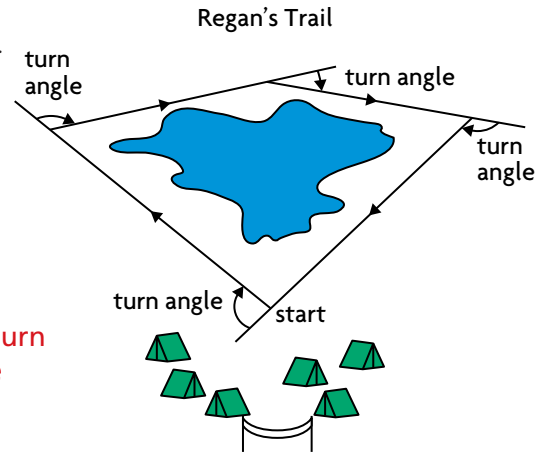
7.2 Angle Properties of Polygons

GOAL

Apply the exterior and interior angle properties of polygons.

INVESTIGATE the Math

Regan set up an orienteering trail around the lake at summer camp. Her trail formed a **convex polygon**, instead of a **concave polygon**. She measured the turn angles, or **exterior angles**, on her trail.



? What is the sum of the turn angles for one complete tour of Regan's trail?

A. Draw a rectangle.

B. Measure its interior and exterior angles.

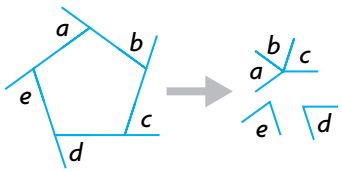
Determine the sum of the exterior angles.

C. Repeat parts A and B for several different convex quadrilaterals.

D. Repeat parts A and B for several different convex polygons with five sides or more.

E. Cut out the exterior angles of each polygon.

Place the angles together so the vertices all touch.



F. What do you notice?

What does this tell you about the sum of the turn angles on Regan's trail?

Reflecting

G. How are the exterior and interior angles at each vertex of a convex polygon related?

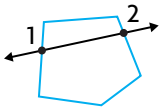
H. What conclusions can you draw about the sum of the exterior angles of any convex polygon?

YOU WILL NEED

- protractor
- scissors
- dynamic geometry software (optional)

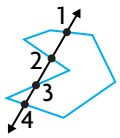
convex polygon

a polygon with every interior angle less than 180°; any straight line through it crosses, at most, two sides



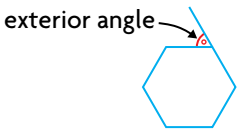
concave polygon

a polygon with at least one interior angle greater than 180°; a straight line through it may cross more than two sides



exterior angle

the angle formed by extending a side of a convex polygon; the angle between any extended side and its adjacent side



Tech Support

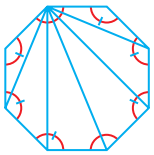
For help on constructing and measuring exterior angles in *The Geometer's Sketchpad*, see Appendix B-22.

APPLY the Math

EXAMPLE 1 Connecting exterior angle sums to interior angles

What is the sum of the exterior angles in a regular octagon?

Jordan's Solution



$$\begin{aligned}\text{sum of interior angles} &= 180° \times (n - 2)\\ &= 180° \times (8 - 2)\\ &= 180° \times 6\\ &= 1080°\end{aligned}$$

There were 8 sides, so there were $8 - 2 = 6$ triangles.

I calculated the sum of the interior angles using the formula $180° \times (n - 2)$ where n is the number of sides.

The sum was 1080°.

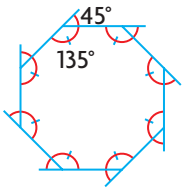
$$\text{measure of one angle} = \frac{1080°}{8} = 135°$$

The interior angles are equal. So, I divided by 8 to determine their measure.

$$\text{exterior angle} + \text{interior angle} = 180°$$

$$\begin{aligned}\text{Therefore, one exterior angle} &= 180° - \text{interior angle}\\ &= 180° - 135°\\ &= 45°\end{aligned}$$

Each exterior angle and adjacent interior angle add to 180°. So, the measure of each exterior angle was 45°.



$$\begin{aligned}\text{sum of exterior angles} &= 8 \times 45°\\ &= 360°\end{aligned}$$

There were 8 exterior angles, so I multiplied by 8.

