8.4 Surface Area of Right Pyramids and Cones

YOU WILL NEED

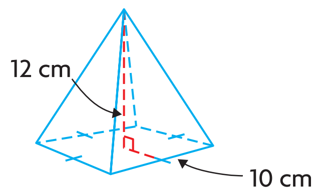
- grid paper

GOAL

Determine the surface area of a pyramid and a cone using a variety of strategies.

LEARN ABOUT the Math

Yvonne is printing slogans on the side of this **right pyramid**. She wants to calculate its surface area.



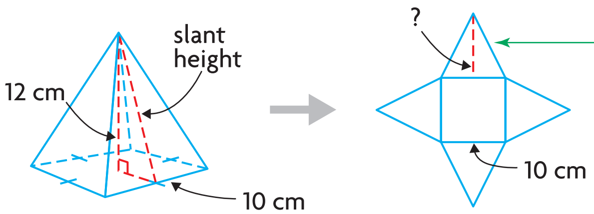
right pyramid

a **pyramid** whose base is a regular polygon and whose top vertex is directly above the centre of the base

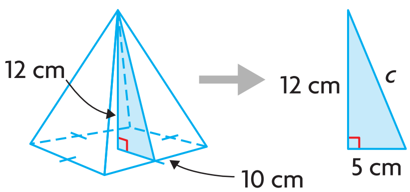
? How can Yvonne determine the area for slogans?

EXAMPLE 1 Calculating surface area using a net and slant height

Yvonne's Solution



I visualized the box's net. It had four identical triangles for the sides and a square base.



$$c^2 = a^2 + b^2$$

I calculated the slant height.

$$c^2 = 12^2 + 5^2$$
$$c^2 = 144 + 25$$
$$c^2 = 169$$
$$c = \sqrt{169}$$
$$c = 13$$

The height of the pyramid, *a*, was 12 cm. The distance, *b*, from the centre of the base to the side was half of 10 cm, or 5 cm. I visualized the 12 cm and 5 cm lengths as legs of a right triangle. The slant height, *c*, was the hypotenuse.

$$A_{\text{base}} = s^2 = 10 \times 10 = 100 \text{ cm}^2$$

$$A_{\text{triangle}} = \frac{1}{2}bh = \frac{1}{2} \times 10 \times 13 = 65 \text{ cm}^2$$

I calculated the area of the square base and of each triangular face.

$$SA_{\text{pyramid}} = A_{4 \text{ triangles}} + A_{\text{base}} = 4 \times 65 + 100 = 360 \text{ cm}^2$$

I calculated the total surface area.

Reflecting

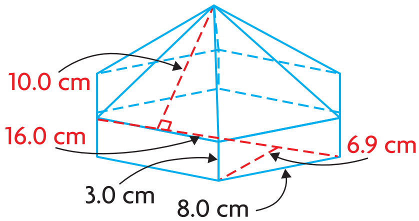
A. How did Yvonne use what she already knew about the area of 2-D shapes to determine the area for slogans on the box?

B. How would you explain to a friend how to calculate the surface area of a pyramid?

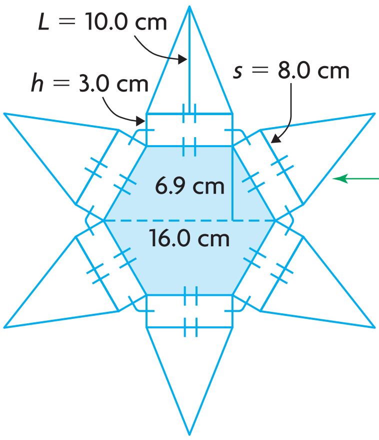
APPLY the Math

EXAMPLE 2 Solving a surface area problem using nets

Judy found a new box. It is a pyramid with six triangular faces on top of a hexagonal prism. What is its surface area?



Judy's Solution



I used a net to see all the faces.

