7.1 Exploring Interior Angles of Polygons

YOU WILL NEED

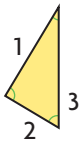
- grid paper
- protractor
- dynamic geometry software (optional)

GOAL

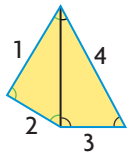
Investigate the sum of the interior angles of polygons.

EXPLORE the Math

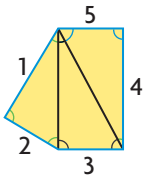
Denise created a triangle on the computer.



She began a pattern of polygons by adding non-overlapping right triangles.

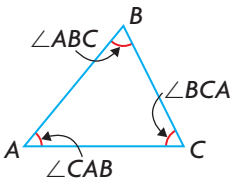


Denise thought, "I know the sum of the **interior angles** of a triangle is 180°. I wonder if I can determine the sum of the angles of any polygon using non-overlapping triangles."



interior angle

the angle formed inside each vertex of a polygon (e.g., ΔABC has three interior angles: $\angle ABC$, $\angle BCA$, and $\angle CAB$)



Communication *Tip*

An *n*-sided polygon is often called an *n*-gon. So, a 20-sided polygon is called a 20-gon.

? How can you determine the sum of the interior angles of a 20-gon?

A. Draw a quadrilateral.

B. Estimate the sum of the interior angles and confirm it by measuring.

C. Draw as many non-overlapping diagonals as you can inside the figure.

D. Calculate the sum of the angles of all the triangles.

Compare to your answer from part A.

E. Repeat parts A to D for each polygon in the table on the next page.

Polygon	Number of Sides	Number of Triangles	Sum of Interior Angles	Sketch of Polygon
triangle	3	1	180° (180° × 1)	
quadrilateral	4	2	360° (180° × 2)	
pentagon	5	3	540° (180° × 3)	
hexagon	6			
heptagon	7			
octagon	8			
n-gon	*n*			

F. Complete the table on the right.

Graph the ordered pairs in it.

x: Sides	*y*: Sum of Interior Angles
3	180°
4	
5	

G. What relationship do you see in your graph?

Explain whether you would join the points.

H. Write the equation of the line.

What is the slope?

What is the *y*-intercept for the linear relationship?

I. Determine the sum of the interior angles of any 20-gon using your equation from part H.

Reflecting

J. Why must the triangles in parts C and D not overlap?

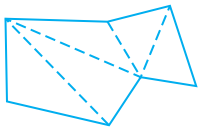
K. What is the formula for the sum of the interior angles of any polygon? Write the formula two ways.

Tech *Support*

For help on constructing a line segment, a triangle, or a polygon; measuring interior angles; or performing a calculation in *The Geometer's Sketchpad*, see Appendix B-16, B-20, B-21, and B-23.

In Summary

Key Ideas



- You can draw non-intersecting diagonals to divide the interior of an n-gon into $n - 2$ non-overlapping triangles.
- The sum of the interior angles of an n-gon is $(n - 2) \times 180°$.

Need to Know

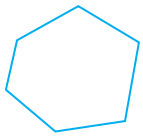
- The sum of the interior angles of a triangle is 180°.
- The sum of the interior angles of a quadrilateral is 360°.

FURTHER Your Understanding

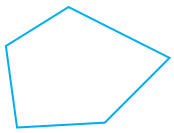
1. Copy the following polygons. Draw as many non-intersecting diagonals as possible to create non-overlapping triangles. What is the sum of the interior angles in each case?

a)

b)



c)



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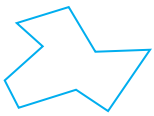
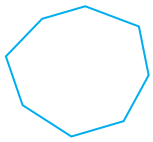


2. Calculate the sum of the interior angles of each polygon.

a)



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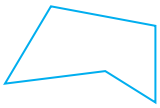

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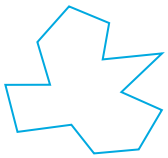
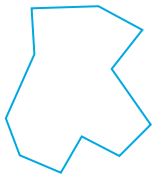


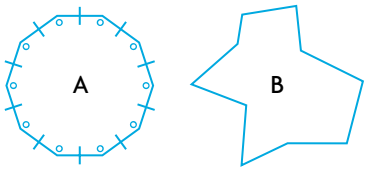
d)



Communication Tip

Regular polygons have equal sides and equal interior angles. Irregular polygons do not.

3. Polygon A is a regular 10-gon and polygon B is an irregular 10-gon. Are the sums of their interior angles equal? Explain.



4. What is the measure of each interior angle of a regular 14-gon?

5. The sum of the interior angles in a polygon is 1440°. How many sides does the polygon have?