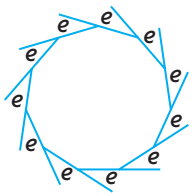
The sum of the exterior angles was 360°.

In any regular n-gon the exterior angles are equal. This helps you determine their measure if you know the value of n.

EXAMPLE 2 Determining exterior angles using reasoning

Determine the measure of each exterior angle in a regular 11-gon.

Lakmini's Solution



$$e = \frac{360°}{n}$$
$$= \frac{360°}{11}$$
$$\doteq 33°$$

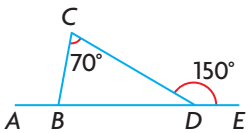
I knew the exterior angles are equal and add to 360° in any regular convex polygon.

I divided 360° by the number of exterior angles.

Each exterior angle is about 33°.

EXAMPLE 3 Solving a problem using angle properties

Determine the measure of $\angle CBA$.



Regan's Solution

$$\angle CDB = 180° - \angle CDE$$
$$= 180° - 150°$$
$$= 30°$$

I saw that $\angle CDB$ and $\angle CDE$ were the interior and exterior angles at vertex D. So, they add to 180°.

$$\angle CBD + \angle CDB + \angle BCD = 180°$$
$$\angle CBD + 30° + 70° = 180°$$
$$\angle CBD + 100° = 180°$$
$$\angle CBD = 80°$$

I knew the sum of the interior angles in a triangle is 180°.

$$\angle CBA = 180° - \angle CBD$$
$$= 180° - 80°$$
$$= 100°$$

$\angle CBA$ and $\angle CBD$ add to 180°. This is because they were the interior and exterior angles at vertex B.

In Summary

Key Ideas

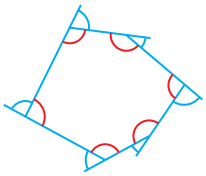
- You can determine unknown angles in polygons using angle properties.
- The sum of the exterior angles of a convex polygon is 360°.

Need to Know

- You can form an exterior angle for a convex polygon by extending a side past its endpoint.
- An exterior angle and its adjacent interior angle are supplementary; they add to 180°.

CHECK Your Understanding

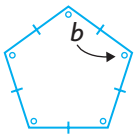
1. What is the relationship between the interior angle and the exterior angle at each vertex of a polygon?



2. Determine the measure of each missing angle.

a)

b)



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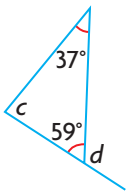

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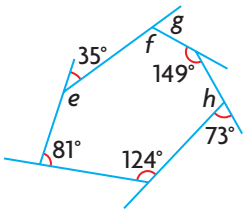
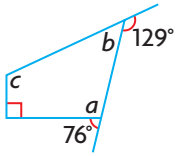


PRACTISING

3. Determine the measure of each missing angle.

a)



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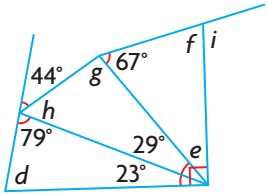
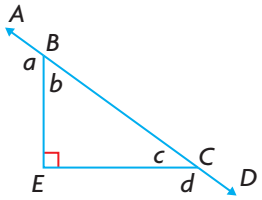
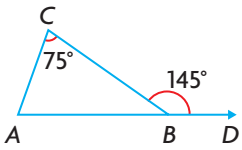


4. In this diagram, $\angle E$ in ΔBEC is a right angle. What is the sum of angles a and d?



5. What is the measure of $\angle CAB$ in this diagram?

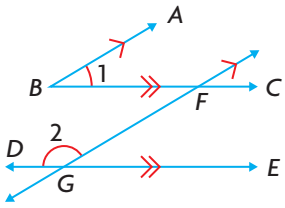


6. In the diagram, *AB* is parallel to *FG* and *BC* is parallel to *DE*.

a) What is the relationship between $\angle 1$ and $\angle 2$?

b) Use *The Geometer's Sketchpad* or several examples to support your answer in part a).