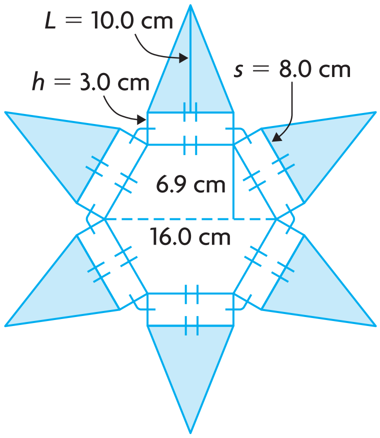
There were six identical triangular faces and six identical rectangular faces. The base was a hexagon, so I divided it into two trapezoids.

$$A_{trapezoid} = \frac{1}{2}(\text{base}_1 + \text{base}_2) \times 6.9$$
$$= \frac{1}{2}(8.0 + 16.0) \times 6.9$$
$$= 82.8 \text{ cm}^2$$

I calculated the area of one trapezoid.

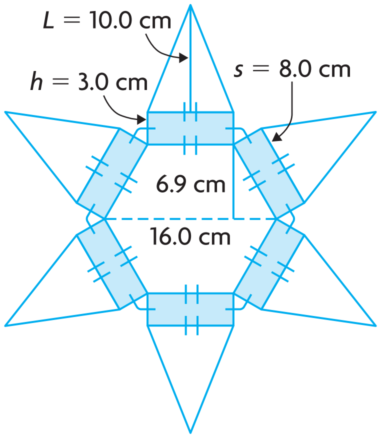
$$A_{hexagon} = 2 \times A_{trapezoid}$$
$$= 2 \times 82.8$$
$$= 165.6 \text{ cm}^2$$

I doubled that area to calculate the area of the base.



$$A_{6\ \text{triangles}} = 6 \times \frac{sL}{2}$$
$$= 6 \times \frac{8.0 \times 10.0}{2}$$
$$= 240.0\ \text{cm}^2$$

I calculated the surface area of the six triangular faces using the base side length *s* and slant height *L*.



$$A_{6\ \text{rectangles}} = 6 \times sh$$
$$= 6 \times 8.0 \times 3.0$$
$$= 144.0\ \text{cm}^2$$

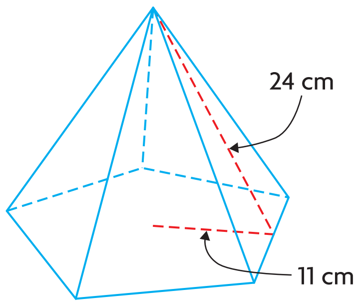
I calculated the surface area of the six rectangular faces using the base side length *s* and rectangle height *h*.

$$SA = A_{\text{base}} + A_{\text{sides}}$$
$$= A_{\text{hexagon}} + A_{6\ \text{triangles}} + A_{6\ \text{rectangles}}$$
$$= 165.6 + 240.0 + 144.0$$
$$= 549.6\ \text{cm}^2$$

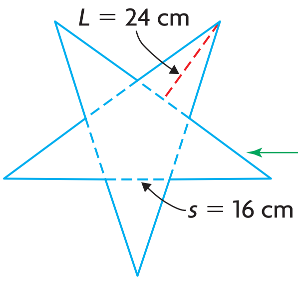
I calculated the total surface area of the box.

EXAMPLE 3 Using reasoning to develop a formula for surface area of a pyramid

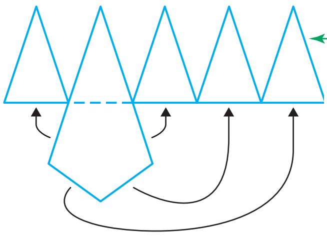
Sarah wants to calculate the surface area of this pyramid. The perimeter of its base is 80 cm.



Sarah's Solution



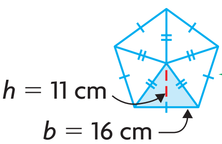
I created a net. I labelled it with the dimensions that I needed. The base side length, s, was $\frac{1}{5}$ of 80 cm, or 16 cm, and the slant height, L, was 24 cm.



There were five triangles, one for each side of the base.

$$A_{5\text{ triangular faces}} = 5 \times \frac{sL}{2}$$
$$= 5 \times \frac{16 \times 24}{2}$$
$$= 960\text{ cm}^2$$

I multiplied the area of one triangle by the number of sides on the base.



