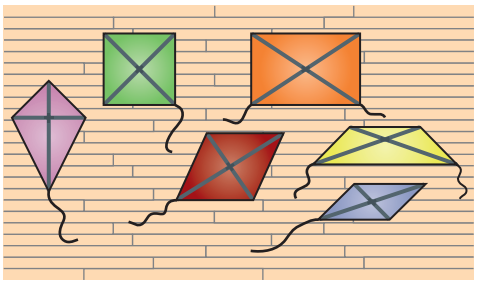
7.3 Exploring Quadrilateral Diagonal Properties

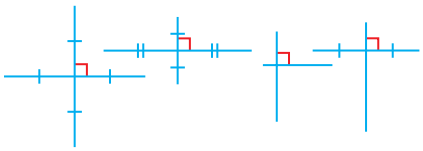
GOAL

Create and test conjectures about properties of quadrilaterals.

EXPLORE the Math

Santos was making flying **kites** of different shapes with two cross pieces. He made a **conjecture** that the shape of each kite depended on how he arranged the diagonals. He started with perpendicular diagonals.





? How can Santos predict the shape of a quadrilateral by using line and angle properties of the diagonals?

A. Draw two intersecting perpendicular line segments of any length. An example is shown to the right.

B. Create a shape using the endpoints of the segments as vertices and the segments as diagonals. An example is shown to the right.

C. Describe the quadrilateral you created.

D. Form a conjecture about what types of quadrilaterals you can construct with perpendicular diagonals.

E. Sketch and label an example of each type of quadrilateral.

F. Draw two intersecting non-perpendicular line segments of any length.

G. Create a quadrilateral using the segments as diagonals.

H. What types of quadrilaterals can you construct?

I. Sketch and label an example of each type.

YOU WILL NEED

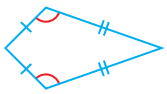
- grid paper
- dynamic geometry software (optional)

Tech *Support*

For help on constructing a midpoint or a segment perpendicular to another segment in *The Geometer's Sketchpad*, see Appendix B-25 and B-26.

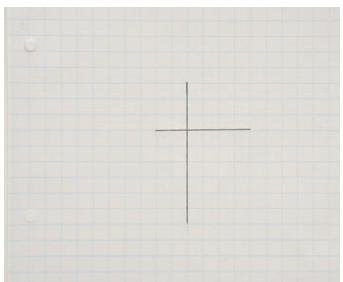
kite

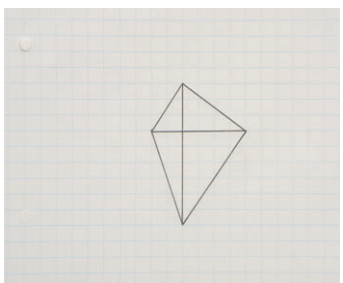
a quadrilateral that has two pairs of equal sides with no sides parallel



conjecture

a guess or prediction based on limited evidence





Communication Tip

"Isosceles" means having two equal sides. Triangles can be isosceles, and so can trapezoids.

isosceles trapezoid

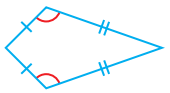
trapezoid

Communication Tip

The word "kite" has several meanings, including a flying toy that can be any shape, or a geometric figure.

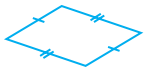
A flying kite

A geometric kite



J. What is the arrangement of the diagonals for each shape?

parallelogram



rectangle

rhombus

square

irregular quadrilateral

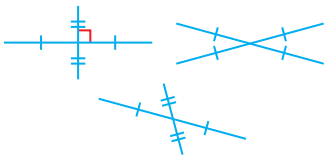
trapezoid

isosceles trapezoid

kite

Reflecting

K. Could you form a square, a rectangle, a rhombus, and a parallelogram using these diagonals? Explain how you know.



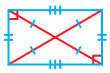
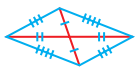
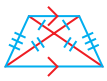
L. Explain why a square is always a rhombus but a rhombus is not always a square. Refer to diagonals in your answer.

M. How do the relationships between the diagonals help you predict the shape of a quadrilateral?

In Summary

Key Idea

- The diagonals of certain quadrilaterals have special properties:

Type of Quadrilateral	The diagonals...	The diagonals form angles that are...	Diagram
square	are equal and bisect each other.	all 90°.	
rhombus (not a square)	are not equal and bisect each other.	all 90°.	
rectangle (not a square)	are equal and bisect each other.	equal when opposite and supplementary when adjacent.	
parallelogram (not a rectangle or rhombus)	are not equal and bisect each other.	equal when opposite and supplementary when adjacent.	
isosceles trapezoid (not a rectangle or rhombus)	are equal and intersect to form two pairs of equal line segments.	equal when opposite and supplementary when adjacent.	
kite	may or may not be equal and only one is bisected by the other.	all 90°.	

Need to Know

- You can identify the type of quadrilateral by using its diagonal properties.

FURTHER Your Understanding

1. Each quadrilateral ABCD below has these three vertices: $A(0, 0)$, $B(3, 4)$, and $C(8, 4)$. Use diagonal properties to identify the coordinates of the fourth vertex D in each case. Explain your method.

a) rhombus

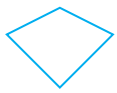
b) isosceles trapezoid

c) kite

2. Match each pair of diagonals with its quadrilateral. Explain your reasoning.

a) 

c) 

e) 

b) 

d) 

f) 

i) 

iii) 

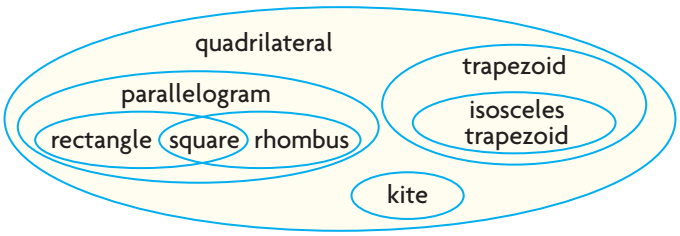
v) 

ii) 

iv) 

vi) 

3. Explain why the quadrilaterals are in different parts of the Venn diagram. Refer to the properties of sides, angles, and diagonals of quadrilaterals.



4. The diagonals and the sides of a quadrilateral form four triangles. Complete the table for the triangles formed by these quadrilaterals. Draw diagrams to support your answers.

Quadrilateral	Number of Congruent Triangles
square	
rhombus	
rectangle	
parallelogram	
isosceles trapezoid	
kite	