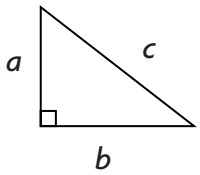
8.3 The Pythagorean Theorem

YOU WILL NEED

- grid paper
- scissors

GOAL

Solve problems using the Pythagorean theorem.



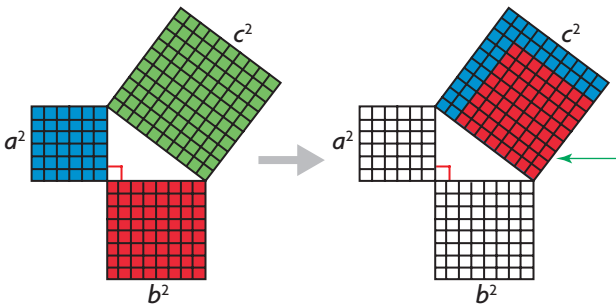
LEARN ABOUT the Math

Julie is tutoring her friend, Annie, on using the Pythagorean theorem to solve problems. To help Annie understand, Julie creates a geometric representation of the theorem using a picture.

? What geometric model can Julie use to represent the Pythagorean theorem?

EXAMPLE 1 Representing the Pythagorean theorem geometrically

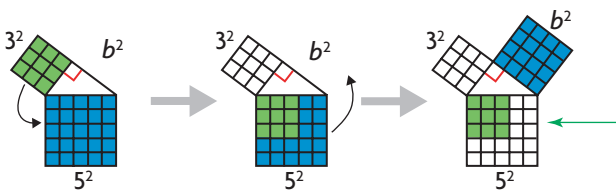
Julie's Solution



I used grid paper. I drew a right triangle with legs of 6 cm and 8 cm. I cut out a square to fit on each side of the triangle. I coloured the squares blue, red, and green.

I rearranged the blue and red squares on top of the green square on the long side or **hypotenuse**. The hypotenuse square had the same area as the sum of the two other squares.

$$\begin{aligned}
\text{square of } a + \text{square of } b &= \text{square of } c \\
a^2 + b^2 &= c^2 \\
6^2 + 8^2 &= c^2 \text{ or } c^2 = 6^2 + 8^2 \\
c^2 &= 36 + 64 \\
c^2 &= 100 \\
\sqrt{c^2} &= \sqrt{100} \\
c &= 10 \text{ cm}
\end{aligned}$$



I tried another triangle. I discovered that the square on the side with a length of 3 had an area of 9 and the square on the side with a length of 5 had an area of 25. I subtracted 9 from 25 to get 16, which is 4 squared. I decided that side b had a length of 4.

Reflecting

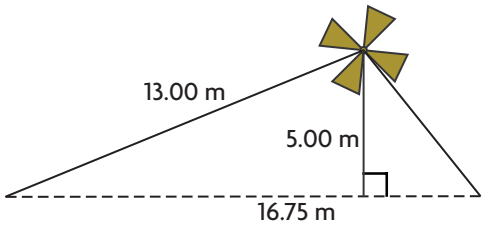
A. How did a geometric model help to represent the Pythagorean theorem?

B. How can you use known sides of a right triangle to calculate an unknown side?

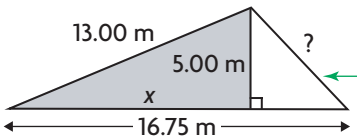
APPLY the Math

EXAMPLE 2 Applying the Pythagorean theorem to calculate a length

Anil is constructing a 5.00 m tall windmill supported by wires. One wire must be 13.00 m long and the distance between the wires must be 16.75 m. Anil wanted to know what length to cut for the other wire.



Anil's Solution

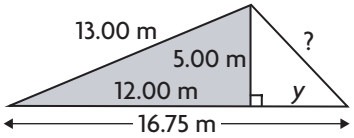


I divided the triangle into two right triangles. I started with the left-hand triangle since I knew two of its dimensions.