I divided the base into five congruent triangles. Each triangle had a base length of 16 cm and a height of 11 cm.

$$A_{\text{triangle}} = \frac{bh}{2}$$
$$= \frac{11 \times 16}{2}$$
$$= 88\text{ cm}^2$$

I calculated the area of one triangle.

$A_{base} = 5 \times 88$ ← There were five triangles, so I multiplied the area by 5.

$= 440 \text{ cm}^2$ ← The area of the base was 440 cm².

$SA = A_{triangular\ sides} + A_{base}$

$= 960 + 440$

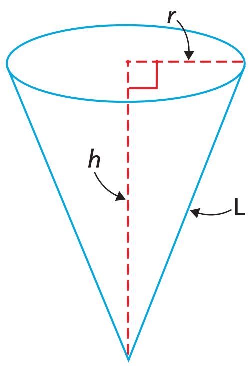
$= 1400 \text{ cm}^2$ ← I calculated the total surface area.

The pyramid has a surface area of 1400 cm².

EXAMPLE 4 Using reasoning to develop surface area of a cone

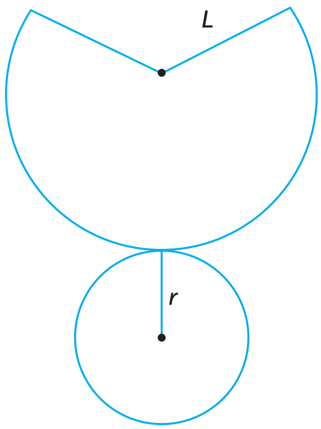
a) Develop a formula for calculating the surface area of any cone with radius *r*, height *h*, and slant height *L*.

b) Use the formula to calculate the surface area of a cone with a radius of 3 cm and a height of 7 cm.



Melinda's Solution

a)



I drew a net for any cone. It is made up of two shapes. The base is a circle with a radius of *r*. The curved surface opens up to form a sector of the circle with a radius of *L*, the slant height of the cone.

The surface area of a cone is the sum of the areas of these two shapes.

$A_{base} = \pi r^2$ ← I used the formula for area of a circle to represent the area of the base of the cone.

$$\frac{\textit{Area of curved surface}}{\textit{Area of circle}(\textit{radius} = L)} = \frac{\textit{Circumference of cone}}{\textit{Circumference of circle}(\textit{radius} = L)}$$

$$\frac{\textit{Area of curved surface}}{\pi L^2} = \frac{2\pi r}{2\pi L}$$

$$\pi L^2 \times \frac{\textit{Area of curved surface}}{\pi L^2} = \pi L^2 \times \frac{2\pi r}{2\pi L}$$

$$\overset{1}{\cancel{\pi L^2}} \times \frac{\textit{Area of curved surface}}{\underset{1}{\cancel{\pi L^2}}} = \overset{1\ L}{\cancel{\pi L^2}} \times \frac{\overset{1}{\cancel{2}}\pi r}{\underset{1\,1\,1}{\cancel{2}\cancel{\pi}\cancel{L}}}$$

$$\textit{Area of curved surface} = \pi r L$$

For the curved surface, I reasoned that its arc length must be equal to the circumference of the circular base. I used proportional reasoning to write two equal ratios that compare areas to circumferences.

I want to find the *Area of curved surface*, so I multiplied both sides of the equation by πL^2. Then, I simplified.

$$\begin{aligned}\textit{Surface area of a cone} &= \textit{area base} + \textit{area of curved surface} \\ &= \pi r^2 + \pi r L\end{aligned}$$

I added the two areas that make up the surfaces. This gave me the formula where r = radius of the circular base and L = the slant height of the cone.

b) $r = 3$ cm
$h = 7$ cm
$L = ?$

I know the radius of the cone and its height but I need to find the slant height L to calculate the surface area.

$$\begin{aligned} r^2 + h^2 &= L^2 \\ 3^2 + 7^2 &= L^2 \\ 9 + 49 &= L^2 \\ 58 &= L^2 \\ \sqrt{58} &= L \end{aligned}$$

r, h, and L are sides in a right triangle, so I used the Pythagorean Theorem to calculate L.

$$\begin{aligned}\textit{Surface area of a cone} &= \pi r^2 + \pi r L \\ &= (3.14)(3)^2 + (3.14)(3)(\sqrt{58}) \\ &= 28.26 + 9.42(\sqrt{58}) \\ &\doteq 100 \text{ cm}^2\end{aligned}$$

I substituted the values into the formula, and then, calculated the answer.

