7.4 Reasoning About Triangle and Quadrilateral Properties

YOU WILL NEED

- protractor
- dynamic geometry software (optional)

GOAL

Form and test conjectures about properties of quadrilaterals.

LEARN ABOUT the Math

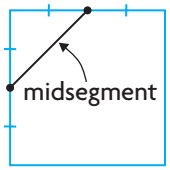
Jafar created square and parallelogram display boards for an art gallery. He made a border for the text area of each board by joining the **midpoints** with string to create the **midsegments.**

The shape inside the square board looked like a square and the shape inside the parallelogram looked like a parallelogram.

? What figure is formed by the midsegments of a quadrilateral?

midsegment

a line segment connecting the midpoints of two adjacent sides of a polygon

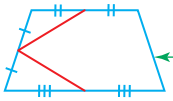


EXAMPLE 1 Forming and testing a conjecture

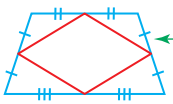
Jafar, Maria, and Elani had different conjectures. They tested them in different ways.

Jafar's Solution: Rejecting a conjecture

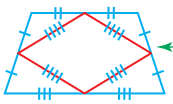
Conjecture: The shape formed by the midsegments of a quadrilateral has the same shape as the original quadrilateral.



I tried an isosceles trapezoid. I thought I would get a trapezoid. I joined the midpoints of the first two sides.



I joined the midpoints of the last two sides. The shape in the middle was a rhombus, not a trapezoid.



I found a **counterexample** to my conjecture. So, I would have to test other quadrilaterals, and then form a new conjecture.

My conjecture was incorrect.

counterexample

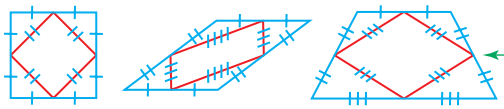
an example that proves that a hypothesis or conjecture is false

Reasoning *Checklist*

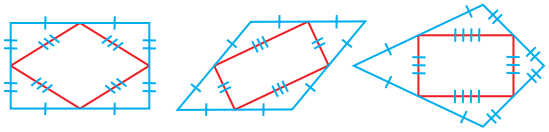
- ✔ Did you explain your reasoning clearly?
- ✔ Are your conclusions reasonable?
- ✔ Did you justify your conclusions?

Maria's Solution: Revising a conjecture

I will test some quadrilaterals to revise Jafar's conjecture.



I joined the midpoints of some quadrilaterals. I always got a parallelogram. I thought this might happen for any quadrilateral.



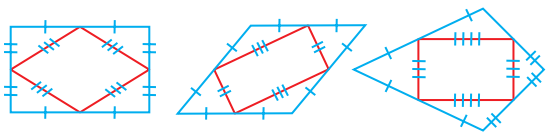
I checked more quadrilaterals. The shape in the middle was always a parallelogram.

Conjecture: All the interior shapes formed by the midsegments are parallelograms.

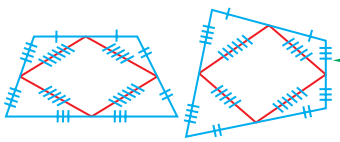
I still had to determine if this held for any quadrilateral. I could create more examples, but I could not be fully sure.

Elani's Solution: Supporting a conjecture

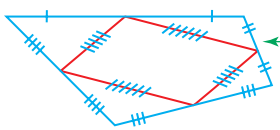
Conjecture: The shape formed by the midsegments of a quadrilateral is a parallelogram.



I joined the midpoints of these quadrilaterals. They confirmed my conjecture.



I checked more quadrilaterals. Each time, the shape in the middle was a parallelogram.



I joined the midpoints of another quadrilateral. The new shape was a parallelogram.

My examples support my conjecture.

Each interior shape formed by the midsegments is a parallelogram.

I reasoned my conjecture was very likely true. However, I could not be fully sure. There might be a counterexample.

Reflecting

A. Explain how Jafar determined that his conjecture was incorrect.

B. Should Maria have tested other quadrilaterals? Explain.

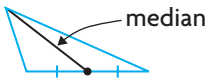
C. Explain how Elani's examples supported the conjecture she tested but did not prove it.

APPLY the Math

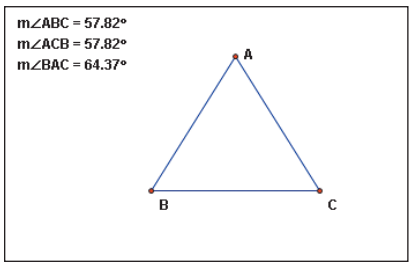
EXAMPLE 2 Confirming or denying a conjecture

Sven's sister saw him doing geometry homework on isosceles triangles. All isosceles triangles have two equal angles and a third angle. She told him that the **median** through the third angle is never perpendicular to the base. Sven decided to test her conjecture.