$a^2 + b^2 = c^2$
$x^2 + 5.00^2 = 13.00^2$
$x^2 + 25.00 = 169.00$
$x^2 = 169.00 - 25.00$
$x^2 = 144.00$
$x = \sqrt{144.00}$
$x = 12.00 \text{ m}$

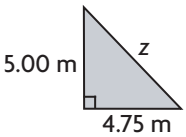
I determined the distance, x, from the windmill to the base of the left-hand wire. I substituted the sides I knew into the Pythagorean theorem and solved for x.

The distance was 12.00 m.



I calculated the distance, y, from the windmill to the right-hand wire's base.

$y = 16.75 - 12.00$
$= 4.75 \text{ m}$



The hypotenuse, z, is the length of wire needed. I calculated the hypotenuse using the legs.

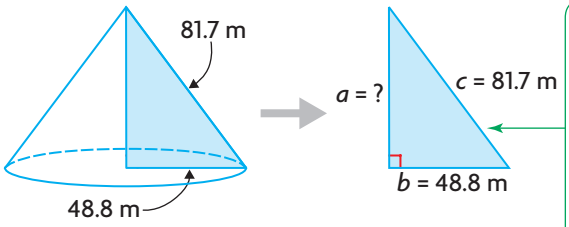
$z^2 = 5.00^2 + 4.75^2$
$z^2 = 47.56$
$z = 6.90 \text{ m}$

The other wire should be 6.90 m long.

EXAMPLE 3 Solving a problem modelled by a right triangle

The Saamis Teepee in Medicine Hat, Alberta, is the tallest teepee in the world. In 2007, a windstorm damaged the teepee, reducing its height. Each beam originally was 81.7 m long and touched the ground 48.8 m from the centre of the base. What was the original height of the teepee?

Dave's Solution



I assumed that the Teepee was a cone and I visualized a right triangle inside it.

The slant height of the cone is 81.7 m, and the radius of the base is 48.8 m.

I used the slant height, c, for the hypotenuse and the base radius for the horizontal leg, b.

Height of cone:

$a^2 + b^2 = c^2$

So,

$a^2 = c^2 - b^2$

$a^2 = 81.7^2 - 48.8^2$

$a^2 = 6674.89 - 2381.44$

$a^2 = 4293.45$

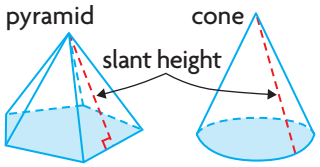
$a = \sqrt{4293.45}$

$a = 65.5$ m

I calculated the original height of the teepee, a, using the Pythagorean theorem. It is 65.5 m, to one decimal place.

slant height

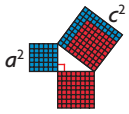
the distance from the top to the base, at a right angle, along a slanted side of a **pyramid** or **cone;** it is measured to the midpoint of the base side for a pyramid



In Summary

Key Idea

- The Pythagorean theorem describes both a numerical and a geometric relationship between the three sides of a right triangle.



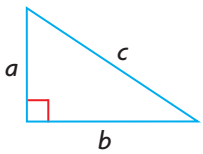
geometric relationship

$a^2 + b^2 = c^2$

numerical relationship

Need to Know

- The formula for the hypotenuse of a right triangle is $c = \sqrt{a^2 + b^2}$, where a and b are the lengths of the legs.



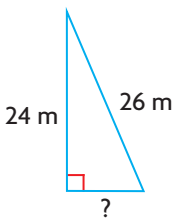
- The formula for the length of a leg of a right triangle is $a = \sqrt{c^2 - b^2}$, where c is the length of the hypotenuse and b is the length of the other leg.

CHECK Your Understanding

Give your answers to the same number of decimal places as in the original measurements.