In Summary

Key Idea

- To calculate the surface area of a right pyramid, add the area of the base and the area of the faces.
- To calculate the surface area of a cone, add the area of the circular base and the area of the curved surface.

Need to Know

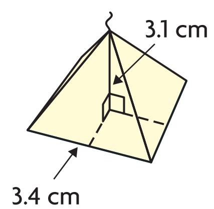
- The slant height of a right pyramid is the height of the triangular faces.
- To calculate the slant height of a right pyramid, use its height, the side length of the base, and the Pythagorean theorem.
- To calculate the area of the base of a right pyramid, divide it into isosceles triangles by drawing lines from the centre of the base to each vertex.
- The surface area of a 3-D figure is the combined area of the 2-D shapes in its net.
- The formula for the surface area of a square-based prism is $SA = A_{4\text{triangles}} + A_{\text{base}}$ or $2bL + b^2$, where b is the base side length and L is the slant height.
- The height of a cone is the distance from the top of the cone to the centre of its circular base.
- To calculate the slant height of a cone, use its radius and height and the Pythagorean theorem.
- The formula for the surface area of a cone is $SA = \pi r^2 + \pi rL$, where r is the radius of the circular base and L is the slant height.

CHECK Your Understanding

Give your answers to the same number of decimal places as in the original measurements.

1. Calculate the surface area of each type of candle.

a)



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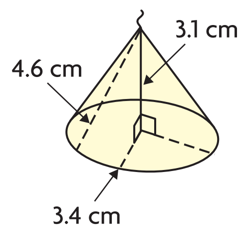
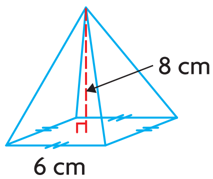


2. Calculate the surface area of each shape.

a)



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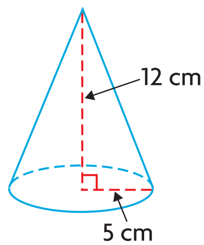
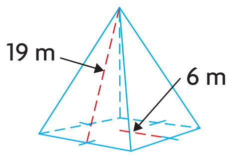


PRACTISING

3. Calculate the surface area.

K **a)**



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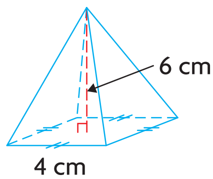

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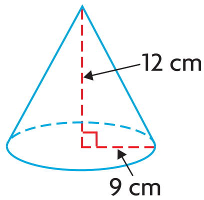


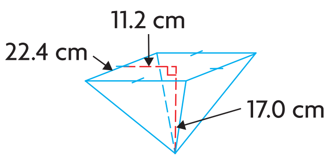
4. Determine the surface area of a square pyramid with a height of 11.0 cm and a base area of 36.0 cm^2.

5. **a)** Determine the slant height of a cone with a height of 8 cm and a radius of 4 cm.

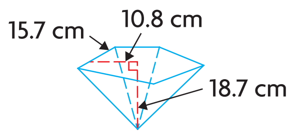
b) Calculate the cone's surface area.

6. There are two shapes of snow-cone cups at the Fall Fair. Which cup uses less material? Assume that the bases are regular polygons.

cup A

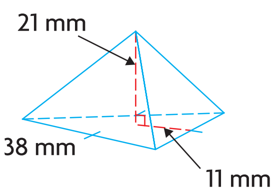


cup B



7. Calculate the surface area of each regular pyramid.

a)



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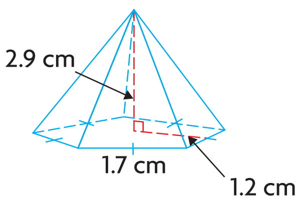

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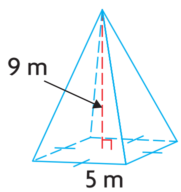
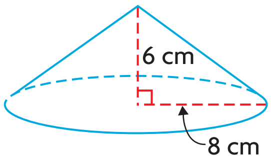


8. Calculate the surface area of each cone.

a)