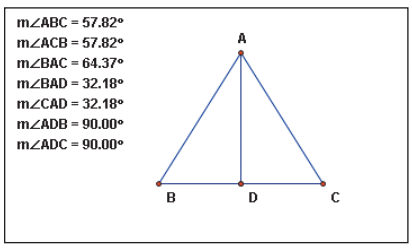
median
the line drawn from a vertex of a triangle to the midpoint of the opposite side



Sven's Solution

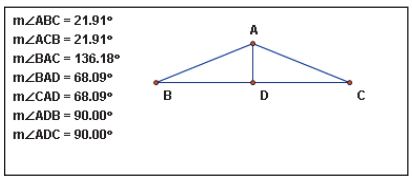


I constructed an isosceles triangle using *The Geometer's Sketchpad*.



I drew the median through the third angle. I measured the angle at the base. It was 90°, so the median and base were perpendicular.

I reasoned that this triangle was a counterexample to my sister's conjecture.



To check my counterexample, I dragged the vertices of the triangle to form different isosceles triangles. The median was always perpendicular to the base.

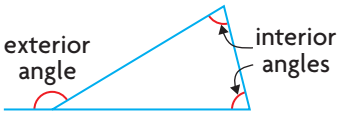
My sister's conjecture was incorrect.

Tech *Support*

For help on constructing and labelling a triangle, measuring an angle, or constructing a midpoint in *The Geometer's Sketchpad*, see Appendix B-20, B-21, and B-25.

EXAMPLE 3 Testing and revising a conjecture

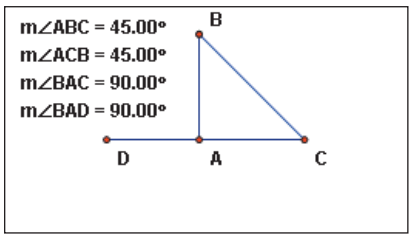
Make a conjecture about the relationship between the exterior angle of a triangle and the two interior angles opposite it. Then test your conjecture.



Aisha's Solution

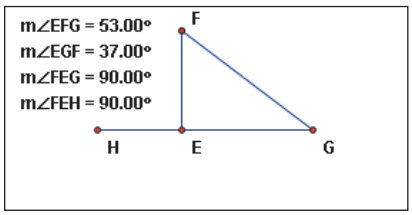
Conjecture: The sum of an exterior angle of a triangle and the two interior angles opposite it is 180°.

I noticed that the sum of the exterior angle of a triangle and the two interior angles opposite it was sometimes 180°.



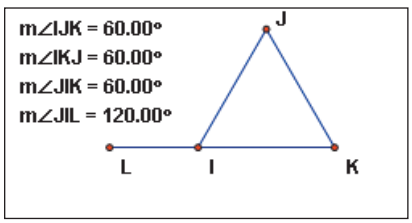
I tested my conjecture with an exterior angle for a right isosceles triangle using *The Geometer's Sketchpad*.

The example confirmed my conjecture.



I tested an exterior angle for a triangle with interior angles of 90°, 37°, and 53°. The sum of $\angle FEH$, $\angle EFG$, and $\angle EGF$ was 180°.

The example confirmed my conjecture.



I tested an equilateral triangle. The sum of $\angle JIL$, $\angle IJK$, and $\angle IKJ$ was 240°.

My conjecture was false. This was a counterexample to my conjecture.

New conjecture: The exterior angle of a triangle is equal to the sum of the two interior angles opposite to it.

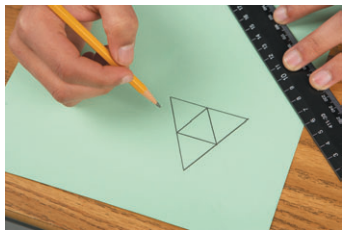
In each example, the exterior angle was equal to the sum of the two interior angles opposite it. I revised my conjecture. The new conjecture needed testing.

Tech *Support*

For help on measuring an exterior angle in *The Geometer's Sketchpad*, see Appendix B-22.

EXAMPLE 4 Testing then confirming a conjecture

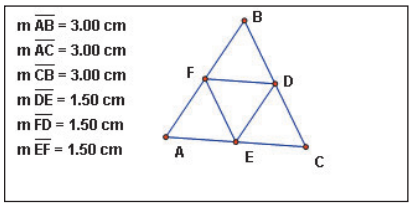
Jeff was making designs. He drew triangles and then formed the midsegments. He thought he saw a relationship between each midsegment and its opposite side.



Jeff's Solution

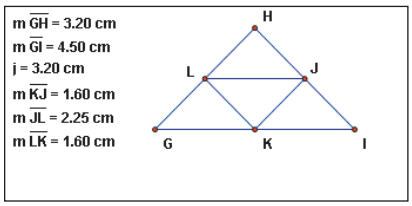
Conjecture: A midsegment of a triangle is half as long as its opposite side.

