

$$V_{\text{pyramid}} = \frac{1}{3} Ah$$

$$= \frac{1}{3} \times 58 \times 8.1$$

$$= \frac{1}{3} \times 469.8$$

$$= 156.6 \text{ cm}^3$$

I calculated the volume of the pyramid.

The volume was 156.6 cm^3.
The capacity was 156.6 mL.

$$750 \div 156.6 = 4.8$$

You can fill the lamp completely four times with one bottle of oil.

I determined how many times you can fill the lamp.

<div style="background:#f9e0b0">EXAMPLE **2** | Selecting a strategy to calculate volume</div>

A conical paper cup has a radius of 4 cm and a height of 10 cm. A cylindrical glass has a radius of 4 cm and a height of 20 cm. How many times do you need to fill the paper cup and pour it into the glass to fill the glass?
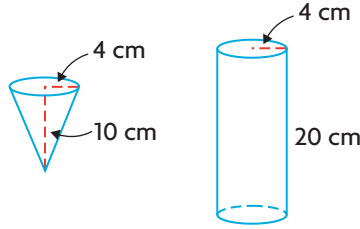
Marcy's Solution: Determining volume using a formula

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$ ← I calculated the volume of the paper cup.

$$V_{\text{cylinder}} = \pi r^2 h$$ ← I calculated the volume of the glass.

$$V_{\text{cylinder}} \div V_{\text{cone}}$$ ← I divided the volume of the glass by the volume of the cup.

You have to fill the paper cup 6 times to fill the glass.

Wyatt decided he could solve the problem without any calculations.

Wyatt's Solution: volume using reasoning

If the glass were the same height as the paper cup, I would need to fill the paper cup 3 times. But, the glass is 20 cm high, so 2 cylinders with a height of 10 cm each will fit inside it. This means that I need to fill the paper cup 2 × 3 or 6 times.

The glass has the same radius as the cup, and twice the height of the cup.

Tech | *Support*

You can store answers from several calculations using the STO▸ and ALPHA keys on a graphing calculator. Select a different letter for each answer. To perform calculations using these answers, use the ALPHA key and each letter in place of the numbers in your expression.

In Summary

Key Ideas

- The volume of a pyramid is $\frac{1}{3}$ the volume of a prism with an identical base and height.

- The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with an identical base and height.
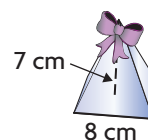
Need to Know

- The formula for the volume of a pyramid is $V = \frac{1}{3}Ah$ where A is the area of its base and h is its height.

- The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$ where r is the radius of its base and h is its height.
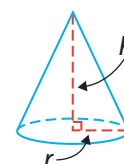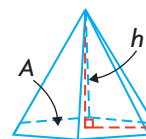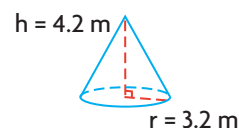
CHECK *Your Understanding*

Give your answers to the same number of decimal places as in the original measurements.
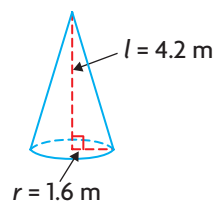
1. Calculate the volume of the gift box.

7 cm
l
8 cm

2. Calculate the volume of the cone.

h = 4.2 m

r = 3.2 m

PRACTISING

3. Determine the volume of sand that would fill a cone with a base radius of 6.5 cm and a height of 12.0 cm.

4. Sammy has a regular octagonal-based pyramidal paperweight filled with coloured liquid. It has a distance of 4.2 cm from the centre of its base to the midpoint of each side, a base perimeter of 19.0 cm, and a height of 6.0 cm. Determine the volume of the pyramid.

5. Calculate the volume of the cone.

K

l = 4.2 m

r = 1.6 m

6. Sand for icy roads is stored in a conical pile 14.2 m high with a base diameter of 34.4 m.
   a) Calculate the volume of the pile.
   b) One sander can take 6.9 m^3 of sand. How many sanders can be filled from the pile?

7. A square-based pyramid has a volume of 100 cm^3 and a base area of 40 cm^2. What is its height?

8. Candles in the shape of square-based pyramids are sold in three volumes: 75 cm^3, 150 cm^3, and 175 cm^3. The base side length of each candle is 5 cm. What are the heights of the candles?

9. A pyramid and a prism with the same height both have a base area of
   T 64 cm^2. How do their volumes compare?

10. Each conical paper cup for a water fountain has a height of 9 cm and a
    A radius of 3 cm. An average of 45 cups of water is drunk each day. What volume of water is drunk each week?

11. Describe the problem-solving process you would use to compare the
    C volume of a square-based pyramid and a cone with the same height.

Extending

12. For each right pyramid, the base is a regular polygon with $a = 4$ cm and $h = 10$ cm.

4-sided   8-sided   12-sided   20-sided
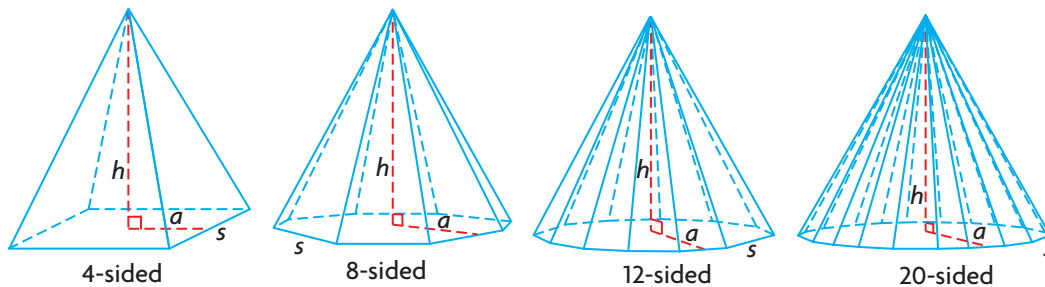
   a) Develop a formula to compute the volume of each pyramid in terms of a and h.
   b) Use your result to state a formula for the volume of a cone with a height of 10 cm and a radius of a.