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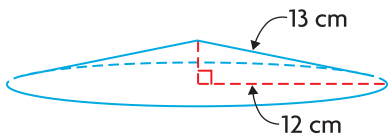

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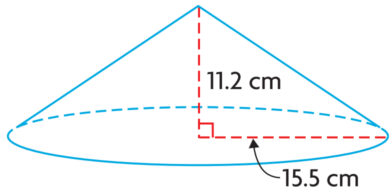
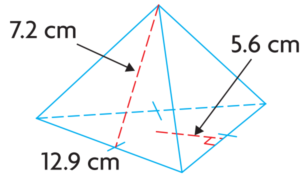


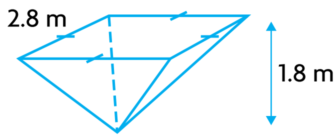
9. **A** The local party store sells pyramid-shaped gift boxes. They have either a square base with a side length of 10 cm or a regular octagon base with a distance of 6 cm from the centre of the base to the midpoint of each side. Both boxes have a base perimeter of 40 cm. Each box has a height of 8 cm. Which box requires more wrapping paper? Explain your solution.

10. Calculate the surface area of this pyramid.

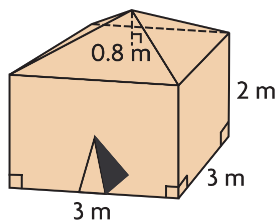


11. Dennis bought a paperweight shaped like a regular hexagonal pyramid for his sister's birthday. It has a measure of 2.6 cm from the centre of its base to the midpoint of each side, a base perimeter of 18 cm, and a height of 4 cm. He wants to know if he has enough wrapping paper for it. Determine the pyramid's surface area.

12. Salt is stored in a bin shaped like an inverted square-based pyramid. The sides of the base are 2.8 m long. The bin is 1.8 m high. Determine the surface area of the bin, including the square base.



13. Determine the surface area of the tent. Include the floor in your calculation.



14. The Great Pyramid of Cheops was originally 147 m high. Its square base had a side length of 230.4 m.

a) Calculate the surface area of the Great Pyramid, including its base.

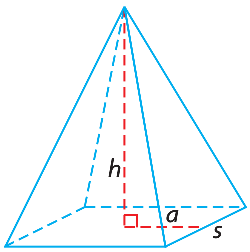
b) The outside surface of each block in the Great Pyramid is 2.3 m by 1.8 m. Estimate the number of blocks that make up the outside facing of the Great Pyramid.

15. Two regular octagonal pyramids are 8 cm high. Pyramid A has a surface area of 318.08 cm^2 and a measure of 6 cm from the centre of its base to the midpoint of each side. Pyramid B has a measure of 15 cm from the centre of the base to the midpoint of each side. What is the surface area of pyramid B?
T

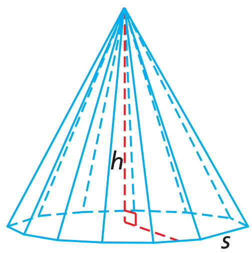
16. Sketch a pyramid and label its dimensions. Show how to calculate its surface area in at least two different ways.
C

Extending

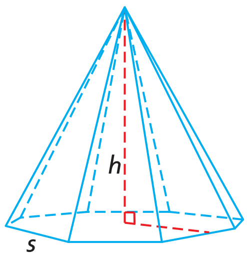
17. Each of these regular pyramids is 10 cm high and measures 4 cm from the centre of the base to the midpoint of each side. Which pyramid do you think has the greatest surface area? Explain.

A.


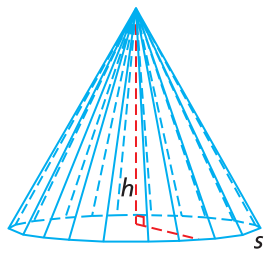
4-sided

C.


12-sided

B.


8-sided

D.


20-sided

18. a) This shape is composed of two identical regular pyramids. They each have a height of 5 cm and a base side length of 7 cm. Determine the surface area.

b) Another identical pyramid is joined to the shape on one of its triangular faces, as shown. Determine the new surface area.

c) Write a formula for the surface area of a shape with n pyramids joined in this way.