I made a conjecture. I tested it by constructing examples.



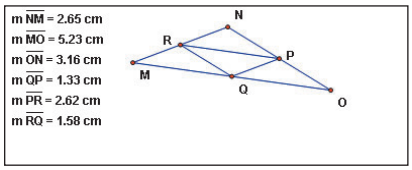
I tested an equilateral triangle. I determined the lengths of the sides and midsegments using *The Geometer's Sketchpad*.

The lengths in the example supported my conjecture.



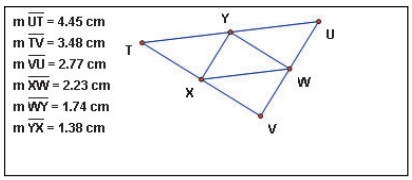
I tested an isosceles triangle.

The lengths in the example supported my conjecture.



I tested two scalene triangles.

The lengths in the examples supported my conjecture.



My examples confirm my conjecture. The length of the midsegment is half the length of the opposite side.

I reasoned my conjecture was very likely correct. I could not be fully sure; a counterexample might still exist.

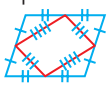
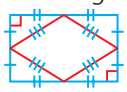
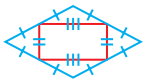
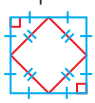
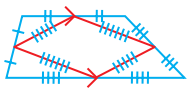
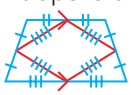
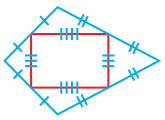
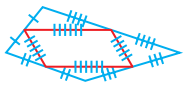
In Summary

Key Ideas

- Examples can support a conjecture about a geometric relationship, but do not prove it.
- You only need one counterexample to disprove a conjecture about a geometric relationship.

Need to Know

- The midsegments of any quadrilateral form a parallelogram:

Midsegments Form a Parallelogram	Midsegments Form a Rhombus	Midsegments Form a Rectangle	Midsegments Form a Square
• parallelogram 	• rectangle 	• rhombus 	• square 
• trapezoid 	• isosceles trapezoid 	• kite 	
• irregular quadrilateral 			

- The median through the angle formed by the two equal sides of an isosceles triangle is perpendicular to the third side.
- The exterior angle at a vertex of a triangle equals the sum of the two interior angles opposite it.
- The length of a midsegment in a triangle equals half the length of the side opposite it.

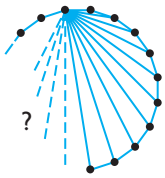
CHECK Your Understanding

1. Test this conjecture: "Midsegments in a triangle are always parallel to the side opposite to them." Support your reasoning with examples.
2. Test this conjecture: "If a quadrilateral has perpendicular diagonals, then it is a square." Support your reasoning with examples.

PRACTISING

3. Predict whether a polygon's sides are all equal if its interior angles are all equal. Support your conjecture with examples or disprove it with a counterexample.

4. Predict whether a polygon's interior angles are all equal if its sides are all equal. Support your conjecture with examples or disprove it with a counterexample.

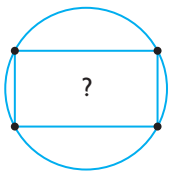


5. Create a conjecture to predict the number of diagonals from any one vertex of a convex polygon with n sides. Support your conjecture with examples or disprove it with a counterexample.

6. T Test this conjecture: "If the midsegments of a quadrilateral form a square, then the quadrilateral is itself a square."

7. Test this conjecture: "The medians of a triangle always intersect at exactly one point."

8. Create a conjecture to predict the ratio of the area of a triangle to the area of the shape formed by its midsegments. Support your conjecture with examples or disprove it with a counterexample.



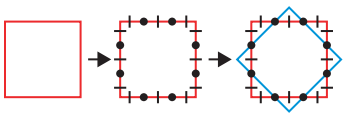
9. Test this conjecture: "It is always possible to draw a circle through all four vertices in a rectangle."

10. Geometric relationships and properties are often discovered using conjectures and counterexamples. Describe the process you use to solve geometric problems using words or diagrams, such as a flow chart.

Extending

11. Create a conjecture to predict when the midsegments of a pentagon form a regular pentagon. Support your conjecture with examples or disprove it with a counterexample.

12. Draw the inner quadrilateral of a square using its midsegments. Then, draw a new inner quadrilateral inside that one, and then another inside the second. Do the same for a rectangle, a rhombus, a parallelogram, a kite, and a trapezoid. Do you notice any patterns? Begin with a conjecture, then either support it with examples or disprove it with a counterexample.



13. To trisect a line segment, divide it into three equal parts. Musim trisected each side of the red square. Then he drew lines to create the blue quadrilateral. What do you notice about it? Form a conjecture about the quadrilateral that is created when you trisect the sides of other quadrilaterals. Support your conjecture with examples or disprove it with a counterexample.