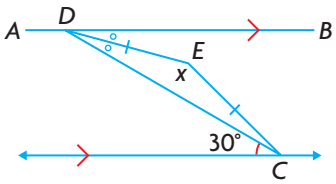
c) Write an expression for your answer.



7. Determine the measure of each missing angle.

K

a)



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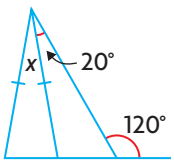

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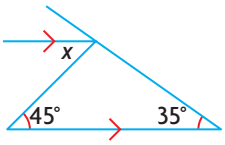

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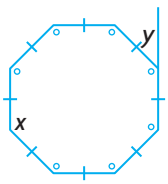
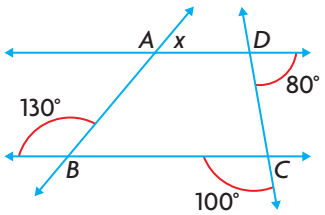


8. For each diagram, state the equation that expresses the relationship needed to solve the problem. Then, determine the measure of each variable. Show the steps in the solution.

C

a)



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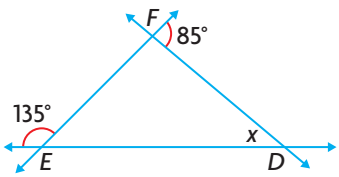

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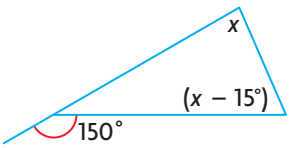

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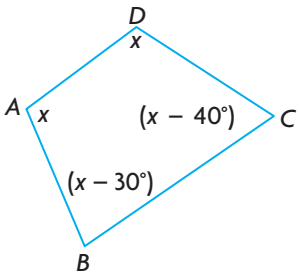


9. An interior angle of a parallelogram is the measure of the exterior angle adjacent to it multiplied by 4. Determine the measure of each interior angle. Draw the parallelogram.

10. A In ΔABC, the measure of $\angle B$ is 21° less than the measure of $\angle A$ multiplied by 4. The measure of $\angle C$ is 1° more than the measure of $\angle A$ multiplied by 5. Determine the measure of each interior angle and each exterior angle of ΔABC.

11. T In a regular polygon, the ratio of the measure of the exterior angle to the measure of its adjacent interior angle is 1 to 4. How many sides does the polygon have?

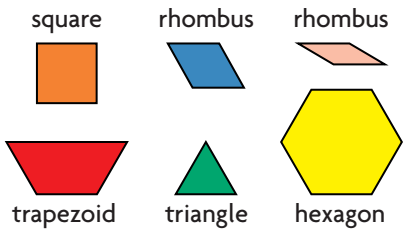
12. For any regular n-gon, develop a formula for calculating the measure of each interior angle.

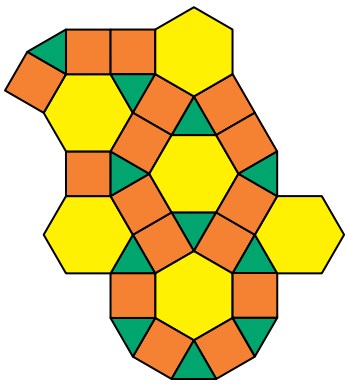
13. Why is the sum of the interior angles of a convex polygon usually greater than the sum of its exterior angles? Explain with an example.

Extending

14. When pattern blocks are used to tile a surface, they have to fit together to join along sides and vertices.

Pentagons were not included in the set of pattern blocks. Explain why pentagons cannot be used to tile a surface.





15. **a)** Suppose you are going to tile a floor with tiles shaped like an octagon and one other shape. What other shape can you use?

b) Determine two other tile shapes you can use to tile a floor.