1. Determine the missing length.

a)



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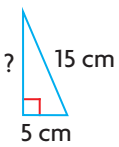

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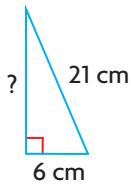

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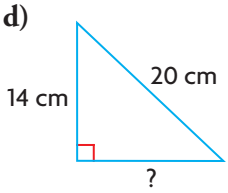
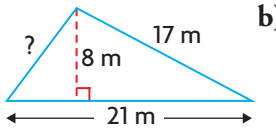


2. What is the length of the direct flight path from Desaulniers to Callander?

PRACTISING

3. Calculate the missing length.

a)



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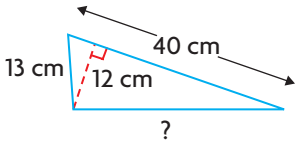

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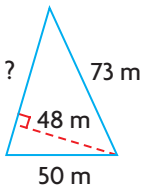
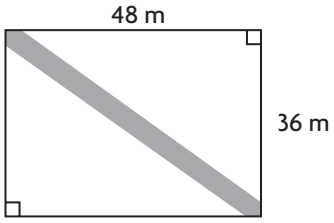


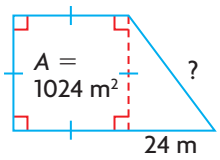
4. A path is being constructed between the corners of the school playground, as shown. Determine the length of the path.

K



5. Determine the length of the hypotenuse.

a)



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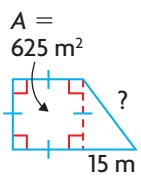

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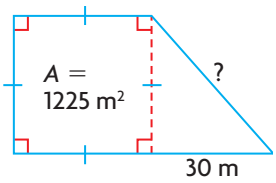
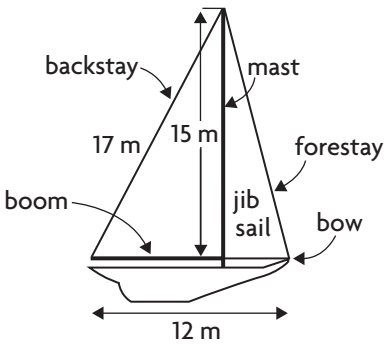
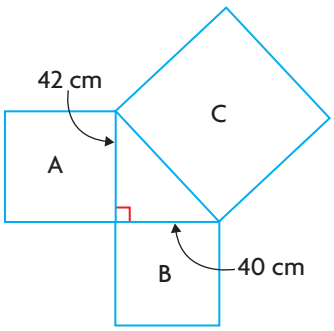


6. Determine the lengths of the boom and the forestay to one decimal place.



7. Determine the area of each square.



8. The outside play area of a daycare centre is shown. Show how you can use the Pythagorean theorem to ensure that the fence corners are at right angles. **A**

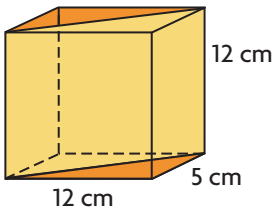
9. A Pythagorean triple is a group of three whole numbers that can represent the lengths of the sides of a right triangle. The smallest Pythagorean triple is 3, 4, 5. Which of the following are Pythagorean triples? **T**

a) 7, 24, 25 **b)** 3, 6, 8 **c)** 9, 21, 23 **d)** 31, 35, 38

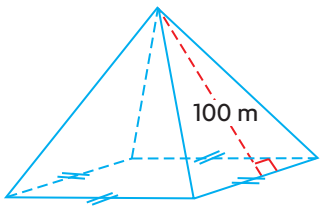
10. Create a geometric problem that you would have to solve using the Pythagorean theorem. Write the problem and its solution, with diagrams. **C**

Extending

11. A box is 12 cm long, 5 cm wide, and 12 cm high. A cardboard rectangle is inserted along the diagonal to divide the box vertically into two equal spaces. Determine the dimensions of the cardboard rectangle.



12. A square-based pyramid has a slant height of 100 m. Determine two possible sets of dimensions for the height and side length of the pyramid.



13. The red triangle shown is not right-angled. Explain how you know that the combined area of squares A and B does not equal the area of square C.